The ABC of complementary products mergers*

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Abstract

We present a simple model where mergers benefit consumers, harm outsiders and, depending on the shape of demand, can be profitable for insiders (and where mergers do not involve cost synergies).

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1 Introduction

Outsiders to a merger frequently complain about the negative effects of a proposed merger. However, in standard oligopoly models outsiders typically benefit from a merger. Moreover, in the classic Cournot merger paradox (Salant, Switzer and Reynolds 1983) the merging parties are often worse off after the merger while outsiders are better off unless the merger involves substantial cost synergies (Farrell and Shapiro 1990). Under Bertrand competition with differentiated products, merging parties profit, but outsiders typically profit even more, so that outsiders have no reason to complain about negative effects of a proposed merger (Deneckere and Davidson 1985).

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We present a simple model of airline competition without cost synergies where mergers benefit consumers, harm outsiders and can be profitable for insiders. There are two ways for passengers to get from origin to destination. They can either fly directly with airline 0 or with a one-stop flight on airlines 1 and 2. Assuming that passengers are heterogeneous in their dislike of changing planes these two options are differentiated products, and the distribution of consumers' nuisance costs determines demand for each airline. While airlines 1 and 2 are complements to each other, as a bundle they are a substitute for airline 0. We show that a merger, or an alliance, between airline 1 and 2 always harms airline 0, the outsider, and always benefits consumers. Whether or not the merger benefits the merging parties depends on the nuisance cost distribution. If this distribution is exponential (uniform), a merger is always (never) profitable.

At the center of our model is the result that a merger between complementary firms internalizes the pricing externality (Cournot (1971 (1838)) and Ellet (1966 (1839)). Unlike the classic monopoly case where two complementary firms merge to one, we allow for a competitor producing a differentiated substitute.

As in Economides and Salop (1992) and Choi (2003), complementarities arise from the demand side. Alternatively, complementarities can arise from bundling strategies (McAfee, McMillan and Whinston 1989, Matutes and Regibeau 1992, Gans and King 2006) or through the cost structure. In the airline industry, for example, cost complementarities arise from economies of traffic density (Brueckner and Spiller 1991). Flores-Fillol and Moner-Colonques (2007) study the formation of airline alliances (mergers) in a model that is related to ours. However, since our model does not presume any cost synergies, the profitability of the merger depends entirely on the shape of the demand function. Therefore, our paper identifies a new channel that makes pro-competitive mergers that harm outsiders profitable for insiders.

2 Model

There are three cities, A, B and C. Passengers are only interested in travelling from A to C. There are three airlines: Airline 0 flies directly from A to C, airline 1 operates the leg from A to B, while airline 2 serves the leg from B to C. Costs are zero for all airlines. Passengers stopping at B to change planes incur a cost c. These costs are distributed according to the atomless distribution function G on $[\underline{c}, \overline{c}]$ with $\underline{c} \leq 0 < \overline{c}$.[1] We make the standard monotone hazard rate assumptions, i.e.

Assumption 1. *The distribution function G and its differentiable density g are such that $\frac{1-G(c)}{g(c)}$ and $\frac{g(c)}{G(c)}$ are both non-increasing in c.*

These conditions are equivalent to requiring that g is logconcave, a relatively mild assumption, and implies that profit functions are quasiconcave. In particular, the uniform and the exponential

[1] We can alternatively think of c as a net product match with Airline 0: hence a negative c can be thought of as a net positive match value for the 1-2 route.

distribution satisfy Assumption 1. In order to assure that more passengers dislike changing planes, we also make

Assumption 2. *For the majority of consumers, the nuisance cost is positive, i.e.* $G(0) < \frac{1}{2}$.

