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Abstract

We provide a canonical and tractable model of a trade platform enabling buyers and sellers to transact. The platform charges a percentage fee on third-party product sales and decides whether to be "hybrid", like Amazon, by selling its own product. It thereby controls the number of differentiated products (variety) it hosts and their prices. Using the mixed market demand system, we capture interactions between monopolistically competitive sellers and a sizeable platform product. Using long-run aggregative games with free entry, we endogenize seller participation through an aggregate variable manipulated by the platform's fee. We show that a higher quality (or lower cost) of the platform's product increases its market share and the seller fee, and lowers consumer surplus. Banning hybrid mode benefits consumers. The hybrid platform might favor its product and debase third-party products if the own product advantage is sufficiently high. We also provide some tax policy implications.

JEL Classification: D42, L12, L13, L40, H25

Keywords: Trade platform, hybrid business model, Antitrust Policy, Tax policy

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Hybrid Platform Model*

Simon Anderson† Özlem Bedre-Defolie ‡

June 9, 2021

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1 Introduction

Business to consumers (B2C) e-commerce raised \$431.6 billion in the US in 2020, which is 8.8 percent of US retail revenue (Statista, 2021b,c). Amazon is the dominant e-commerce platform in the US with a market share of 38.7 percent and also in most European countries, with a market share of 30 percent in the UK (Statista, 2020) and 35 percent in Germany (Skeldon, 2019). Amazon's e-commerce activities raised \$386.06 billion worldwide in 2020 (Statista, 2021a). Amazon is a "hybrid platform," a marketplace enabling interactions between buyers and sellers while simultaneously being a retailer (reseller) of its own products (either private label products, like AmazonBasics, or branded products).[1] Several other platforms have adopted a hybrid business model. For example, app stores Apple and Google Play sell their own applications, like Apple Music, Apple TV, or Google TV, Games, Podcasts, and Analytics along with third-party applications.

The hybrid business model of dominant ("gatekeeper") platforms has raised significant antitrust concerns in the last couple of years.[2] The European Commission (EC) is currently investigating whether Amazon's practices violate the antitrust treaty that prohibits the abuse of a dominant market position. Two key concerns are whether Amazon limits access of third-party products to its consumer base and whether Amazon favors its own products to the detriment of third-party products by, for example, making its own products more prominent and thus steering consumers from third-party products to its own.[3] The EC is also investigating Apple's App Store following

[1]In 2020, 55 percent of paid units were sold by third-party sellers on Amazon, see Amazon Q4 2020 Press Release, page 18.

[2]The US House Majority Report (2020) notes that "As Amazon, Apple, Facebook, and Google have captured control over key channels of distribution, they have come to function as gatekeepers. A large swath of businesses across the U.S. economy now depend on these gatekeepers to access users and markets." The report accuses each of these platforms of "using its gatekeeper position to maintain its market power." The European Commission (2020) notes that "A few large platforms increasingly act as gateways or gatekeepers between business users and end users and enjoy an entrenched and durable position, often as a result of the creation of conglomerate ecosystems around their core platform services, which reinforces existing entry barriers." It specifically defines gatekeepers as "core platform services that exceed a number of size thresholds, e.g., more than 45 million active monthly end users or more than 10,000 active yearly business users." This definition effectively captures GAFAM (Google, Apple, Facebook, Amazon and Microsoft).

[3]See The EC's Press Release on Amazon investigation, Nov 10, 2020. Another major concern is whether Amazon uses third-party sales data to benefit its reseller channel. We will not investigate this question in this paper, but our framework could be used to analyze this and other concerns related to hybrid business model of a dominant trade platform.

a complaint by Spotify against Apple's rules.[4] In the US, there is an ongoing antitrust investigation of Amazon, Google Android's Google Play, and Apple iOS App Store rules.[5]

Another policy debate addresses how governments should tax the revenues of dominant digital platforms. In August 2019, the French government introduced a 3 percent tax on Amazon marketplace revenues from purchases placed via its French website. Amazon quickly responded to the French tax by raising the fee that it collects from third-party product sales on Amazon.fr.[6]

Despite the prevalence of platforms hosting third parties selling consumer goods, there is surprisingly little attempt in the literature to provide a descriptive model of buyers and sellers that tracks the main details. Once we allow for the platform itself to join in as a reseller as well (the hybrid model), there is no such extant model. Our first contribution is to deliver such a model. A model of a platform, such as Amazon, needs to embody several key features of the market. First, in each market segment, consumers make a discrete choice from a range of differentiated products. Second, many small sellers decide whether to enter the platform and those that join the platform make positive sales in equilibrium. Third, the platform has a dominant position both in setting seller fee and, in hybrid mode, pricing its own product with this product attracting a significant fraction of total sales. Finally, to analyze the hybrid business mode, the model needs to allow incentives for both own-products and third-party products to survive and be remunerative for the platform. This entails having some product differentiation.

These considerations drive our choice of framework and lead us to adopt a discrete choice model of differentiated products with a monopolistically competitive "fringe" of small sellers. We microfound our model on two key recent conceptual innovations. First, we engage a mixed oligopoly demand model (Neary, 2010; Shimomura and Thisse, 2012; Parenti, 2018; Helpman and Niswonger, 2020) to capture interactions between a large player (the platform product) and monopolistically competitive third-party sellers. The difference of our mixed oligopoly framework from the previous literature is that the large firm can also collect a percentage fee from sales of small

[4] The EC's Press Release on June 16, 2020.

[5] See "12 Accusations in the Damning House Report on Amazon, Apple, Facebook and Google," The New York Times, October 6, 2020, The House Antitrust Report on Big Tech, The New York Times, 2020, and Reuters, February 4, 2020, Business Insider, June 24, 2020.

[6] See Forbes, August 19, 2019 and Tax Foundation, August 6, 2019.

firms. Second, we obtain a tractable setup by engaging recent insights on long-run aggregative games to model entry of third-party sellers. The hybrid platform sets both its own price and the seller fee so as to control entry of fringe sellers. It thereby indirectly controls the number of differentiated products (variety) it hosts and their prices, that is, it modulates both competition for its own product and the revenues it gets from sales of third-party products, which are its rivals in the product market. Its role is that of a long-run Stackelberg leader with two instruments controlling entry (seller fees and its product market share), which leads to novel takeaways. The market structure that we describe effectively melds the dominant firm-fringe paradigm (Forchheimer, 1908) to price leadership of the dominant firm in a long-run context. This resembles that of Etro (2006) and Anderson et al. (2020).[7] The additional ingredient here is that the leader (the platform) collects fees from sales of the followers.

There is fluid seller entry; sellers can come to the platform anytime when they anticipate large enough profits to cover their fixed costs of operating on the platform and leave the platform if they cannot be profitable. The platform is a gatekeeper; sellers do not have an alternative access to consumers. This model captures the competition on a dominant ("gatekeeper") hybrid trade platform, like Amazon, particularly well.[8] We deliver a fully-fledged two-sided market structure when we allow for endogenous consumer participation via the device of heterogeneous participation costs, where the platform's tools (seller fee and own-product price) control not only seller participation and transactions on the platform, but also the extent of buyer participation.

Using our framework, we study how the dual role of a monopoly hybrid platform affects prices, variety, and consumer welfare, and deliver policy implications for antitrust policy, regulation, and taxation of dominant hybrid platforms. We show that the hybrid mode leads to a higher platform fee for third-party sellers, less variety on the platform, and lower consumer welfare, compared to when the platform is a pure marketplace. Thus, banning the hybrid mode benefits consumers if the platform becomes a pure marketplace after the ban. On the other hand, the ban harms

[7]Etro (2006) implicitly invoked the structure of long-run aggregative games to study price leadership, as elaborated upon by Anderson et al. (2020); the dominant firm chooses its price while internalizing the entry response of (monopolistically competitive) fringe firms.

[8]The US House Majority Report (2020) notes that "Amazon has 2.3 million active third-party sellers on its marketplace worldwide, and a recent survey estimates that about 37% of them – about 850, 000 sellers – rely on Amazon as their sole source of income."

consumers if the platform becomes a pure reseller after the ban.[9] With regards to steering concerns we find that the hybrid platform wants to favor its own product to the detriment of third-party products if the platform product has a sufficiently high quality or a sufficiently low cost. For tax policy, we show that a percentage tax on third-party sales of a pure marketplace is neutral for seller fees (unless the platform has marginal costs of processing third-party sales). However, a percentage tax on third-party sales on a hybrid platform leads to higher seller fees and so lower consumer surplus. Conversely, taxing only the hybrid platform's own-product sales leads to lower seller fees and so higher consumer surplus.

To prove the result that the hybrid mode harms consumers we show that an improvement in the quality of the hybrid platform's product or the reduction of its cost raises the platform's equilibrium fee for third-party products, thus lowering consumer welfare. This finding is in surprising contrast to a common finding in industrial economics: a firm selling a higher-quality or a lower-cost product is good for consumers. This does not hold true for a hybrid platform, which makes revenues from sales of its own product and from fees collected from third-party product sales.

When the platform has a better product (arising from a higher quality or a lower cost), the platform generates more sales from its own product relative to the sales of third-party products. This induces the platform to raise the fee for third-party products, which in turn leads to fewer third-party products joining the platform (both due to the better platform product and due to the higher platform fee) and ultimately leads to less variety for consumers. In this way, the reduced variety due to fewer third-party products neutralizes the benefit the consumers get from the platform product's improvement. The higher fee to third-party sellers leads to higher prices, lowering consumer surplus. The analysis also predicts a higher market share for the platform's own product when its product improves, and thus a positive relation between the platform product's market share and its fee on third-party sales.

When we endogenize the platform's choice of which product to sell, the platform always chooses to sell the product with the greatest quality minus cost. Thus, considering endogenous product choice of the platform makes the hybrid mode worse for consumers.

The platform prefers the hybrid business model if its own product has zero fixed

[9] This happens when the platform product's advantage is at the high end of the hybrid mode region.

costs: the hybrid mode enables the platform to improve the variety of products on the platform (due to product differentiation between products) and so to make more revenue (via collecting fees on third-party sales). If the platform's product has a positive fixed cost, the platform prefers to be a pure marketplace when the own product is not good enough (due to low quality or high cost). It prefers to be hybrid for products with intermediate levels of quality or cost. When the own product is sufficient quality, the platform turns itself into a pure reseller by a prohibitive seller fee.

We next summarize our relation to the literature. Section 2 describes our general model. In Section 3 we assume that all consumers visit the platform and provide our main results in the simplest version of our model. In Section 3.5 we derive policy implications from this analysis. In Section 4 we confirm our main results in a framework with two-sided network effects by allowing for ex ante consumer heterogeneity in visiting costs and generating elastic consumer participation. We discuss our methodological contribution in Section 5. All formal proofs are in the Appendix.

1.1 Related Literature

Our paper contributes to the literature on the economics of multi-sided platforms by providing a canonical and tractable model of a buyer-seller trade platform. Most of this literature has focused on pure membership or pure transaction models while while mainly analyzing pricing of two symmetric sides of the market (see Caillaud and Jullien (2003), Rochet and Tirole (2003), Rochet and Tirole (2006), Armstrong (2006) for seminal contributions).

One distinctive feature of our model is to allow the platform to affect not only consumer prices but also variety provision (the number of differentiated products) on the platform via its choices of seller fee and whether to sell its own product (hybrid mode). We therefore contribute to the literature on variety provision on platforms (Nocke et al., 2007; Hagiu, 2009; Galeotti and Moraga-González, 2009). This literature, which is synthesized in Belleflamme and Peitz (2019), assumes that the platform charges only membership (entry) fees to buyers and sellers. We instead allow the platform to charge percentage commissions on seller revenue and this makes it necessary to specify seller price competition which usually complicates the analysis due to the pass-through of platform commission on prices.[10] Our monopolistically competitive fringe gives a tractable pass-through function of percentage commissions for differentiated seller competition and the free entry condition ties down the endogenous number of products. In our model, the consumer value from one more seller (cross-group externality per seller) is endogenous since the platform's seller fee affects the seller price and so the unit consumption value on the platform. Going beyond the previous literature, we analyze variety provision incentives of a hybrid platform and also deliver a fully-fledged two-sided market structure when we allow for endogenous consumer participation via the device of heterogeneous participation costs.

By considering differentiated competition between sellers we contribute to the literature analyzing implications of within-group negative externalities in markets with multi-sided platforms, like Belleflamme and Peitz (2018), Belleflamme and Peitz (2019), Halaburda et al. (2018), and Karle et al. (2020). This literature focuses on the implications of within-group negative externalities on competition between platforms, whereas we focus on understanding how third-party seller competition affects the optimal choices of a trade platform and its business mode choice.[11]

We also contribute to recently growing literature on hybrid platform business models (Jiang et al., 2011; Hagiu et al., 2020; Hervas-Drane and Shelegia, 2021; Etro, 2020; Zennyo, 2020). Different from this literature, we consider fluid seller entry (an endogenous number of fringe products joining the platform), differentiated products, and heterogenous consumers in their match values for products. In our model, the platform's product and third-party products all have some positive demand in equilibrium due to different tastes of consumers. Besides, we focus on understanding how the existence of the platform's product (hybrid mode) affects the variety on the platform (number of differentiated third-party sellers joining) and the seller fee the platform charges in equilibrium. We further discuss our contribution and key differences relative to this literature in Section 3.4.

Our paper is also related to the literature analyzing incentives to sell private label products in retailing and their impact on retailers' incentives to stock third-

[10]Different from us, Galeotti and Moraga-González (2009) assume a fixed number of sellers, uniform taste distribution for products (as opposed to our Gumbel), and show that, in equilibrium, all buyers and sellers join the platform which extracts all the market surplus.