Airlines 0, 1, and 2 set prices p_0, p_1, and p_2, respectively. A consumer chooses airline 0 iff $u - p_0 > u - p_1 - p_2 - c$. The benefit u from flying is assumed to be high enough that all consumers fly. Define $\hat{c} \equiv p_0 - p_1 - p_2$, so that passengers with nuisance cost $c > \hat{c}$ fly with airline 0. Demand for airline 0 is thus $1 - G(\hat{c})$ and demand for airlines 1 and 2 is $G(\hat{c})$. The first order conditions for profit maximization are $p_0 = \frac{1-G(\hat{c})}{g(\hat{c})}$ for airline 0 and $p_1 = p_2 = \frac{G(\hat{c})}{g(\hat{c})}$ for airlines 1 and 2, respectively. Plugging these three equations into the definition of $\hat{c}$ gives the equilibrium condition

$$\hat{c}^* = \frac{1 - 3G(\hat{c}^*)}{g(\hat{c}^*)} \tag{1}$$

which defines equilibrium prices via

$$p_0^* = \frac{1 - G(\hat{c}^*)}{g(\hat{c}^*)} \text{ and } p_1^* = p_2^* = p^* = \frac{G(\hat{c}^*)}{g(\hat{c}^*)}. \tag{2}$$

The next Lemma proves that the profit functions are quasiconcave, so that the first order conditions that determine (1) and (2) indeed characterize profit maxima, and that the equilibrium is unique. Figure 1 depicts $\hat{c}^*$ and the equilibrium prices p_0^* and p^*.

FIGURE 1 HERE.

Lemma 1. *There exists an unique interior equilibrium given by the solution to equations* (1) *and* (2). *Equilibrium profit for airline* 0 *is* $\Pi_0^* = \frac{[1-G(\hat{c}^*)]^2}{g(\hat{c}^*)}$ *and equilibrium profits for airlines* 1 *and* 2 *are* $\Pi^* = \frac{G(\hat{c}^*)^2}{g(\hat{c}^*)}$ *each.*

Proof. We first show that profit functions are quasiconcave and then prove that an unique fixed point $\hat{c}^*$ satisfying (1) exists and is interior. Consider airline 0 first. Evaluating the second derivative of the profit function, $-2g - p_0 g'$, where the first-order condition holds yields the expression $-2g - (1 - G)\frac{g'}{g}$. This is negative if $\frac{d[(1-G)/g]}{dc} < 1$, which is implied by Assumption 1. Equivalently, evaluating the second derivative of i's profit function, $-2g + p_i g'$ where the first order condition holds yields $-2g + G\frac{g'}{g}$ for $i = 1, 2$. This is again guaranteed negative by Assumption 1. Since any turning point is a local maximum, and hence a global one, the profit functions are quasiconcave.

To see that the fixed point $\hat{c}^*$ is unique and interior, notice first that Assumption 1 implies

$$\frac{d(1 - nG)/g}{dc} < 1 \text{ for all } n \geq 1, \tag{3}$$

which is easily seen be to true by rewriting $\frac{1-nG}{g} = \frac{1-G}{g} - (n-1)\frac{G}{g}$ and then differentiating. Therefore the slope of the right-hand side in (1) is less than the slope of the left-hand side. Thus, the two functions intersect at most once on $(\underline{c}, \overline{c})$. To see that they do, note that $\frac{1-3G(\underline{c})}{g(\underline{c})} = \frac{1}{g(\underline{c})} > 0$ and $\frac{1-3G(\overline{c})}{g(\overline{c})} < 0$ and hence $\frac{1-3G(\underline{c})}{g(\underline{c})} > \underline{c}$ while $\frac{1-3G(\overline{c})}{g(\overline{c})} < \overline{c}$. Consequently, there exists an unique $\hat{c}^* \in (\underline{c}, \overline{c})$. □