[11]Another important difference from this literature is that we allow for endogenous cross-group transaction externalities between buyers and sellers by allowing the platform to charge seller fees on transactions.

party products (Mills, 1995; Berges-Sennou et al., 2004; Meza and Sudhir, 2010; Tiboldo et al., 2021). Hybrid platforms are different from retail stores because they do not own third-party products, so do not pay for their purchasing costs and do not directly control their prices.[12] Hybrid platforms use the agency model instead of the wholesale model - they tax transactions of third-party sellers and let sellers determine their product prices.[13] Hybrid platforms might look similar to store-within-a-store retailing in the offline world (Jerath and Zhang, 2010, 2019). We contribute to this literature by studying the determinants of why a pure marketplace wants to sell its own product (switching to hybrid mode) and how this decision affects the number of differentiated third-party product sellers available on the platform, fees charged to sellers, and final prices.

2 Model

Consider a platform enabling interactions between buyers and sellers. There is a continuum of differentiated sellers ("fringe sellers"), each of which sells a single product and decides whether to enter the platform. Each fringe seller needs to incur a fixed cost, K, to be present on the platform. This fixed cost might include costs of entering into a contract with the platform or setting up necessary logistics to be able to sell on the platform. The platform collects a percentage fee, t, on revenue generated by each seller on the platform. In the pure marketplace mode, the platform sells only third-party fringe products. When we consider a hybrid mode, the platform also sells its own version of the product along with the products of the fringe sellers. The platform's product has a fixed cost of K_A.

Products have different qualities. The platform's product has "quality" level v_A and a fringe seller has quality level v. We assume symmetric fringe firms:

Assumption 1 *All fringe products have the same quality, v, marginal cost, c, and fixed cost, K.*

[12]See Hagiu and Wright (2015b) for detailed discussion of key differences between a retailer and a marketplace. Hagiu and Wright (2015a) study trade-offs involved when choosing one mode over the other.

[13]See Johnson (2017) for key differences in the economics of these two business models and their implications for final prices.

Each consumer gets utility u_i from purchasing one unit from fringe product i:

$$u_i = v - p_i + \mu\epsilon_i, \tag{1}$$

where v denotes the consumption value from fringe product i, p_i denotes its price, and ϵ_i is the idiosyncratic match value. Product differentiation is measured by parameter μ, which is assumed to be positive. In a hybrid mode, a consumer also has an option to buy the platform's product and get utility u_A:

$$u_A = v_A - p_A + \mu\epsilon_A, \tag{2}$$

where v_A denotes the consumption value from the platform's product, p_A denotes its price, and ϵ_A is the idiosyncratic match value. We allow for an (exogenous) outside option for buyers by assuming that buyers get u_0 if they do not buy any products available on the platform: $u_0 = \mu\epsilon_0$, where ϵ_0 is the match value for the outside good. We assume that match values, ϵ_i, ϵ_A, and ϵ_0, are independently and identically distributed with Gumbel (Type I Extreme Value) distribution across products.

There is a unit mass of consumers who decide whether to join the platform. Consumers face a cost of visiting the platform, s, and they are heterogenous in s. Costs s are assumed to follow a cumulative distribution function, $F(\cdot)$, over interval $[0, \overline{s}]$. Consumers do not know their match value for each product before visiting the platform and they learn their match only once they are on the platform (incur s).

To model the competition between a big player (the platform) and a continuum of small firms (fringe firms), we adopt the "mixed oligopoly" framework used in the literature (Neary, 2010; Shimomura and Thisse, 2012; Parenti, 2018; Helpman and Niswonger, 2020). Following Neary (2010) and Parenti (2018), we model the big firm (the platform) supplying a continuum of varieties (of measure M), whereas each small firm supplies a single variety. The varieties of the big firm have the same deterministic quality v_A, but they differ among each other so that each has an i.i.d match value draw from the Gumbel distribution. An alternative motivation, following Shimomura and Thisse (2012), is to assume that each small firm's demand is "minuscule" and the big firm's demand has a mass $M > 1$. The difference of our mixed oligopoly framework from the previous literature is that the large firm can also collect a percentage fee from the sales of small firms.

The timing of the interactions is the following:

1. The platform sets the percentage fee, t. In the hybrid mode the platform also sets the price of its product, p_A.
2. Fringe sellers choose whether to enter. If they do enter, they incur the fixed cost, K, and choose their price, p_i.
3. Consumers observe everything apart from their match values with each product and decide whether to pay their intrinsic costs of joining the platform.
4. Consumers who join the platform learn all match values ϵ_i, ϵ_A, and ϵ_0, and buy the product which gives them the highest utility on board. They buy nothing on the platform if the outside option is better.

We look for a Subgame Perfect Nash Equilibrium of this game. We start by solving the simplest version of the model when consumers' visiting costs s are zero (or negligible), so that all consumers visit the platform (Section 3). This version does not embody two-sided network effects between fringe products and consumers because the number of participants on the consumer side is fixed and so does not depend on the number of fringe sellers. Within this version we first look at a pure marketplace where the platform does not sell its own product and sells only third-party products (products of fringe sellers). We next consider a hybrid platform where the platform sells both its product and third party products. We then look at the endogenous choice of the platform if it is allowed to choose between different regimes (pure marketplace, hybrid, pure reseller). The objective is to see how allowing the platform sell its product (hybrid mode) affects consumer welfare. We look at the full model with two-sided network effects by introducing positive and binding visiting costs in Section 4.

3 All consumers participate

We start by characterizing the equilibrium outcome in the benchmark where all consumers join the platform (non-binding search costs).

3.1 Pure marketplace

If the platform is a pure marketplace, it does not sell its own product and sells only third-party (fringe) products. By the choice of its fee t the platform can control the number of fringe sellers and affect the price chosen by fringe sellers.

The platform is *viable* if it is able to attract some fringe firms at the lowest fee, $t = 0$. As we shall see below, this is the case when the fixed cost of a fringe firm is sufficiently low:

Assumption 2 $K < \mu \exp\left(\frac{v-c-\mu}{\mu}\right)$.

3.1.1 Fringe sellers' choices

Given the fee set by the platform, t, consider first the monopolistic competitors (fringe sellers). Seller i's demand when all other sellers set price p is

$$q_i(p_i, p) = \frac{\exp\left(\frac{v-p_i}{\mu}\right)}{n \exp\left(\frac{v-p}{\mu}\right) + 1}, \tag{3}$$

where n denotes the number of fringe firms on the platform. Each fringe seller's demand depends on its own price, p_i, and the market price, p, which is the common one set by each fringe firm in equilibrium. We call the denominator of the fringe seller's demand expression the "Aggregate" (following its use in the theory of aggregative games). The aggregate is independent of the price p_i since each fringe seller is infinitesimal. The aggregate is denoted below as A:

$$A \equiv n \exp\left(\frac{v-p}{\mu}\right) + 1. \tag{4}$$

We first characterize the equilibrium price of fringe sellers. The variable profit of fringe seller i is

$$(p_i (1-t) - c) \frac{\exp\left(\frac{v-p_i}{\mu}\right)}{A}, \tag{5}$$

where the seller gets a fraction $(1-t)$ of the revenue from its sales on the platform and $\frac{\exp\left(\frac{v-p_i}{\mu}\right)}{A}$ is demand for seller i's product. The fringe seller sets its price to maximize its profit, which is the same as the maximizer of

$$\max_{p_i} \left(p_i - \frac{c}{1-t}\right) \exp\left(\frac{v-p_i}{\mu}\right),$$

so that each fringe seller sets the same price:

$$p(t) = \frac{c}{1-t} + \mu. \tag{6}$$

The equilibrium price of a fringe firm is equal to the effective marginal cost of selling on the platform, $\frac{c}{1-t}$, plus the logit markup, μ. Thus, a higher platform fee implies a higher fringe product price, as does more differentiation between fringe firms (higher μ).

The equilibrium net profit of each fringe seller is the equilibrium variable profit minus the fixed cost of entry

$$\pi(t) = \mu\,(1-t)\,\frac{V(t)}{A(t)} - K,$$

where $\mu\,(1-t)$ is the fringe profit per unit, $\frac{V(t)}{A(t)}$ is the equilibrium demand per firm, and we have defined

$$V(t) \equiv \exp\left(\frac{v - p(t)}{\mu}\right) = \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right). \tag{7}$$

The zero-profit entry condition together with the fringe profit expression, $\pi(t) = 0$, ties down the equilibrium value of the aggregate:

$$A(t) \quad = \quad \frac{\mu\,(1-t)}{K} V(t), \tag{8}$$

as long as this exceeds 1, otherwise t or K is too large to allow any fringe entry. The right hand-side of (8) is decreasing in t and K, so the equilibrium aggregate decreases in the fee set by the platform and in the seller fixed cost of entry. The highest number of fringe firms is achieved when $t = 0$, so the condition for the platform to be viable at some $t \geq 0$ is that $A(0) > 1$, which is Assumption 2. The equilibrium size of the fringe, $n(t)$, solves

$$A\,(t) = nV(t) + 1. \tag{9}$$

After replacing $A\,(t)$ from (8), we rewrite this as

$$n(t) \quad = \quad \frac{\mu\,(1-t)}{K} - \frac{1}{V(t)}.$$

There is no fringe firm if the fee is too high, that is, if $t \geq \hat{t}$ such that $A(\hat{t}) = 1$ or

$$\frac{\mu\left(1-\hat{t}\right)}{K} V(\hat{t}) = 1.$$

Note that $\hat{t} < 1$, since if the fee exceeded 100 percent, any active fringe firm would make a loss, so the platform is viable for all $t \in \left(0, \hat{t}\right)$.

3.1.2 The platform's problem

The platform profit is the total amount of revenue generated from fees collected from third-party (fringe) sales:

$$\Pi = tp(t)\frac{A\left(t\right)-1}{A\left(t\right)}, \tag{10}$$

where $A(t)$ is the aggregate and $\frac{A(t)-1}{A(t)}$ is the demand share of the fringe products after deducting the share of the outside good. An optimal fee sets to zero the marginal profit from raising t:

$$\frac{d\Pi}{dt} = \frac{1}{A(t)}\left[\left(tp(t)\right)'\left(A\left(t\right)-1\right) + tp(t)\frac{A'\left(t\right)}{A\left(t\right)}\right] = 0. \tag{11}$$

The first term is extra revenue on the base of fringe output from raising t. The second is lost revenue from fringe contraction (fewer fringe firms joining the platform). We next show the existence of an equilibrium fee:

Lemma 1 *There exists an optimal fee $t^* \in (0, \hat{t})$, which satisfies the optimality condition in (11), which we rewrite as the platform's fundamental pricing formula:*

$$A(t) = 1 - \frac{\epsilon_{A(t)}}{\epsilon_{tp(t)}}, \tag{12}$$

where $\epsilon_{A(t)} = t\frac{A'(t)}{A(t)} < 0$ is the elasticity of the aggregate and $\epsilon_{tp(t)} = t\frac{(tp(t))'}{tp(t)} > 0$ is the elasticity of fee revenue per unit with respect to the fee.[14]

[14] An alternative elasticity representation of the maximizing fee comes from writing $\Pi = tp(t)G(t)$ with $G(t) \equiv \frac{A(t)-1}{A(t)}$ as the demand for fringe. The corresponding elasticity condition is $\epsilon_{tp(t)} = -\epsilon_{G(t)}$. Intuitively, when the platform increases its fee, it gains more per-unit revenue (measured by the elasticity of fee revenue per unit, $\epsilon_{tp(t)}$), but it loses profits due to the reduced number of fringe products entering the platform, which is measured by the elasticity of the fringe sellers' demand, $\epsilon_{G(t)}$. We return to this perspective in Section 4

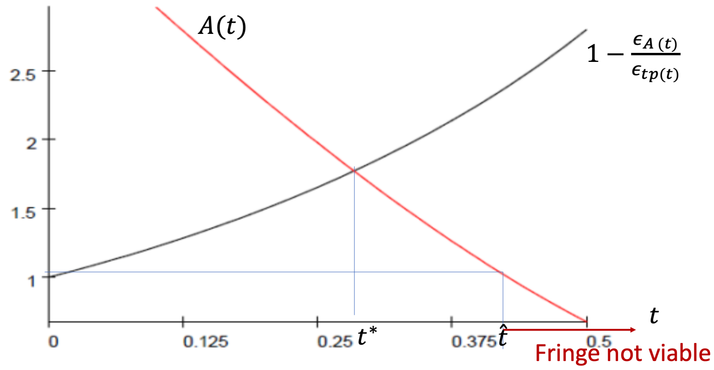

Figure 1: The optimal platform fee satisfying (12) is the point where the red line crosses the black line, at t^*, for parameter values $\mu = v = c = 1$ and $K = 1/10$.

Figure 1 shows an example of the equilibrium solution with all parameters being equal to 1 except that $K = 1/10$. The red line corresponds to $A(t)$ (the left-hand side (LHS) of equation (12)) which is decreasing in t. The black line corresponds to the right-hand side (RHS) of (12). An increasing function on the RHS coupled with the decreasing function on the LHS results in a unique crossing, which implies a unique maximizing fee.[15]Notice that if the LHS is lower than the RHS at $t = 0$, the platform is not viable, which happens for K at too high a point. We prevent this from happening by Assumption 2.

Comparative statics can be drawn directly from the picture. For example, a lower entry cost, K, moves up the red curve and more fringe firms enter the platform. As we shall see, the platform then sets a higher fee to fringe sellers.

3.2 Hybrid platform

In the hybrid mode, the platform sells its own product competing alongside third-party products. The platform sets both its own price and the fee, so it controls entry of fringe sellers and controls both competition for its own product and the fees it gets from sales of third-party products (its rivals in the product market). It is like a long-run Stackelberg leader controlling entry; this feature leads to some novel takeaways.

For the sake of presentation in the rest of the analysis, we denote the platform

[15]Appendix B gives a sufficient condition for the platform's profit to be strictly quasi-concave.

product's advantage, that is, its value minus its cost by x_A, so $x_A = v_A - c_A$, and its markup by m_A, so $m_A = p_A - c_A$. Given c_A, choosing price p_A pins down the markup m_A, so in the hybrid platform analysis we consider the platform choosing its markup m_A and the fee t.

3.2.1 Fringe sellers' choices

Given the platform product's markup, m_A, and the fee set by the platform, t, consider first the monopolistic competitors (fringe sellers). Seller i's demand is

$$q_i(p_i, p, m_A) = \frac{\exp\left(\frac{v-p_i}{\mu}\right)}{A_h}, \tag{13}$$

where the aggregate differs from the one of the previous section as it incorporates the platform's product:

$$A_h \equiv n \exp\left(\frac{v-p}{\mu}\right) + M \exp\left(\frac{x_A - m_A}{\mu}\right) + 1.$$

Given the aggregate is independent of p_i, each fringe seller sets the same price (6) as for the pure marketplace:

$$p(t) = \frac{c}{1-t} + \mu.$$

The equilibrium total profit of each fringe seller is analogous to the profit in the pure marketplace:

$$\pi_h(t) = \mu\,(1-t)\,\frac{V(t)}{A_h} - K.$$

The zero-profit entry condition together with the latter profit expression ties down the equilibrium value of the aggregate, which is the same expression as (8) for the analysis of pure marketplace. Hence, we prove the following:

Lemma 2 *Given the platform's seller fee t, the equilibrium aggregate of a hybrid platform is the same as the aggregate of the pure marketplace:*

$$A_h(t) = A(t) = \frac{\mu\,(1-t)}{K} V(t), \tag{14}$$

which is independent of the platform product markup, m_A, or its advantage, x_A.

Hence, we use $A(t)$ to denote the equilibrium aggregate in the rest of the analysis.

The size of the fringe, $n_h(t, m_A)$, is the solution to

$$A\left(t\right) = n_h \exp\left(\frac{v - p(t)}{\mu}\right) + M \exp\left(\frac{x_A - m_A}{\mu}\right) + 1. \tag{15}$$

After replacing $A\left(t\right)$ by (14), we rewrite the latter condition determining $n_h(t, m_A)$:

$$n_h(t, m_A) = \frac{\mu\left(1 - t\right)}{K} - \frac{M \exp\left(\frac{x_A - m_A}{\mu}\right) + 1}{V\left(t\right)}.$$

At a given commission t there will be fewer fringe firms in the hybrid platform case than the case of pure marketplace: $n_h(t, m_A) < n(t)$, since the platform's product diverts demand from the fringe products, lowering their variable profit from entering. When the platform's product becomes more attractive (m_A decreases or x_A increases) keeping t constant, the number of fringe firms entering the hybrid platform decreases.

3.2.2 The platform's choices

First we find the optimal markup m_A for any choice of t and work from this composition relation to solve the problem. We split the problem of the platform in this way since, given t, the aggregate is fixed at $A(t)$. Then the platform choosing m_A tells us how much of $A(t) - 1$ the platform wants to divert to its own product.

Recall that the platform's revenue from third-party products per unit sold is $tp(t) = t\left(\frac{c}{1-t} + \mu\right)$. The platform's profit is:

$$\Pi_h = m_A \frac{V_A}{A(t)} + t\left(\frac{c}{1 - t} + \mu\right)\frac{A\left(t\right) - V_A - 1}{A(t)}, \tag{16}$$

where V_A is defined as

$$V_A \equiv M \exp\left(\frac{x_A - m_A}{\mu}\right). \tag{17}$$

The first term in (16) captures the platform's net profit from sales of its own product and the second term captures the platform's revenues as a marketplace from third-party product sales. The profit (10) of the platform when it was a pure marketplace corresponds to the hybrid platform's profit if we set $V_A = 0$, which would be the case when the platform's product's advantage goes to minus infinity, $x_A \to -\infty$. Thus, the platform's hybrid platform profit approaches its pure marketplace profit when

$x_A \to -\infty$.

Choosing m_A: The platform balances two margins when choosing its product's markup. A higher own-product markup will bring more fee revenue from third-party product sales (fringe products) since there will be more fringe products when competing against a more expensive platform product (though each fringe product has the same demand for a given aggregate). A lower own-product markup will bring a larger share of the total product demand $\frac{A(t)-1}{A(t)}$ to the platform's product. Because this total demand is independent of the platform product's markup, the platform may want more demand for its own product using a low markup. This conjecture comes from what we know for price leadership under endogenous entry in long-run aggregative games. Indeed, if there were no seller fees, $t = 0$, $v_A = v$, and $c_A = c$, then a price leader under endogenous entry would set a lower price than followers to take a larger share of the fixed market (Etro, 2006; Anderson et al., 2020) if followers were oligopolistic competitors. However, in the current model, the followers are monopolistically competitive and in that context Anderson et al. (2020) show that the leader's price is the same as if it were monopolistically competitive itself, for in both situations the firm takes the aggregate as given. Thus, for $t = 0$, $v_A = v$, and $c_A = c$, the platform would set the same price as the fringe. Now add in the fact that the price leader (the platform) collects fees from the followers (fringe sellers) for $t > 0$. Then, the opportunity cost of selling another unit of the platform's product is the lost commission on a supplanted fringe unit, $tp(t)$. Thus, the platform's product might be sold at a higher price. We next elaborate when this happens allowing v_A to differ from v and c_A to differ from c.

When choosing m_A, the platform maximizes its profit (16). Given that the aggregate, $A(t)$, is constant in m_A, the optimality condition for the platform's markup is:

$$m_A(t) = p_A(t) - c_A = \mu + tp(t) = \mu + t\left(\frac{c}{1-t} + \mu\right). \tag{18}$$

Intuitively, the platform sets the "standard" markup of μ plus its opportunity cost of lowering m_A, that is, the lost revenue per unit from third-party product sales, $tp(t)$, when the platform sells one more unit of its product. Comparing the platform product's price to the fringe products' price, we obtain a necessary and sufficient condition under which the platform sets a higher price for its product:

$$p_A(t) = \mu + t\left(\frac{c}{1-t} + \mu\right) + c_A > p(t) = \frac{c}{1-t} + \mu \text{ if and only if } c_A + \mu t > c.$$

When unit costs are the same, $c_A = c$, the platform's product is always more expensive than the fringe products. The platform's product is cheaper than the fringe products if, and only if, the platform has a sufficiently lower cost than the fringe sellers, $c_A < c - \mu t$. Thus, the platform's product is cheaper than the fringe products when the platform's fee and differentiation between products are sufficiently low.

choosing t: We first define the platform product's fraction of the aggregate when the platform sets its markup optimally for given t:

$$V_A(t) \equiv M \exp\left(\frac{x_A - m_A(t)}{\mu}\right),$$

so the demand for the platform product for given t is $\frac{V_A(t)}{A(t)}$. In order to make the problem of a hybrid platform interesting, we assume that the platform is able to attract some fringe firms at the lowest fee, $t = 0$. This is the case when the fixed cost of a fringe firm is sufficiently low such that $A(0) - V_A(0) > 1$. The following assumption ensures that this is the case:

Assumption 3 $K < \frac{\mu \exp\left(\frac{v-c-\mu}{\mu}\right)}{1+M\exp\left(\frac{x_A-\mu}{\mu}\right)}$.

Observe that Assumption 3 is more demanding than the one we made for a pure marketplace, Assumption 2. This is because it is easier for a pure marketplace to attract a positive number of fringe firms than a hybrid platform. Let us define the minimum fee above which the fringe disappears. We denote this fee by $\hat{t}_h$, which is a solution to $A\left(\hat{t}_h\right) = V_A\left(\hat{t}_h\right) + 1$. We next show that this solution exists, it is unique, strictly positive, and strictly less than 100 percent:

Lemma 3 *For any x_A there is a unique $\hat{t}_h \in (0,1)$ such that some fringe firms join the platform if $t_h \in [0, \hat{t}_h)$ and there is no fringe if $t \geq \hat{t}_h$.*

An optimal platform fee sets the marginal profit from raising t_h at zero:

$$\frac{d\Pi_h}{dt} = \frac{1}{A(t)}\left[(tp(t))'\left(A(t) - V_A - 1\right) + tp(t)\frac{A'(t)}{A(t)} - (m_A - tp(t))V_A\frac{A'(t)}{A(t)}\right] = 0. \quad (19)$$

The first term is the per-unit extra revenue from raising t_h on the base of fringe output. The second is the lost revenue from contraction of the total demand due to fewer fringe firms entering the platform. The third is the gains from sales of the own product when the total demand shrinks, since then the platform has a larger share of the total demand. Recall that per-unit gains from more sales of the own product is the markup of the own product, $m_A = p_A - c_A$, minus the opportunity cost of selling one unit of the own product, $tp(t)$. Given the equilibrium markup $m_A(t) = \mu + tp(t)$ (18), we write the optimal fee as the solution $t = t_h^*$ to

$$(tp(t))' \frac{(A(t) - V_A(t) - 1)}{A(t)} + \frac{A'(t)}{A(t)^2} (tp(t) - \mu V_A(t)) = 0.$$

The second term in the optimal fee equation represents the lost profit from fringe exit due to higher commission.[16] To see why, consider the effects on platform profit from a single fringe firm exiting due to a particular commission hike. The lost revenue on that single exit is $tp(t)\frac{V(t)}{A(t)}$ where $\frac{V(t)}{A(t)}$ is the demand per fringe firm. The platform product's demand increases by $\frac{V(t)V_A(t)}{A^2(t)}$ as the aggregate is reduced by $V(t)$. Each unit increase carries a markup of $\mu + tp(t)$. Moreover, the platform also gains from increased sales by remaining fringe members; demand for each of these rises by $\frac{V^2(t)}{A^2(t)}$, for a total extra sales over n fringe members of $n\frac{V^2(t)}{A^2(t)}$ worth $tp(t)$ per unit to the platform. Adding up these gains and losses gives us a profit decrease of

$$\frac{V(t)}{A(t)} \left(tp(t) \left(1 - \frac{nV(t)}{A(t)} \right) - (\mu + tp(t)) \frac{V_A(t)}{A(t)} \right),$$

or

$$\frac{V(t)}{A^2(t)} \left(tp(t) \left[A - nV(t) - V_A(t) \right] - \mu V_A(t) \right),$$

which concurs with the lost profit expression $\frac{A'(t)}{A(t)^2} (tp - \mu V_A)$ when $A(t)$ goes down by one fringe firm counting for a change of size $V(t)$.

To obtain the platform's fundamental pricing formula we rewrite the optimal fee

[16] The astute reader will note that the dimensionality of the 2 terms in parentheses looks wrong: the first (tp/A) is \$/unit lost revenue on each fringe sale, while $\mu V_A(t)/A$ is platform product profit in excess of imputed fee revenue on those sales, so in \$. This seeming inconsistency is redressed by recalling that we normalized the outside good's value in the aggregate to 1. If we give the outside good a "quality" of v_o to consumers and run through the derivative, then the term $tp(t)$ is multiplied by $V_o \equiv \exp(v_o/\mu)$ and then the corrected expression $tp(t)V_o/A$ is in \$.

condition at equilibrium markup $m_A(t)$ (18) as

$$\begin{aligned} A(t) - V_A(t) &= 1 - \frac{tp(t)}{(tp(t))'}\frac{A'(t)}{A(t)} + \mu V_A(t)\frac{1}{(tp(t))'}\frac{A'(t)}{A(t)}, \\ &= 1 - \left(1 - \frac{\mu V_A(t)}{tp(t)}\right)\frac{\epsilon_{A(t)}}{\epsilon_{tp(t)}}, \end{aligned} \tag{20}$$

where we used the definitions of elasticities for the second equality. Recall that the elasticity of the aggregate is negative and the elasticity of fee revenue per unit is positive.

Lemma 4 *In any hybrid regime, the platform sets fee $t_h^* > 0$ satisfying the optimality conditions (18) and (19) (or (20)). At the optimal fee there are some fringe firms (third-party products), that is, $t_h^* < \hat{t}_h$ if the platform product's advantage is low enough:*

$$x_A < \mu + \frac{c\hat{t}_h}{(1-\hat{t}_h)} + \hat{t}_h\mu + \mu In\left(\frac{\hat{t}_h}{M} + \frac{c\hat{t}_h}{\mu M(1-\hat{t}_h)}\right) \equiv \tilde{x}_A. \tag{21}$$

A hybrid platform compares its profit from sales of the own product to its revenue per unit from the sale of a third-party product. It becomes profitable for the platform to sell at least one unit of third-party product if the unit revenue from a third-party product is greater than the profit from its own sales at $\hat{t}_h$ (the minimum fee above which there is no fringe entry). This is the case if the platform's product has an advantage below $\tilde{x}_A$ given in (21). As we show in Section 3.3.2, the equilibrium mode is hybrid if, and only if, $x_A < \tilde{x}_A$. Otherwise, the platform is a pure reseller.

We next show how the advantage of the platform's product affects its commission on third-party products:

Proposition 1 *Suppose $x_A < \tilde{x}_A$. The hybrid platform's optimal fee on third-party product sales increases in the advantage of the platform's product, $x_A = v_A - c_A$.*

To prove the proposition we calculate how the marginal profit of raising the fee changes in the advantage of the platform's product, that is, the sign of $\frac{\partial^2 \Pi_h}{\partial t \partial x_A}$ at equilibrium prices.