3 Airlines 1 and 2 Merge

When airlines 1 and 2 merge they set a price $p^m_{1,2}$ for the full trip from A to C via B. It is assumed that the merged airline maintains its stop at B. Otherwise, it becomes a perfect substitute for airline 0, resulting in Bertrand competition with all prices equal to zero. The merged airline gets all passengers for whom $p_{1,2} + c < p_0$. The indifferent passenger between airline 0 and the merged airline is now given by $\hat{c} = p_0 - p_{1,2}$. Consequently, all passengers with $c > \hat{c}$ choose airline 0, whose demand is therefore $1 - G(\hat{c})$. All others choose the merged airline, whose demand is $G(\hat{c})$. At the equilibrium prices $p^m_{1,2}$ and p^m_0, the first order conditions are:

$$p^m_0 = \frac{1 - G(\hat{c}^m)}{g(\hat{c}^m)} \quad \text{and} \quad p^m_{1,2} = \frac{G(\hat{c}^m)}{g(\hat{c}^m)} \tag{4}$$

and so the critical nuisance cost $\hat{c}^m$ satisfies

$$\hat{c}^m = \frac{1 - 2G(\hat{c}^m)}{g(\hat{c}^m)}. \tag{5}$$

As the structure of the problem is identical to the pre-merger problem, an analog to Lemma 1 applies. That is, equations (4) and (5) constitute an unique equilibrium.
Note that $\frac{1-nG(c)}{g(c)} \leq \frac{1-G(c)}{g(c)}$ for all $n > 0$ (and c). Hence, because $\hat{c}^*$ is implicitly defined by $\hat{c}^* = \frac{1-3G(\hat{c}^*)}{g(\hat{c}^*)}$, it is smaller than $\hat{c}^m$ which satisfies $\hat{c}^m = \frac{1-2G(\hat{c}^m)}{g(\hat{c}^m)}$:

$$\hat{c}^m > \hat{c}^*. \tag{6}$$

Consequently, equilibrium demand for the route through B increases if airlines 1 and 2 merge. Lemma 2 reports the effect of the merger on prices and Figure 1 illustrates the post- and pre-merger equilibrium.

Lemma 2. *Pre- and post-merger prices:*

(i) The price charged by the merged airlines 1 and 2 is weakly smaller than airline 0's price: $p^m_0 > p^m_{1,2}$*;*

(ii) The price of airline 0 is smaller after the merger by airlines 1 and 2 than before: $p^m_0 \leq p_0$*;*

(iii) *The pre-merger price for flying from A to C via B ($2p^*$) is higher than the post-merger price for the same flight ($p^m_{1,2}$) which in turn is weakly greater than the pre-merger price for each single flight leg: (p^*): $2p^* > p^m_{1,2} \geq p^*$.*

Proof. (i) Prices for airlines 1 and 2 are given by (4) and can be rewritten as

$$p^m_{1,2} = \frac{G(\hat{c}^m)}{g(\hat{c}^m)} = \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)} - \frac{1-2G(\hat{c}^m)}{g(\hat{c}^m)} = \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)} - \hat{c}^m$$

where the last equality follows from the optimality condition (5). Since $G(0) < \frac{1}{2}$ by Assumption 2, (5) implies $\hat{c}^m > 0$. From (4), airline 0's price is $p^m_0 = \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)}$. Since $\hat{c}^m > 0$, $p^m_0 > p^m_{1,2}$ immediately follows.

(ii) By Assumption 1, $\frac{1-G}{g}$ is non-increasing and hence $c^m > c^*$ implies $p^m_0 = \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)} \leq \frac{1-G(\hat{c}^*)}{g(\hat{c}^*)} = p_0$.

(iii) By Assumption 1, $\frac{G}{g}$ is non-decreasing. Hence $\hat{c}^m > \hat{c}^*$ implies $p^m_{1,2} \equiv \frac{G(\hat{c}^m)}{g(\hat{c}^m)} \geq \frac{G(\hat{c}^*)}{g(\hat{c}^*)} \equiv p^*$. Since $p^* = \frac{G(\hat{c}^*)}{g(\hat{c}^*)}$ and $\hat{c}^* = \frac{1-G(\hat{c}^*)}{g(\hat{c}^*)} - 2\frac{G(\hat{c}^*)}{g(\hat{c}^*)}$, the pre merger price for the flight over B is $2p^* = \frac{1-G(\hat{c}^*)}{g(\hat{c})^*} - \hat{c}^*$. Since $(1-G)/g$ is non-increasing and $\hat{c}^m > \hat{c}^*$, $\frac{1-G(\hat{c}^*)}{g(\hat{c}^*)} \geq \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)}$ holds. Consequently, $2p^* = \frac{1-G(\hat{c}^*)}{g(\hat{c}^*)} - \hat{c}^* > \frac{1-G(\hat{c}^m)}{g(\hat{c}^m)} - \hat{c}^m = p^m_{1,2}$.