We rewrite the platform's profit (16) as the sum of the platform's margin over the opportunity cost of selling the own product times the demand of the own product plus the platform's fee revenue from sales of third-party products:

$$\Pi_h = (m_A - tp(t))\frac{V_A}{A(t)} + tp(t)\frac{A(t)-1}{A(t)},$$

where $V_A = M\exp\left(\frac{x_A - m_A}{\mu}\right)$ and $p(t) = \frac{c}{1-t} + \mu$. Using the envelope theorem, we first differentiate the profit with respect to the platform product's advantage, x_A:

$$\frac{\partial \Pi_h}{\partial x_A} = \frac{1}{A(t)}\left[(m_A - tp(t))\frac{V_A}{\mu}\right] > 0. \tag{22}$$

We next calculate how this changes in fee:

$$\frac{\partial^2 \Pi_h}{\partial x_A \partial t} = -\frac{A'(t)}{A^2(t)}\left[(m_A - tp(t))\frac{V_A}{\mu}\right] + \frac{1}{A(t)}\left[(-tp(t))'\frac{V_A}{\mu}\right]. \tag{23}$$

From the free-entry condition of fringe firms, (14), we have $A(t) = \frac{\mu(1-t)}{K}V(t)$ where $V(t) = \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right)$, so the semi-elasticity of the aggregate is

$$\frac{A'(t)}{A(t)} = -\frac{\frac{c}{1-t} + \mu}{\mu(1-t)}.$$

We also derive how unit fee revenue changes in fee, $(tp(t))' = \mu + \frac{c}{(1-t)^2}$. Substituting these and the optimal markup $m_A(t) = \mu + tp(t)$ in (23), we show that the marginal profit of raising the fee increases in the platform product's advantage at equilibrium prices:

$$\frac{\partial^2 \Pi_h}{\partial x_A \partial t} = \frac{V_A(t)}{A(t)}\frac{t}{1-t} > 0. \tag{24}$$

Intuitively, when the platform product's quality increases, keeping t constant, the demand for the platform's own product increases. The aggregate is constant in the advantage of the platform's product (see Lemma 2). This implies that the demand for monopolistic sellers goes down, so there will be fewer fringe sellers on the platform. This induces the platform to increase its commission, since now the platform puts greater weight on the profits generated from its own product and less weight on the revenue generated from sales of third-party products, that is, in (16) the first term's weight has increased and the second term's weight has decreased.[17]

[17] Note that we proved the existence of an optimal fee in Lemma 4. Proposition 1 is valid for any optimal fee if we have multiple fees that solve the optimality condition in (19). By the super-

Thus, as the platform product gets better, for given t the platform takes a larger share of the total demand. We also know that the platform raises its optimal t as its product's advantage rises (Proposition 1). Thus, both of these effects point to the same conclusion:

Corollary 1 *The equilibrium share of the hybrid platform's product increases when its own product has a higher advantage,* $x_A = v_A - c_A$.

Consumer surplus is given by the log-sum formula due to the logit demand model (Anderson et al., 2020):

$$CS(t) = \ln A(t). \tag{25}$$

The platform's optimal fee is higher when its own product has a higher advantage, see Proposition 1. This implies that the equilibrium aggregate, $A(t_h^*)$, goes down and so consumer surplus falls when x_A increases. This gives us the main result in the hybrid platform analysis:

Proposition 2 *Consumer surplus and the number of fringe products on a hybrid platform decrease when the platform's product is stronger (better quality or a lower unit cost).*

Normally a stronger (a higher quality or a lower cost) product is good news for consumers. We know that when a monopolist has a better quality product, consumers benefit as long as the price does not outweigh the benefit, which is the "usual" case, that is, when demand is log-concave. This is true for oligopoly with fixed numbers of firms, too. Here, different from a standard oligopoly framework, a stronger platform product causes the platform to rebalance its business model. If it were to keep t fixed, the effect would simply be neutral; a stronger platform product just crowds out some fringe firms (to restore their marginal profits to zero) and the lost variety is offset exactly by a larger market base for the stronger product. But t is not fixed; the platform takes advantage of the stronger product to raise t and earn more from fees on third-party sales, as well as from sales of the own product. The fringe shrinks through the twin reasons of stronger on-platform (in-house) competition and induced

modularity of the profit function in fee and the advantage of the own product, which we show in the proof of Proposition 1, any increase in own-product advantage leads to an increase in equilibrium fee.

higher prices stifling demand and profits. Consumers are hurt by the loss of fringe seller variety and from the higher prices they charge.

We can also allow the hybrid platform to choose which product to sell from various different x_A possibilities. It is straightforward to show that the platform prefers to sell the product with the highest advantage $x_A = v_A - c_A$. To see this, we simply refer to (22), which shows how the platform's equilibrium profit at equilibrium prices changes in x_A. It increases in x_A because $m_A(t) = \mu + tp(t)$, that is,

$$\frac{\partial \Pi_h}{\partial x_A} = \frac{V_A(t)}{A\,(t)} > 0.$$

Note that the platform always has the option to raise the price by the same amount as the quality rise and keep demand constant. Thus, a one \$ decrease in cost delivers a one \$ rise in profit on the demand base $V_A(t)/A(t)$. Likewise, a one \$ rise in quality raises profit by one \$ on the base. By the envelope theorem, the same expression holds true when evaluated at the optimal fee, t_h^*, that is, the equilibrium profit of the platform increases in its advantage by $V_A(t_h^*)/A(t_h^*)$. The direct implication of this, together with Propositions 1 and 2, is:

Corollary 2 *Allowing endogenous product choice of the hybrid platform leads to a higher advantage for the platform product, raises the equilibrium fee on third-party sales, and lowers consumer surplus.*

3.3 Optimal business model of the platform

In this section we will analyze the endogenous business model choice of the platform by ranking the platform's highest profit in each structure: a pure marketplace, a hybrid platform and a pure reseller. First, we will characterize the platform's choice between a pure marketplace, and a hybrid platform. Next, we will characterize the platform's choice between a hybrid platform and a pure reseller. We conclude with conditions under which each business mode emerges endogenously.

3.3.1 Pure marketplace vs. hybrid

Consider the platform's choice between hybrid and pure marketplace. The platform selects the model that generates more profit. Profit functions under the two business models are those shown in the previous sections (recall that the platform has to pay an

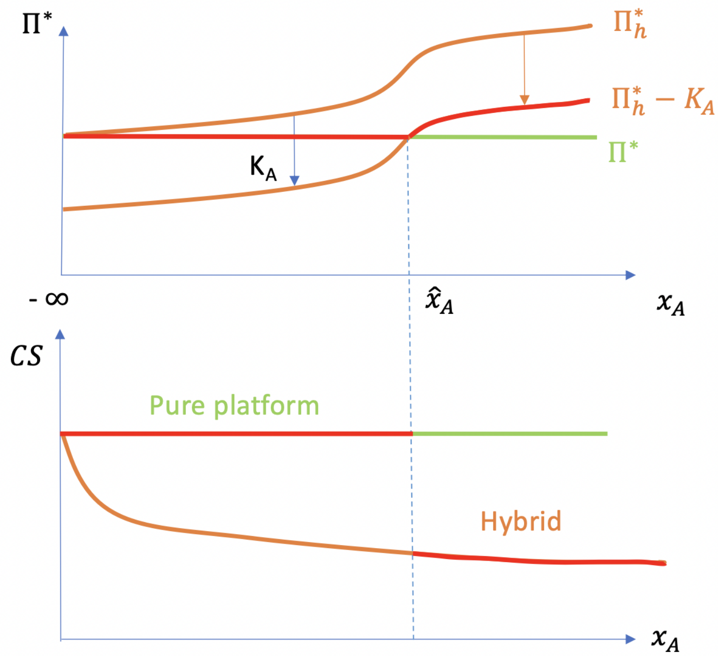

Figure 2: Equilibrium profits and consumer surplus

entry fee of K_A to sell its own product). If the maximum profit as a pure marketplace, Π^*, is bigger than the maximum profit as a hybrid platform, $\Pi_h^* - K_A$, the platform prefers to be a pure marketplace. If the opposite holds, it prefers to be a hybrid platform. The equilibrium choice is determined by the advantage of the platform's product, $x_A = v_A - c_A$, and the level of its fixed cost, K_A.

The upper panel in Figure 2 shows how the platform chooses its business model depending on its product's advantage, x_A. The maximized profit under the pure marketplace model is independent of x_A. On the other hand, the maximized profit under the hybrid model is increasing in x_A. As the pure marketplace is a limiting case of hybrid, (see (16) when $x_A \to -\infty$), the profit under the hybrid model starts from that of the pure marketplace model. With the entry cost, the platform operates as a pure marketplace if its product's advantage is less than a cutoff, $\hat{x}_A$, since the net profit is bigger there. The cutoff point, $\hat{x}_A$, is the advantage of the platform's product where the two business models generate the same profits:

$$\Pi^* \;=\; \Pi_h^* - K_A. \tag{26}$$

Note that $\hat{x}_A$ is uniquely defined, given that Π^* is constant in x_A, Π_h^* is continuously increasing in x_A and $\Pi_h^* \to \Pi^*$ when $x_A \to -\infty$. Moreover, when the platform's product fixed cost, K_A, increases, the cutoff $\hat{x}_A$ increases: it only becomes attractive to sell its own product along with the third-party products if the own product has a high enough advantage. Note also that when $K_A \to 0$, the platform prefers the hybrid mode to the pure marketplace mode for any x_A even when the platform product's advantage is lower than the third-party products' quality: $x_A < x$ where $x = v - c$. We thereby show the platform's optimal choice between the pure marketplace mode and the hybrid mode:

Proposition 3 *The platform prefers the hybrid mode to the pure marketplace mode when the advantage of the platform's product is higher than a cutoff value: $x_A \geq \hat{x}_A$. For $x_A < \hat{x}_A$, the platform prefers the pure marketplace mode. The cutoff $\hat{x}_A$ increases in the fixed cost of platform product, K_A.*

The lower panel in Figure 2 draws consumer surplus for the two modes. The aggregate is a sufficient statistic for consumer welfare. As for optimal profits, the aggregate with a pure platform is independent of x_A, so consumer surplus is constant in x_A in that mode. However, the aggregate under the hybrid mode decreases in x_A starting from the aggregate under the pure marketplace, since the equilibrium fee on third-party products is rising (Proposition 1). Through this, consumers are always better off with the pure marketplace mode compared to the hybrid mode whatever the advantage of the platform's product.

Corollary 3 *When $x_A \geq \hat{x}_A$, banning the hybrid platform mode benefits consumers if the platform switches to the pure marketplace mode. When $x_A < \hat{x}_A$, the ban has no effect on the equilibrium outcome.*

3.3.2 Hybrid vs. pure reseller

In Lemma 4 we documented the existence of a cutoff advantage for the platform's product, $\tilde{x}_A$, above which the hybrid platform prefers to set a prohibitive fee such that there is no fringe product on the platform. In this case, the platform effectively becomes a pure reseller. Note that when $x_A = \tilde{x}_A$, the hybrid platform's equilibrium fee converges to the level where there is no entry by fringe sellers, $t_h^* = \hat{t}_h$ (point O in Figure 3). Figure 3 is drawn for the case where the advantage of the platform's

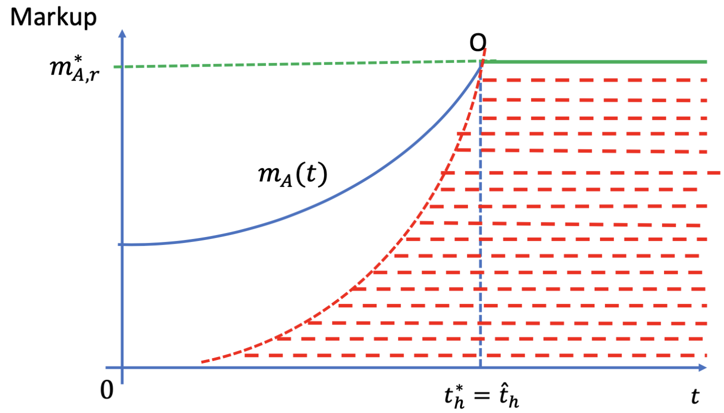

Figure 3: Markup of the platform's product in the hybrid mode, $m_A(t)$, and in the pure reseller mode, $m_{A,r}^*$, when the platform's product's advantage is $x_A = \tilde{x}_A$. The red shaded area is where there is no entry by fringe sellers, so the platform is a pure reseller.

product is at the cutoff $\tilde{x}_A$. The red zone in the figure shows the combinations of the platform's product markup and fee level where fringe firms find it optimal not to enter the platform (pure reseller mode). Formally, the boundary of the red zone is where $A(t) = V_A(m_A) + 1$ or

$$\frac{\mu(1-t)}{K}V(t) = M\exp\left(\frac{x_A - m_A}{\mu}\right) + 1. \qquad (27)$$

The RHS of (27) is decreasing in m_A and the LHS is decreasing in t, so when the platform raises its markup, the fee at which the fringe disappears increases.

Figure 3 also shows the optimal markup $m_A(t) = tp(t) + \mu$ (18) conditional on the hybrid regime, that is, when $t < \hat{t}_h$ and positive fringe firms entry given t: $m_A(t)$ is an increasing function. The boundary of the red zone increases in the markup up to its intersection with $m_A(t)$ at $m_A(\hat{t}_h)$, which is where the fringe is just driven out at the optimal markup (there is a single intersection at $\hat{t}_h$, see the proof of Lemma 3 in the Appendix). When $t \geq \hat{t}_h$, there is no more fringe firms on the platform and the platform's profit becomes the profit of a reseller monopolist, which we denote by Π_r:

$$\Pi_r = m_{A,r} \frac{V_{A,r}}{V_{A,r}+1}, \tag{28}$$

where $V_{A,r} = M \exp\left(\frac{x_A - m_{A,r}}{\mu}\right)$ and $m_{A,r} = p_{A,r} - c_A$. Let $m^*_{A,r}$ denote the optimal markup of the pure reseller, so $m^*_{A,r}$ solves $\frac{d\Pi_r}{dm_{A,r}} = 0$ or $m^*_{A,r} = \mu(V^*_{A,r}+1)$.[18]

Lemma 4 also shows that at $x_A = \tilde{x}_A$, the hybrid platform prefers to set $t^*_h = \hat{t}_h$, and so $m_A(\hat{t}_h)$ corresponds to the optimal markup when the platform effectively becomes a pure reseller: $m_A(\hat{t}_h) = m^*_{A,r}$ (point O in Figure 3). Thus, at $x_A = \tilde{x}_A$, the maximized profit of the platform under the hybrid mode corresponds to the maximized profit under the pure reseller mode: $\Pi^*_h = \Pi^*_r$ at $x_A = \tilde{x}_A$.

The key question we address in this section is whether the hybrid platform wants to switch to the pure reseller mode when its product's advantage is below $\tilde{x}_A$. Let us consider the effects of lowering the advantage of the platform's product below $\tilde{x}_A$. Figure 4 illustrates these effects. Compared to the case of $x_A = \tilde{x}_A$ in Figure 3, the red zone in Figure 4 is further right. This is because a lower advantage for the platform's product increases the total demand for the fringe firms, and so more fringe firms enter the platform. Thus, the cutoff point, $\hat{t}_h$, at which $m_A(t)$ hits the red zone, also moves right. Another effect of lowering the advantage of the platform's product below $\tilde{x}_A$ is that the platform's optimal commission, t^*_h, decreases (by Proposition 1). This is illustrated in Figure 4 as a move from point O to point C. Note that $m_A(t)$ in Figure 3 stays the same in Figure 4, since $m_A(t)$ is constant in x_A. On the other hand, the optimal markup of the pure reseller, $m^*_{A,r}$, goes down as the product's advantage decreases. Let us denote by $\tilde{t}$ the platform's commission where $m^*_{A,r}$ hits the red zone. The previous discussion illustrates why $t^*_h < \tilde{t} < \hat{t}_h$ (the equilibrium fee decreases, $m^*_{A,r}$ goes down, and the red zone moves right when x_A goes below $\tilde{x}_A$).

The platform's profit is maximized at t^*_h (point C) conditional on being in the hybrid zone. Thus, the platform's profit at point B (where the seller fee is at $\tilde{t}$) is

[18] In Figure 3 the platform's optimal markup $m_A(t)$ is continuous in t, so there is a smooth switchover in m_A as t rises through $\hat{t}_h$, the point at which fringe disappears. Formally, the claim is

$$\lim_{t \to \hat{t}_h} m_A(t) = m^*_{A,r} = \mu(V^*_{A,r}+1).$$

To prove the claim we use the platform's optimal fee condition (20), when the platform product's advantage is at the cutoff: $x_A = \tilde{x}_A$, so at $t^*_h = \hat{t}_h$ we have $\hat{t}_h p\left(\hat{t}_h\right) = \mu V_A\left(\hat{t}_h\right)$. Thus, at the optimal fee $\hat{t}_h$, the optimal markup conditional on hybrid regime becomes $m_A\left(\hat{t}_h\right) = \hat{t}_h p\left(\hat{t}_h\right) + \mu = \mu\left(V_A\left(\hat{t}_h\right)+1\right)$. But this is just a rewrite of the pure reseller optimal markup $m^*_{A,r} = \mu(V^*_{A,r}+1)$ (which is uniquely determined and is therefore the same).

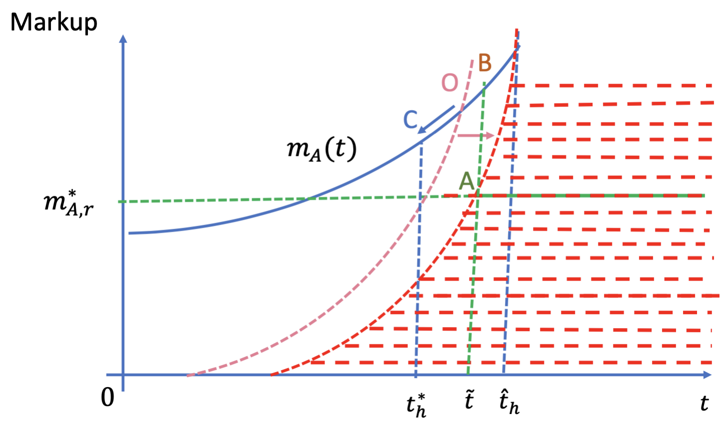

Figure 4: The markup of the platform's product in the hybrid zone and in the pure reseller zone when $x_A < \tilde{x}_A$.

lower than its profit at point C: $\Pi_h(\tilde{t}) < \Pi_h(t^*_h)$. Now we compare the platform's profit at point B to its profit at point A, where $m^*_{A,r}$ hits the red zone in Figure 4. Increasing the markup above $m^*_{A,r}$, while keeping the fee at $\tilde{t}$ (moving from point A to point B in the figure), increases the platform's profit since the platform's profit is quasi-concave in markup and we are moving toward the platform's optimal markup (point B on $m_A(t)$). By transitivity we prove that the platform's maximized profit in the hybrid mode (at point C) is higher than its maximized profit in pure reseller mode (point A) when $x_A < \tilde{x}_A$. This result, together with Lemma 4, determines the platform's optimal choice between the hybrid mode and the pure reseller mode.

Proposition 4 *When $x_A < \tilde{x}_A$ (defined in Lemma 4), the platform prefers the hybrid mode to the pure reseller mode and sets $t^*_h < \hat{t}_h$. Otherwise, the platform prefers to be a pure reseller and sets a prohibitive fee to third-party sellers.*

We next compare the consumer surplus at the equilibrium fee in the hybrid mode (point C) to the consumer surplus at the equilibrium markup of a pure reseller (point A). The consumer surplus at point A is the same as the consumer surplus of the hybrid platform if the platform set its markup at $m^*_{A,r}$ and its seller fee at $\tilde{t}$, since then there would be no fringe sellers on the platform and the platform would generate the same pure reseller profit. Recall that the consumer surplus depends only on the seller fee in the hybrid mode since the aggregate pins down the consumer surplus and the aggregate is independent of the platform product's markup (by Lemma 2), so we have $CS(\tilde{t}) = In(A(\tilde{t}))$. Thus, the consumer surplus at point B is $CS(\tilde{t})$, which is the same as the consumer surplus at point A. Given that the equilibrium seller fee t_h^* is lower than $\tilde{t}$ when $x_A < \tilde{x}_A$, the consumer surplus at the equilibrium of the hybrid mode (point C) is higher than the consumer surplus at point B. Hence, we prove the following:

Corollary 4 *When $x_A < \tilde{x}_A$, the consumer surplus is higher at the equilibrium of the hybrid mode (at point C) than the equilibrium of the pure reseller mode (at point A). Hence, banning the hybrid mode harms consumers if the platform switches to a pure reseller due to the ban.*

Note also when $x_A \geq \tilde{x}_A$, the platform prefers the pure reseller mode to the hybrid mode (by Lemma 4). Thus, in that case, banning the hybrid mode does not affect the equilibrium outcome.

3.3.3 Equilibrium business mode

Proposition 3 shows that below the cutoff point $\hat{x}_A$ the platform prefers the pure marketplace and above this cutoff the platform prefers the hybrid mode. Moreover, $\hat{x}_A$ is continuously and monotonically increasing in K_A and when $K_A \to 0$, $\hat{x}_A \to -\infty$. In Proposition 4 we show that the platform prefers the hybrid mode for $x_A < \tilde{x}_A$ and prefers the pure reseller mode for $x_A \geq \tilde{x}_A$, and that $\tilde{x}_A$ does not depend on K_A (as long as the fixed cost is low enough to guarantee non-negative profit in the hybrid regime when $x_A < \tilde{x}_A$ and in the pure reseller regime when $x_A \geq \tilde{x}_A$). Combining these, we show that there exists $\overline{K}_A > 0$ at which the two cutoff points are equal: $\hat{x}_A(\overline{K}_A) = \tilde{x}_A$ (illustrated as point D in Figure 5). For any $K_A > \overline{K}_A$, $\hat{x}_A(K_A) > \tilde{x}_A$ and for any $K_A < \overline{K}_A$, $\hat{x}_A(K_A) < \tilde{x}_A$.

Let $\overline{x}_A$ denote the advantage of the platform's product when the maximized profit of the pure marketplace is equal to the maximized profit of the pure reseller: $\Pi^* = \Pi_r^*(\overline{x}_A)$. Note that the existence and uniqueness of $\overline{x}_A$ is guaranteed given that the pure marketplace profit is independent of x_A and the pure reseller profit continuously increases in x_A. By definition, $\overline{x}_A$ must go through point D in Figure 5 as illustrated by the red curve.

Figure 5 illustrates the equilibrium business model choice of the platform depending on the fixed cost and the advantage of its product. The platform chooses the pure marketplace mode for $x_A < \hat{x}_A$ and $x_A < \overline{x}_A$ (green area in Figure 5), chooses

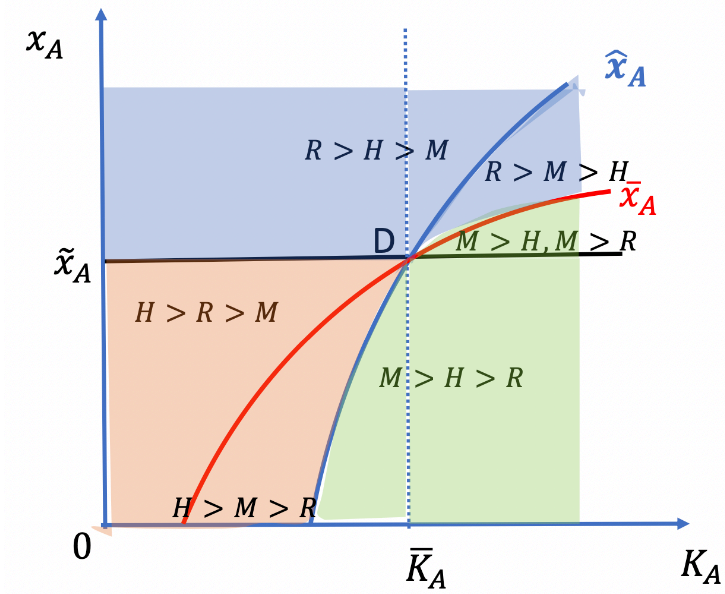

Figure 5: The platform's optimal business model choice between pure marketplace mode (M), hybrid mode (H), and reseller mode (R) depending on its product's advantage x_A and fixed cost K_A.

the hybrid mode for intermediate levels of own-product advantage, for $\hat{x}_A < x_A < \tilde{x}_A$ (orange area in Figure 5), and chooses the pure reseller mode when the own product has a sufficiently high advantage, $x_A > \tilde{x}_A$ and $x_A > \overline{x}_A$ (blue area in Figure 5).

Now we analyze the effects of a ban on the hybrid regime. Consider the area (orange in Figure 5) where the hybrid regime is optimal (since in other areas the ban has no effect on the business mode choice of the platform). If the hybrid mode is banned, the platform chooses the mode that generates the highest profits between pure marketplace mode and pure reseller mode. Figure 6 illustrates the platform's equilibrium business mode choice if the hybrid is banned for a given $K_A < \overline{K}_A$. The platform switches to the pure marketplace mode when $\hat{x}_A < x_A \leq \overline{x}_A$ and switches to the pure reseller mode when $\overline{x}_A < x_A < \tilde{x}_A$. The figure also illustrates how the ban of the hybrid mode affects consumers (using Corollary 3, Figure 2 and Corollary 4 together):

Corollary 5 *A ban on the hybrid mode benefits consumers if the platform switches to the pure marketplace mode (when $\hat{x}_A < x_A \leq \overline{x}_A$) and harms consumers if the platform switches to the pure reseller mode (when $\overline{x}_A < x_A < \tilde{x}_A$). A ban on the*

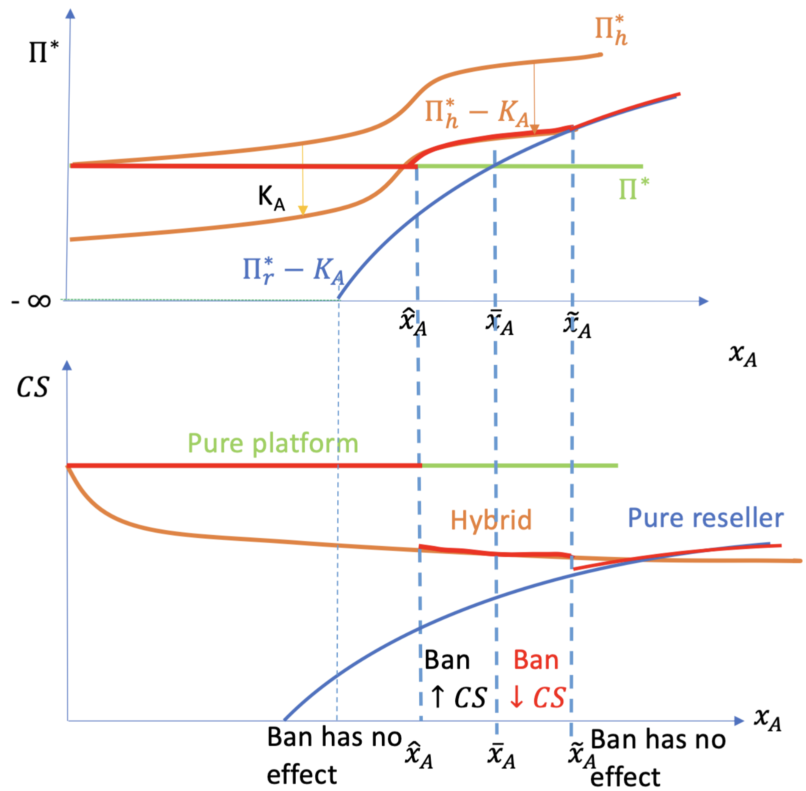

Figure 6: The platform's optimal business model choice and consumer welfare effects when the hybrid mode is banned (for a given $K_A < \overline{K}_A$).

hybrid mode has no effect on the equilibrium outcome otherwise.