□

With these results on prices, we immediately have

Proposition 1. *Both price and demand are lower for airline* 0 *after the merger. Consequently, airline* 0*'s profit after the merger between airlines* 1 *and* 2 *is lower than before.*

Proof. From (6) $\hat{c}^m > \hat{c}^*$, which implies $1 - G(\hat{c}^m) < 1 - G(\hat{c}^*)$, and $p^m_0 < p_0$ by Lemma 2(ii). □

The reason the outsider is always worse off is that by merging, airlines 1 and 2 internalize the negative pricing externalities that arises from complements, as first pointed out by Cournot and Ellet. This makes the merged firms more competitive vis-à-vis their competitor. Proposition 2 below shows that this pro-competitive effect is beneficial to consumers who are always better off after the merger because of the lower prices.

Proposition 2. *Consumers are better off after the merger between airlines* 1 *and* 2*.*

Proof. By Lemma 2, parts (ii) and (iii), all prices are lower after the merger. □

Although the outsider is hurt, the effect on the merged firms' joint profit is not clear. Pre-merger joint profit for airlines 1 and 2 is $2p^*G(\hat{c}^*)$ while post-merger profit is $p^m_{1,2}G(\hat{c}^m)$. Demand for the merged firms increases because $\hat{c}^m > \hat{c}^*$. However, by Lemma 2(ii), the joint price, $p^m_{1,2}$, is smaller than the pre-merger price, $2p^*$. Hence, the effect on the merged firms' profit is ambiguous. In the Cournot-Ellet case (without strategic effects from third parties) it is always profitable to merge. Here, however, competition from a third, substitute firm counteracts the beneficial internalization of the pricing externality and may result in net losses. Proposition 3 shows that there exist distributions of nuisance costs where the merging firms always profit and others where they never profit.

Proposition 3. *For uniformly distributed nuisance costs, all firms' profits decline by the merger. For an exponential distribution of nuisance costs $G(c) = 1 - e^{-\lambda c}$ with $\lambda > 0$ and $c \in [0, \infty)$, however, the merged firms' profit increases. Moreover, $\Pi^m_{1,2}/2 = 1.0986\Pi^*$ for all $\lambda > 0$.*

Proof. Without loss of generality let the nuisance costs be uniformly distributed on $[0, 1]$. Then, we have $\Pi^* = 1/16$ pre merger and $\Pi^m_{1,2} = 1/9$ post merger. So $2\Pi^* > \Pi^m_{1,2}$. For the exponential distribution, $\hat{c}^*$ and $\hat{c}^m$ are given as $2e^{\lambda\hat{c}^*} + \lambda\hat{c}^* = 3$ and $e^{\lambda\hat{c}^m} + \lambda\hat{c}^m = 2$, respectively. Both equations are of the form $e^x + ax = b$ with $a, b > 0$ and $x = \lambda c$. This equation has a unique solution $x^*(a, b)$ and hence $\lambda c(\lambda, a, b) = x^*(a, b)$ is independent of λ. Consequently, $\frac{\Pi^m_{1,2}}{2\Pi^*} = \frac{1}{2}\frac{e^{-\lambda\hat{c}^*}}{e^{-\lambda\hat{c}^m}}\left(\frac{1-e^{-\lambda\hat{c}^m}}{1-e^{-\lambda\hat{c}^*}}\right)^2$ solely depends on $\lambda c(\lambda, a, b)$ and does not change with λ. Setting $\lambda = 1$, we get $\Pi^* = 0.0907$ pre merger and $\Pi^m_{1,2} = 0.1993$ post merger, so that $\Pi^m_{1,2}/(2\Pi^*) = 1.0986$. $\square$

4 Conclusion

By merging, complementary firms reduce their pricing externality and become more aggressive competitors vis-à-vis substitute firms. This reduces prices and thus harms outsiders and benefits consumers. The shape of the demand curve determines whether a merger is profitable to insiders. Examining the role of the degree of demand concavity on insider merger profitability, and allowing for demand on the local markets (i.e. A-B and B-C) is left for future research.