3.4 Our contribution compared to the literature

Similar to our paper, Hagiu et al. (2020) analyze the implications of the hybrid business mode (or dual mode) on third-party sellers' actions and consumers. Their model captures different circumstances to ours: there are three different types of products (a superior product, a platform product, and competitive fringe products) that differ in their quality or cost, consumers are homogenous in their valuations of products, and there is always some (exogenous number of) consumers that prefer to buy directly from third-party sellers. Hybrid mode is profitable in equilibrium only if there are some consumers who directly buy from third-party sellers,[19] whereas in our paper the profitability of the hybrid mode relies on product differentiation and consumers' heterogenous tastes for products. In their paper, like ours, the platform charges higher seller fees in the dual mode than in the pure marketplace mode and when its advantage (benefits from buying on the platform) increase. They show that increased competition in the dual mode[20] more than outweighs the effect of high commissions and so leads to higher consumer surplus. Besides, at the hybrid mode equilibrium, only the superior seller product is purchased, so the platform product and fringe products have zero demand (see their Proposition 3). On the other hand, in our setup, increasing the platform's product quality or hybrid mode relative to pure marketplace mode lowers variety (leads to fewer fringe firms joining the platform), the latter reduced variety effect neutralizes the former higher quality effect, and higher prices resulting from higher seller fees lead to lower consumer surplus. In our hybrid mode equilibrium, the platform product and all fringe sellers have some positive demand on the platform. The reasons behind these different results are, therefore, us considering differentiated products, heterogenous consumers' match values for products, and elastic seller entry depending on the platform's actions (seller fee and own-product price).

Hervas-Drane and Shelegia (2021) address the concern that a platform can learn from the success of third-party sellers to muscle in on lucrative product market categories. They consider independent product categories that differ in their value, inelastic demand (homogenous consumers) and a single independent seller in each market. The platform has an exogenous number of categories it can enter (its capacity). It only knows about the existence and value of a fraction of categories: entry by an independent seller informs it of the value of those entered. There are then three types of category. The most profitable of the pre-known ones are served by the platform alone. The least profitable of the ones entered by third parties are left alone (due to the capacity constraint) as pure marketplaces. In the most profitable ones entry drives a subgame equilibrium in which the platform gives itself the "BuyBox"

[19] The superior seller sometimes prefers to exploit its direct consumers rather than lowering its direct price to compete against the platform product. Thus, direct consumers prevent head-to-head competition between on-platform and off-platform purchases. Without direct consumers, the platform does not host third-party products and acts as a reseller.

[20] The platform product's existence constrains the superior seller's product price in asymmetric Bertrand equilibrium.

and sells at the category reservation price to (exogenous) inattentive buyers who pay no attention to the cheaper third-party seller. The platform's problem is to set the commission rate to induce third parties to enter unknown markets and either earn commission on them or else enter too and earn both commission and own-product profit (it is assumed that the platform cannot change the commission after entry tells it market strength, else it would monopolize the market, but then third-parties would not enter). Another difference from our setup is their price timing (in the hybrid mode): first, the third-party seller sets its price and then the platform chooses its price. Due to this timing, they obtain limit pricing by the third-parties setting their price just high enough to induce the platform's product to just take the inattentive consumers rather than undercutting and taking all. Given their different assumptions, they come up with a different conclusion to ours: banning the hybrid mode is likely to lower welfare. This is quite intricate because of the various different effects of a ban on seller fees and on the range of products that are offered to consumers. The answer depends on the parameters: how attractive is the marketplace compared to reselling (how valuable is information acquisition and the level of capacity constraints) as well as the fraction of inattentive consumers.

Etro (2020) analyzes the incentives of a platform to sell a product as a private label or first-party reseller, or else host third-party sellers. In each (independent) market, the product is homogeneous up to a proportional demand shift, so the platform makes a mutually exclusive choice and is never hybrid. Etro (2020) compares the platform's incentives to the socially optimal choice conditional on platform pricing. The market architecture we have in mind for our model is for slightly broader product categories (say "river shoes" or "Bluetooth portable speakers") than Etro (2020) who looks at unique products (like Toy Story 4) on the Amazon platform (which have Amazon Standard Identification Numbers, or ASINs). Nonetheless, Etro (2020) assumes demand is independent for products with different ASINs, although, even within an ASIN, one observes true hybrid selling of an Amazon product alongside multiple third-party sellers. Etro (2020) parameterizes a platform cost advantage in logistics over third-party sellers but a demand disadvantage. Etro (2020) shows that, in general, the private and social incentives are not aligned, but they coincide for specific demand functions, for which consumer surplus is proportional to profit (e.g., linear demand, isoelastic demand, and log-linear demand). Due to this property, the option maximizing profit is also the one maximizing consumer surplus, and so too total surplus. Etro (2020) extends these alignment results to when the platform also internalizes consumer participation, as we do in our two-sided market analysis. This Etro (2020) does through a device similar to ours, where consumers have heterogeneous outside options analogous to our cost of joining the platform.[21] Again, from the property that consumer surplus is proportional to profit for Etro (2020)'s key demand specifications, the profit-maximizing choice aligns with what is good for consumers, and so is the best instrument for encouraging participation too. As expected, the platform should sell those products where its logistics cost advantage is large enough relative to the demand advantage of third parties. These results fully accord with our finding that the reseller mode emerges for large enough advantage of the platform product and a pure platform is chosen when it is too small. Our main interest though is in the fully-fledged hybrid model that flourishes between these extreme regimes.

In short, the previous papers deliver important insights about when and why platforms prefer hosting to reselling, but the solution is "bang-bang", that is, either one or the other. What they cannot address, because of product homogeneity, is an equilibrium involving a true hybrid platform with the trade-offs that an active hybrid faces: how much revenue to extract from the two sources that compete simultaneously.

Zennyo (2020) focuses on the incentives of a hybrid platform to bias product search (in a simultaneous search setup) towards its own good. Like us, Zennyo (2020) allows for free entry of differentiated sellers and free entry of consumers, but, different from our setup, the number of sellers entering (variety) does not affect consumer surplus due to consumers randomly sampling a fixed number of products. Thus, there is no variety effect of platform entry on equilibrium commission, prices, and consumer welfare when fewer sellers enter the platform. Zennyo (2020) gets the interesting result that welfare rises when there is a biased search (so the platform's product is always considered) because, in the hybrid (or "encroachment") mode, commissions are lower to attract more consumers to the platform and to the larger base of the platform's own product.[22]

[21] Etro (2020)'s assumption of a uniform distribution does not seem instrumental to the results.

[22] Jiang et al. (2011) analyze the threat of entry by the platform into the third-party seller market and its effect on sellers' incentives to exert effort. Zhu and Liu (2018) provide empirical evidence that Amazon is more likely to enter successful product markets (with high demand and high prices).

3.5 Policy Implications

Various accusations of potentially anti-competitive behavior have been levelled against hybrid platforms. Courts may enact rules to proscribe some of these. Our key finding is that the hybrid business model may itself be intrinsically anti-competitive because a pure platform mode may benefit consumers by increasing the variety of fringe products on the platform and lowering their consumer prices (Corollary 3). Even within the hybrid mode there are various ways the platform may want to distort allocations. One criticism is that the platform may free-ride on efforts by third-party sellers to discover profitable market niches and then muscle in on them with its own product. Hervas-Drane and Shelegia (2021) address this in a well-formulated model with taste uncertainty. Platforms have also been criticized for "steering" consumers toward their own products by recommending them in ratings or web page placement (e.g., Amazon Choice or the earlier Buy Box), and so giving them unfair prominence and promoting them earlier in consumer search (The US House Majority Report, 2020; European Commission, 2020).

Each of these distortions might require a dedicated micro model to investigate in detail. But, in broad brush-strokes we can capture the gist at a simple but informative level by asking two questions sequentially. First, we ask if the hybrid platform has any incentive to actually denigrate third-party products. This is (quite) clearly not the case for a pure platform which obtains all its wherewithal from fringe sellers it hosts. But, in hybrid mode, fringe sellers are direct competitors as well as a revenue source, so it is not *a priori* obvious that the platform might not want to disadvantage them in the online marketplace. As we show, this is not the case, suggesting that the platform would like to improve the quality of fringe products.

Second, we know (from the analysis of Section 3.2) that the platform benefits from improvements in its own allure and, as we will show below, it suffers if fringe quality declines. Arguably, steering has elements of both aspects, at least insofar as demands rebalance toward the platform's product. For example, manipulating product rankings might make it appear to consumers that the platform product is more superior than it really is, and fringe products are more inferior than they are.[23] This leads us to ask whether and when a demand rebalance driven by pushing up the platform's quality and decreasing the fringe quality can raise profit. As we show,

[23] In what follows, we concentrate on the positive economics and side-step the normative economics of dealing with changing consumers' perceived quality valuations from their true values.

this is true as long as the platform's product commands a sufficiently large market share, that is, if its quality is already high enough. The following analysis also serves to highlight the tractability of the current framework in being able to address these questions.

3.5.1 Favoring own product against third party products

The maximized profit of the hybrid platform for given t is

$$\Pi_h(t) = \mu \frac{V_A(t)}{A(t)} + tp(t)\frac{A(t)-1}{A(t)},$$

where $V_A(t) = M \exp\left(\frac{x_A - m_A(t)}{\mu}\right)$, $m_A(t) = \mu + t\left(\frac{c}{1-t} + \mu\right)$, $p(t) = \frac{c}{1-t} + \mu$, and the fringe zero-profit relation $A(t) = \frac{\mu(1-t)}{K}V(t)$. In the profit expression, the only term that depends directly on the quality of the platform's product is $V_A(t)$ and the only term that depends directly on the quality of a fringe product is the aggregate $A(t)$ via $V(t) = \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right)$.

We first analyze whether the hybrid platform has an incentive to lower the quality of third-party (fringe) products, v. From the envelope theorem, the change in the platform's profit from raising v at equilibrium prices and fees is

$$\frac{d\Pi_h(t_h^*)}{dv} = \frac{t_h^* p(t_h^*) - \mu V_A(t_h^*)}{\mu A(t_h^*)}.$$

The platform prefers to have a lower quality fringe product if, and only if, its equilibrium gain from selling its own product is higher than selling one unit of a fringe product: $\mu V_A(t_h^*) > t_h^* p(t_h^*)$. From the optimal fee equation in (19), given $(tp(t))' > 0$ and $A'(t) < 0$, we must have $t_h^* p(t_h^*) > \mu V_A(t_h^*)$. This condition is interpreted below. Furthermore, the first term in (19) vanishes as $t \to \hat{t}_h$, so that $tp(t)$ approaches $\mu V_A(t)$ arbitrarily closely. This implies that there exists $\Delta > 0$ such that $(1+\Delta)\,\mu V_A(t) > tp(t)$. Because $t_h^* \to \hat{t}_h$ for $x_A \to \tilde{x}_A$, then for any $\Delta > 0$ we have:

Lemma 5 *Within the hybrid regime we have $t_h^* p\left(t_h^*\right) > \mu V_A\left(t_h^*\right)$. For any $\Delta > 0$ we have $(1+\Delta)\,\mu V_A\left(t_h^*\right) > t_h^* p(t_h^*)$ for x_A close enough to $\tilde{x}_A$.*

We next ask whether the platform might want to raise the perceived quality of its own product at the cost of lowering the perceived quality of a fringe product, which constitutes a short-hand version of describing the effects of steering consumers toward its own product and away from others via manipulating page placement or reviews. This type of steering can be studied in our framework by looking at how the equilibrium profit of the hybrid platform changes when it raises its quality, say by $\Delta > 0$, at the expense of lowering the quality of a fringe product by one unit. Using the envelope theorem, the consequent change in the platform's equilibrium profit is

$$\Delta \frac{d\Pi_h^*}{dv_A} - \frac{d\Pi_h^*}{dv} = \left[\Delta \frac{V_A\left(t_h^*\right)}{A\left(t_h^*\right)} - \frac{t_h^* p\left(t_h^*\right) - \mu V_A\left(t_h^*\right)}{\mu A\left(t_h^*\right)} \right].$$

Thus, the platform benefits from this change if, and only if, $(1+\Delta)\, \mu V_A(t_h^*) > t_h^* p(t_h^*)$. As shown in Lemma 5 this is true for any $\Delta > 0$ when the platform's product has a sufficiently high advantage, that is, if $x_A = v_A - c_A$ is sufficiently high. In this case, the platform has the incentive to raise the perceived quality of its own product even at the cost of lowering the perceived quality of fringe products. The larger Δ is (i.e., the larger the own-quality benefit at the expense of the others), the smaller the platform's market share needs to be for this to be worthwhile. The following proposition summarizes the hybrid platform's incentives to favor its own product at the cost of lowering the quality of third-party products:

Proposition 5 *The hybrid platform prefers strong third-party products. It has incentives to steer consumers toward its own product if it bears a sufficiently high advantage (high quality, low cost).*

3.5.2 Taxing dominant platforms

In August 2019, French government introduced a 3 percent tax on the marketplace revenue of Amazon from purchases on Amazon's French website (Amazon.fr). Starting on October 1, 2019, Amazon has raised the fee that it collects from third-party product sales on Amazon.fr.[24] In our framework we can study the effect of imposing different forms of taxes on the platform's revenues. If the government imposes a percentage tax on sales revenues of a pure marketplace, this will have no impact on the fees set by the platform unless the platform faces a cost for every third-party transaction. To see this, we can simply consider the equilibrium profit of a pure marketplace

[24]See Forbes, August 19, 2019 and Tax Foundation, August 6, 2019.

from (10) and assume a percentage tax of ω on third-party sales revenues, so

$$\Pi^{tax} = (1-\omega)tp(t)\frac{(A\,(t)-1)}{A\,(t)}.$$

Observe that the tax on the platform revenue from third-party sales will reduce the profit by a fraction and therefore does not affect the equilibrium commission.

Recall that, in our model, the platform has zero marginal cost of processing third-party sales. If the pure marketplace platform has a positive marginal cost, say γ, its profit after tax is

$$\Pi^{tax}(t) = ((1-\omega)tp(t)-\gamma)\,\frac{(A\,(t)-1)}{A\,(t)}.$$

Then, the platform's optimal fee is given by the optimality condition

$$\frac{d\Pi^{tax}}{dt} = \frac{1-\omega}{A(t)}\left[(tp(t))'\,(A\,(t)-1) + \left(tp(t) - \frac{\gamma}{1-\omega}\right)\frac{A'\,(t)}{A\,(t)}\right] = 0.$$

It is then straightforward to show that the equilibrium fee increases in the tax $\frac{dt^*}{d\omega} > 0$ as the tax increases the perceived marginal cost of the platform, $\frac{\gamma}{1-\omega}$. Moreover, the pass-through of the tax increases in the elasticity of the aggregate:

$$\frac{dt^*}{d\omega} = \frac{\frac{\gamma}{(1-\omega)^2}\frac{A'(t)}{A(t)}}{\frac{d^2\Pi^{tax}}{dt^2}} > 0,$$

as $\frac{d^2\Pi^{tax}}{dt^2} < 0$ by the second-order condition (assuming quasi-concavity of the platform profit) and $A'\,(t) < 0$.

Now consider a percentage tax of ω on third-party sales revenue of the hybrid platform in our framework, so the hybrid platform's profit becomes

$$\Pi_h^{tax}(t) = m_A(t)\frac{V_A(t)}{A(t)} + (1-\omega)tp(t)\frac{A\,(t) - V_A(t) - 1}{A(t)},$$

where $V_A(t) = M\exp\left(\frac{x_A - m_A(t)}{\mu}\right)$. As before, the fringe sellers' optimal pricing gives $p(t) = \frac{c}{1-t} + \mu$ and the free-entry condition gives $A(t) = \frac{\mu(1-t)}{K}V(t)$ since the tax does not affect the pricing of fringe sellers and the zero-profit condition given the fee set by the platform. It is then straightforward to show that the hybrid platform's optimal price reflects the tax by lowering the opportunity cost of selling its own product, $(1-\omega)tp(t)$, and lowering revenues from third-party sales. As a result, the percentage sales tax on third-party sales increases the gains from reseller sales and lowers the gains from third-party sales:

$$\Pi_h^{tax}(t) = (m_A(t) - (1-\omega)tp(t))\frac{V_A(t)}{A(t)} + (1-\omega)tp(t)\frac{A\,(t)-1}{A(t)}.$$

This leads to a higher fee for third-party sellers $\left(\frac{dt_h^*}{d\omega} > 0\right)$. The tax, therefore, lowers consumer surplus, $CS(t) = In\,(A(t))$, given that $A(t)$ is decreasing. Thus, our framework can explain why Amazon has reacted to the French tax by raising the fees on third-party sales in the French market and predicts that this tax will lower consumer surplus in the French market.

Suppose now, instead, that a percentage tax of ω were imposed only on own-product sales of the hybrid platform, so the platform's profit becomes

$$\Pi_h^{tax}(t) = (1-\omega)m_A(t)\frac{V_A(t)}{A(t)} + tp(t)\frac{A\,(t) - V_A(t) - 1}{A(t)},$$

where $V_A(t)$, $m_A(t)$, $p(t)$, and $A(t)$ are the same as before. The tax now lowers the margin from the sales of the own product:

$$\Pi_h^{tax}(t) = \left[(1-\omega)m_A(t) - tp(t)\right]\frac{V_A(t)}{A(t)} + tp(t)\frac{A\,(t)-1}{A(t)},$$

and so leads to a lower fee for third-party sellers $\left(\frac{dt_h^*}{d\omega} < 0\right)$. The following proposition summarizes these implications for tax policy:

Proposition 6

- *A percentage tax on third-party sales revenue of a pure marketplace is a pure profit tax on the platform and so has no effect on seller fees (if the platform incurs zero transaction cost of third-party sales).*
- *A percentage tax on third-party sales revenue of a hybrid platform increases the equilibrium seller fee and so lowers consumer surplus.*
- *A percentage tax over own-product revenue of a hybrid platform decreases the equilibrium seller fee and so increases consumer surplus.*
- *If the same percentage tax is imposed on both revenue sources of the hybrid platform, it is again a pure profit tax (no effect on seller fees).*

4 Two-sided market

In the previous analysis we assumed that all consumers visit the platform, so there were no network effects from sellers to consumers. Now we bring in a consumer participation margin by introducing consumer heterogeneity in consumers' (fixed) costs, s, to discover their matches. Costs s are assumed to be distributed with a cumulative distribution function, $F(\cdot)$, over interval $[0, \overline{s}]$. Now only those consumers with a cost less than the expected consumer surplus from visiting will visit the platform. These consumers who visit the platform may leave without buying, and so ex-post the consumer surplus is a random variable.

The expected consumer surplus from visiting the platform is given by (25), that is, $CS = InA(t)$. The fraction of consumers visiting the platform is then

$$\Pr\left[s \leq \ln A(t)\right] = F\left(\ln A(t)\right). \tag{29}$$

We now have two-sided positive network externalities between consumers and firms. Having more sellers on board raises the expected consumer surplus from visiting the platform, and thus attracts more buyers to the platform. Having more buyers increases the expected variable profits from selling on the platform,and thus attracts more sellers to the platform as more sellers will be able to cover the entry cost K.

When choosing whether to visit the platform, consumers do not internalize the impact of their participation decision on seller entry. Similarly, when deciding whether to enter the platform, sellers do not internalize their positive impact on buyers' participation decisions. This coordination failure between buyers and sellers may lead to failure to launch the platform. It is well-known from the literature that this equilibrium with no buyers and no sellers can co-exist with other equilibria with positive participation on both sides (Caillaud and Jullien, 2003). Following most of the literature, we will ignore the zero-zero equilibrium. We instead concentrate on showing the existence and uniqueness of a stable equilibrium with positive participation on both sides. We will show below when the platform can induce a positive participation equilibrium via its choice of the fee. Due to participation externalities, the market becomes two-sided and the platform needs to balance demand of buyers and demand of sellers when choosing its fee. Besides, sellers do not internalize the consumer participation when setting their price, so they will free-ride on the consumer participation margin by charging too high prices from the view point of the platform.

4.1 Participation decisions of buyers and sellers

We will now characterize the subgame equilibrium of buyer and seller participation given the platform's fee (and price of its own product in the hybrid platform case). The subgame analysis given t is the same whether the platform is a pure marketplace or a hybrid platform. This is for the same reason as in the one-sided market: given t the equilibrium price of sellers is $p(t) = \frac{c}{1-t} + \mu$, and the equilibrium aggregate is fixed by the free-entry condition of sellers. Different from the one-sided market analysis, the equilibrium aggregate is now determined by the consumer participation in conjunction with seller participation, and the participation levels depend upon each other.

We refer to F as the consumer participation, since it is the fraction of consumers visiting the platform. We use $\tilde{A}$ to measure the seller participation in the two-sided market analysis since the aggregate $\tilde{A}(t)$ will pin down the number of sellers entering the platform both for a pure marketplace (by plugging $\tilde{A}(t)$ into the LHS of (9)) and also for a hybrid platform (by plugging $\tilde{A}(t)$ into the LHS of (15)). Given the platform's fee we can think of participation "reaction" functions and determine the corresponding stability conditions to select equilibria.

In Figure 7 we represent F on the vertical axis and $\tilde{A}$ on the horizontal axis. On the buyer side, the mass of joining consumers is:

$$F = F\left(\ln \tilde{A}\right), \tag{30}$$

which is increasing and caps out at 1 when $\tilde{A} = \exp(\overline{s})$. When F is log-concave we have F concave in $\tilde{A}$ since then f/F is decreasing, which is the same as lnF concave. The consumer participation condition is represented by the black line, (B), in Figure 7 for the case of log-concave F.[25] Note that line (B) is independent of t in the participation reaction space, by (30).

On the seller side, we have $\tilde{A}$ as a function of F. This is given by the zero-profit condition for entrants, just mildly extending the earlier condition to account for the

[25] Note that an exponential visit cost distribution will deliver a linear function F in $\tilde{A}$ until it reaches its upper bound and then it is flat. A log-convex F distribution delivers a convex function F in $\tilde{A}$ and hits the upper bound at some point. If lnF is convex, then the stable equilibrium solution has full consumer coverage, which is perhaps less interesting, since we effectively revert to the analysis of the one-sided market. We illustrate in a graph the subgame equilibrium of log-convex F in Appendix C

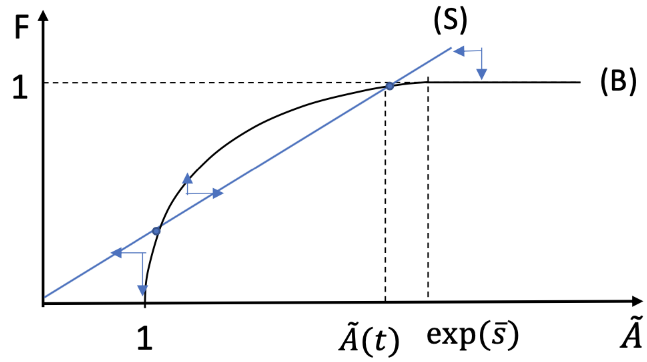

Figure 7: Subgame equilibrium participation condition of buyers (B) and sellers (S) at a given platform commission, t.

buyers' participation margin:

$$K = F\mu\,(1-t)\,\frac{V(t)}{\tilde{A}},$$

where $V(t) = \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right)$ corresponds to the equilibrium share of fringe firms' total demand over the aggregate. In the above, F is consumer participation demand, $\mu\,(1-t)$ is per-seller markup, and $V(t)/\tilde{A}$ is equilibrium demand per seller. Hence, $\tilde{A}$ is linear (through the origin) in F. To commensurate with how we are drawing the axes, we write the seller participation condition as

$$F = \frac{K}{\mu\,(1-t)\,V(t)}\tilde{A}, \tag{31}$$

which is represented by the blue line, (S), in Figure 7.

As long as F does not switch log-concavity sign, there are generically either 2 interior solutions or none (there is also $(0,0)$ point as an equilibrium, where the platform fails to launch). The allocation with the lower number of sellers and buyers is not stable, since, starting from that point, if we increase the number of sellers slightly the equilibrium will move to the allocation with the larger number of sellers and buyers. Buyers will be better-off with a higher number of sellers (buyer participation increases) and more sellers enter the platform. We therefore select the large solution, $\tilde{A}(t)$ in the figure, which is Pareto dominant, that is, better for buyers and the

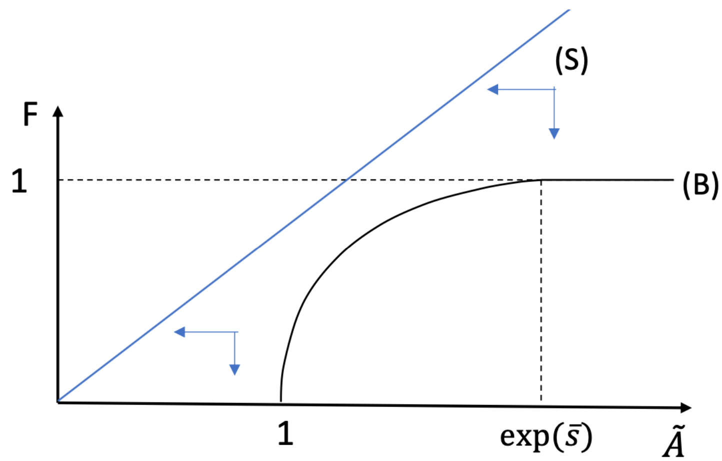

Figure 8: Subgame equilibrium participation condition of buyers (B) and sellers (S) when the platform fails to launch at $t = 0$.

platform (sellers are indifferent as they make zero profit).

Notice that as t increases, $\mu\,(1-t)\,V(t)$ decreases, so that line (S) pivots up as t goes up. This leads to a lower aggregate $\tilde{A}(t)$ in equilibrium, so a lower buyer participation and a lower seller participation.

The $\tilde{A}$-ray (line (S)) for $t = 0$ is the lowest it can go keeping the platform viable. If this locus does not intersect the buyer locus (B), there is no equilibrium with positive numbers of sellers and buyers on board. This necessarily means failure to launch the platform. Figure 8 illustrates this case. Failure to launch can occur if buyer visit costs are too high. Suppose that visit costs s are distributed by $F_1(\cdot)$ in Figure 8. We can think of reducing buyer visit costs in sense of moving from a cdf $F_1(\cdot)$ to $F_2(\cdot)$, which is first-order stochastically dominated by $F_1(\cdot)$. Suppose that, when visit costs are distributed by $F_2(\cdot)$, the market first "appears" when the buyer participation condition (B) is tangent to the lowest $\tilde{A}$-ray. When it does, a mass of agents from both sides join the platform and platform participation goes from zero to a sizeable amount on both sides. Figure 9 illustrates this case.

This analysis illustrates that for a given commission level, the platform might fail to launch (the case in Figure 8) or just manage to launch with positive numbers of buyers and sellers on both sides (the case in Figure 9), or might lead to two intersection points between the buyers' participation condition and the sellers' participation condition, the case in Figure 7, where the larger participation point, $\tilde{A}(t)$, is the stable

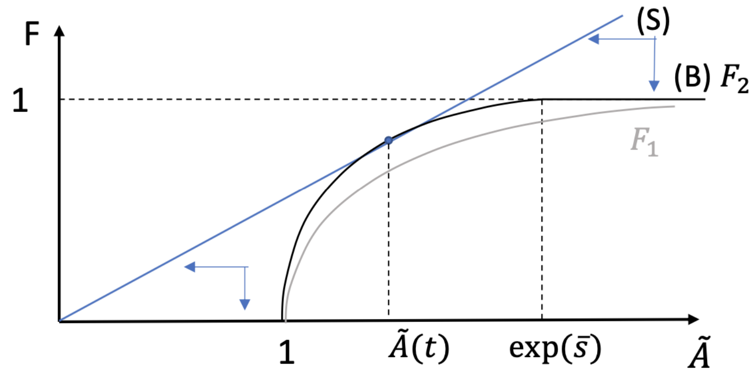

Figure 9: Subgame equilibrium participation condition of buyers (B) and sellers (S) when the platform is just viable at $t = 0$.

subgame equilibrium of the buyer and seller participation conditions.

The equilibrium level of the aggregate, which we denote by $\tilde{A}(t)$, is a different function from the one in the case of one-sided market, (see (8)), since now $\tilde{A}(t)$ is implicitly given by the two participation conditions:

$$\tilde{A}(t) = F(In\tilde{A}(t))\frac{\mu\,(1-t)}{K}V(t), \tag{32}$$

where we plugged in the participation condition of buyers, (30), into the seller one (31).