References

Brueckner, J. K. and P. T. Spiller, 1991, Competition and Mergers in Airline Networks, International Journal of Industrial Organization 8, 323–342.

Choi, C. P., 2003, Antitrust Analysis of Mergers with Bundling in Complementary Markets: Implications for Pricing, Innovation, and Compatibility Choice, NET Institute Working Paper.

Cournot, A., 1971 (1838), Researches into the Mathematical Principles of the Theory of Wealth (Augustus M. Kelly, New York).

Deneckere, R. and C. Davidson, 1985, Incentives to Form Coalitions with Bertrand Competition, RAND Journal of Economics 16(4), 473–486.

Economides, N. and S. C. Salop, 1992, Competition and Integration among Complements, and Network Market Structure, Journal of Industrial Economics 1(40), 105–123.

Ellet, C., 1966 (1839), An Essay on the Laws of Trade in Reference to the Works of Internal Improvement in the United States (Augustus M. Kelly, New York).

Farrell, J. and C. Shapiro, 1990, Horizontal Mergers: An Equilibrium Analysis, American Economic Review 80(1), 107–126.

Flores-Fillol, R. and R. Moner-Colonques, 2007, Strategic Formation of Airline Alliances, Journal of Transport Economics and Policy 23, 427–449.

Gans, J. S. and S. P. King, 2006, Paying for Loyalty: Product Bundling in Oligopoly, Journal of Industrial Economics 54(1), 43–62.

Matutes, C. and P. Regibeau, 1992, Compatibility and Bundling of Complementary Goods in a Duopoly, Journal of Industrial Economics 40(1), 37–54.

McAfee, R. P., J. McMillan and M. D. Whinston, 1989, Multiproduct Monopoly, Commodity Bundling, and Correlation of Values, Quarterly Journal of Economics 104(2), 371–383.

Salant, S. W., S. Switzer and R. J. Reynolds, 1983, Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium, Quarterly Journal of Economics 98(2), 185–199.

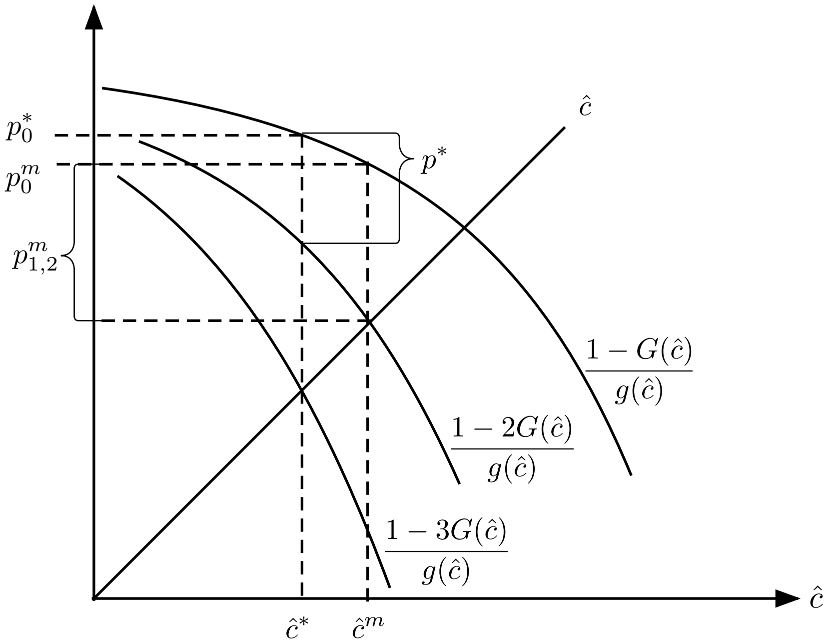

Figure 1: Equilibrium pre and post merger.