We assume that visit costs are not too high, so that the platform can successfully launch by charging a positive seller fee, $t > 0$. To guarantee that we need the slope of the $\tilde{A}$-ray (S) at $t = 0$, $\frac{K}{\mu V(0)}$, to be higher than the slope of the buyer locus (B), $\frac{f\left(In\left(\tilde{A}(0)\right)\right)}{\tilde{A}(0)}$, at the larger intersection point between these lines,

$$\frac{K}{\mu\exp\left(\frac{v-c-\mu}{\mu}\right)} > \frac{f\left(In\left(\tilde{A}(0)\right)\right)}{\tilde{A}(0)} \tag{33}$$

which is equivalent to $\frac{F\left(In\tilde{A}(0)\right)}{f\left(In\tilde{A}(0)\right)} > 1$ using (32) at $t = 0$, given that (32) gives us the equilibrium aggregate for a given t, and $\tilde{A}(0)$ is the equilibrium aggregate at $t = 0$. Observe that the latter inequality holds if $In\left(\tilde{A}(0)\right) > 0$ or if there exists $\tilde{A}(0) > 1$, which is guaranteed if the seller fixed cost is sufficiently low (using (32) at $t = 0$):

Assumption 4 $K < F\left(In\left(\tilde{A}(0)\right)\right)\mu \exp\left(\frac{v-c-\mu}{\mu}\right)$.

4.2 Pure marketplace optimal commission t

Next we analyze the platform's optimal commission if it is a pure marketplace, anticipating the subgame equilibrium characterized previously. The platform's profit is[26]

$$\Pi^{2sm}(t) = tp(t)\frac{\tilde{A}(t) - 1}{\tilde{A}(t)}F(In(\tilde{A}(t))). \tag{34}$$

The first-order condition for an optimal fee is

$$\frac{d\Pi^{2sm}(t)}{dt} = \frac{1}{\tilde{A}(t)}\left((tp(t))'(\tilde{A}(t) - 1)F(\cdot) + tp(t)\frac{\tilde{A}'(t)}{\tilde{A}(t)}\left[F(\cdot) + (\tilde{A}(t) - 1)f(\cdot)\right]\right) = 0. \tag{35}$$

Note that at the optimal fee there are some fringe firms entering the platform, that is, $t^*_{2sm} < \hat{t}_{2sm}$ where $\tilde{A}^{-1}(1) = \hat{t}_{2sm}$ is the fee above which the fringe disappears. Different from the one-sided market, here, consumer participation is elastic. Therefore, any $t \geq \hat{t}_{2sm}$ would lead to zero demand and would never be selected by the platform in equilibrium; at $t \geq \hat{t}_{2sm}$ there will be zero participation by consumers and $F(In\tilde{A}(t)) = F(Ine^0) = F(0) = 0$ given that we assume visit costs s are non-negative.

Using the definitions of the elasticity of fee revenue, $\epsilon_{tp(t)} = t\frac{(tp(t))'}{tp(t)}$, and the elasticity of the aggregate, $\epsilon_{\tilde{A}(t)} = t\frac{\tilde{A}'(t)}{\tilde{A}(t)}$, the latter condition can be rewritten as

$$\frac{d\Pi^{2sm}(t)}{dt} = \frac{F(\cdot)p(t)}{\tilde{A}(t)}\left[(\tilde{A}(t) - 1)\epsilon_{tp(t)} + \epsilon_{\tilde{A}(t)} + (\tilde{A}(t) - 1)\frac{f(\cdot)}{F(\cdot)}\epsilon_{\tilde{A}(t)}\right] = 0. \tag{36}$$

The first term inside the brackets, $(\tilde{A}(t) - 1)\epsilon_{tp(t)}$, reflects the platform's gains in fee revenue over its base when it raises the fee. The second term, $\epsilon_{\tilde{A}(t)}$, is negative and it reflects the loss at the margin of fringe firm entry when the fee increases. The third, $(\tilde{A}(t) - 1)\frac{f(\cdot)}{F(\cdot)}\epsilon_{\tilde{A}(t)}$, is also negative and reflects losses due to fewer consumers coming to the platform when there is less variety of products on the platform. The first two effects were already present in the one-sided market (see (11)). The third term is new

[26]If $F(\cdot)$ were exponential then $\frac{F(\ln A(t))}{A(t)}$ would be a constant, so we would have a simpler-looking problem than the case without visit cost heterogeneity.

and specific to the two-sided market: more fringe products attract more consumers. The platform's optimal fee is at the point where the marginal gains from raising the fee are equal to the marginal losses:

$$\tilde{A}(t) = 1 - \frac{\epsilon_{\tilde{A}(t)}}{\epsilon_{tp(t)}} \left[1 + (\tilde{A}(t) - 1) \frac{f(\cdot)}{F(\cdot)} \right], \tag{37}$$

which is the fundamental platform pricing formula similar to the one we had without visit costs (see (12)).[27]

Proposition 7 *The platform sets a lower commission in the two-sided market than the one-sided market where all consumers visit the platform.*

Intuitively, in the two-sided market, the profit is lower due to some consumers not visiting the platform. The platform wants to staunch consumer leakage by bolstering platform attractiveness, which it does by lowering the seller fee to encourage more sellers to join the platform and hence encourage buyers.

An alternative characterization of the platform's optimal commission: Recall that the platform's profit is

$$\Pi^{2sm}(t) = tp(t) \frac{\tilde{A}(t) - 1}{\tilde{A}(t)} F\left(In\left(\tilde{A}(t) \right) \right), \tag{38}$$

where price of each fringe product is $p(t) = \frac{c}{1-t} + \mu$. To simplify, we define the total buyer demand for products on the platform as $G(t) \equiv \frac{\tilde{A}(t)-1}{\tilde{A}(t)} F\left(In\left(\tilde{A}(t) \right) \right)$. Observe that $G(t)$ incorporates both the participation margin, $F(\cdot)$, and the transaction margin, that is, demand per visitor on the platform: $\frac{\tilde{A}(t)-1}{\tilde{A}(t)}$. So, now the platform's profit becomes $\Pi^{2sm}(t) = tp(t)G(t)$. The equilibrium fee is given by the first-order condition, which implies the elasticity version of the equilibrium condition:

$$\begin{aligned} \frac{d(tp(t))/dt}{tp(t)} &= -\frac{G'(t)}{G(t)} \\ \epsilon_{tp(t)} &= -\epsilon_G, \end{aligned}$$

[27] Observe that if we set the second term in the brackets to zero, the pricing formula has the same expression as the one-sided market.

where the derivative of revenue per unit is $\frac{d(tp(t))}{dt} = \frac{c}{(1-t)^2} + \mu$, the elasticity of per-unit revenue is $\epsilon_{tp(t)} = \frac{\left(\frac{c}{(1-t)^2}+\mu\right)}{\left(\frac{c}{(1-t)}+\mu\right)} > 1$, and we define $\epsilon_G = t\frac{G'(t)}{G(t)}$ as the sub-game equilibrium elasticity of the total buyer demand.

Intuitively, the platform sets its commission at the level where the magnitude of the total buyer demand elasticity, $-\epsilon_G$, is equal to the elasticity of the per-unit revenue the platform generates from sellers of these products, $\epsilon_{tp(t)}$. This condition is similar to the fundamental optimal pricing condition of two-sided markets equating the demand elasticities of the two sides (Armstrong, 2006; Rochet and Tirole, 2006). For instance, in a pure-transaction model of payment card markets, the optimal fee allocation between the two sides equates the elasticity of buyer demand to the elasticity of seller demand (Rochet and Tirole, 2006, p.654). In advertising-financed business models of two-sided markets, the optimal pricing sets the advertising revenue elasticity (with respect to advertising level) equal to the consumer leakage elasticity, that is, the nuisance cost times the elasticity of consumer demand with respect to price (Anderson and Coate, 2005; Anderson and Jullien, 2015). Here, similar to previous models of two-sided markets with positive cross-group externalities, a higher seller fee (commission) lowers the number of sellers entering the platform, which in turn makes it less attractive for consumers to visit the platform.

In the classical models of two-sided markets, per-participant cross-side externalities are assumed to be exogenous.[28] Different from the previous work, here, unit participation externality between sellers and buyers is endogenous and determined by the platform's choice of seller commission (and its own-product price in the hybrid mode). When the platform raises its commission, this leads to lower seller gains from one more buyer (i.e., a lower unit externality from buyers to sellers). A higher seller fee leads to higher prices for consumers, so lowers consumers' benefits from one more seller (lowers unit externality from sellers to buyers). For instance, advertising-financed business models of two-sided markets have nuisance cost (exogenously given)

[28]For example, see Anderson and Coate (2005), Armstrong (2006), Rochet and Tirole (2006). Caillaud and Jullien (2003) allow for unit transaction utility of each side to depend on the sum of transaction fees charged by the matchmaker, however, since they assume ex-ante homogenous agents, there is inelastic participation demand on each side and so in equilibrium all users join only one matchmaker (the dominant firm), the total transaction fee is set at the value of trade (observed by the platform) and platforms compete in fixed membership prices. One notable exception is Edelman and Wright (2015), who consider endogenous consumer participation and variable fees to consumers and sellers, but do not allow for elastic seller participation and focus on understanding the implications of price coherence restrictions imposed by a platform on its sellers.

from advertisers to buyers, and this cost is multiplied by the semi-elasticity of the consumer demand with respect to price. Here, consumers benefit from one more seller (nuisance cost is negative) and this benefit depends on the seller fee, which affects the price consumers pay for a product and the number of sellers (variety) on the platform. By controlling its seller commission, the platform could affect the amount of buyers and sellers joining the platform. Ultimately, the consumer utility from participating depends on the aggregate, $\tilde{A}(t)$, which captures the effect of the seller fee on the price and the number of fringe sellers coming to the platform at a given seller fee. When choosing its optimal seller fee, the platform balances the consumer demand elasticity with respect to the seller fee against the elasticity of its per-unit revenue from sellers. The monetary transfer (product price) between buyers and sellers does not neutralize these two-sided externalities due to the elastic participation demands of buyers and sellers. The platform balances these externalities via its optimal commission choice.

4.3 Hybrid platform's optimal commission t

Now suppose that the platform also sells its own product. The subgame analysis given t is the same as the pure marketplace. The aggregate is the same as the pure marketplace, (32), which is given by the equilibrium participation condition of buyers, (30), and sellers, (31). The only difference is at the first stage when the platform chooses its product's price in addition to the seller fee.

The platform's problem is modified to capture the consumer participation margin, so the platform's profit (own brand profit plus the fees from third-party sellers) becomes

$$\Pi_h^{2sm}(t) = \frac{1}{\tilde{A}(t)} \left[m_A(t) V_A(t) + t \left(\frac{c}{1-t} + \mu \right) \left(\tilde{A}(t) - V_A(t) - 1 \right) \right] F \left(\ln \tilde{A}(t) \right) \quad (39)$$

where $m_A(t) = p_A(t) - c_A$ is the platform product's markup and $F(.)$ is the fraction of consumers that visit the platform. Thus, the platform internalizes the consumer participation when choosing the seller fee and own-product price.

The platform's problem can be addressed as before. In particular, first note that for given t, the earlier "composition" analysis holds and the choice of $m_A(t)$ is the same as (18) given that the aggregate does not depend on $m_A(t)$. Using (32) we re-write the platform's profit as

$$\Pi_h^{2sm}(t) = \left[m_A V_A(t) + t\left(\frac{c}{(1-t)} + \mu\right)\left(\tilde{A}(t) - V_A(t) - 1\right)\right]\frac{K}{\mu\,(1-t)\,V(t)}. \quad (40)$$

Proposition 8 *In the case of two-sided markets (elastic consumer participation), the hybrid platform's optimal commission for third party products increases in the advantage of the platform's product.*

The proof follows the same lines as before, but there are further complications because the aggregate is now determined implicitly by the two participation conditions, and thus, the way the platform fee affects the aggregate requires taking into account how consumer participation reacts to changes in the number of sellers.

Now the consumer welfare is the sum of expected surplus of those consumers visiting the platform minus their visit costs:

$$CS(t) = \int_0^{\ln \tilde{A}(t)} \left(\ln \tilde{A}(t) - s\right) f(s)ds, \quad (41)$$

which is an increasing function of the aggregate, $\tilde{A}(t)$, and so we obtain our key result for consumer welfare:

Corollary 6 *Banning the hybrid platform mode in the market benefits consumers if the platform switches to a pure marketplace due to the ban.*

One can analyze the platform's optimal business model following similar steps to the ones we did in the one-sided market case. In particular, when $x_A \to -\infty$, the hybrid platform profit, (40), approaches close to the pure marketplace profit (38). We would then obtain similar qualitative results to Proposition 3. Similar to Lemma 3, one can prove the existence and uniqueness of a cutoff commission, $\hat{t}_h^{2sm}$, above which there is no fringe entry and show the following:

Lemma 6 *In any hybrid regime, the platform sets fee* $0 < t_h^{2sm*} < \hat{t}_h^{2sm}$ *if the platform product's advantage is low enough.*

We then obtain the qualitative results of Proposition 4 and show that policy implications of a ban of hybrid mode, Corollary 5, hold also for a two-sided platform.

5 Our methodological contribution and concluding remarks

We have brought together several modelling strands to study a novel market structure which delivers a new mechanism to drive our results on hybrid platform performance. The market structure incorporates a dominant firm facing monopolistically competitive fringe firms. The dominant firm is a direct participant in the market as well as collecting royalties (a "percentage seller fee") on fringe sales revenue. We also deliver a fully-fledged two-sided market structure when we allow for endogenous consumer participation via the device of heterogeneous participation costs.

The dominant firm and fringe paradigm originates with Forchheimer (1908), who considers an "incomplete monopolist" which faces the residual demand from a perfectly competitive fringe through its pricing. Different from that framework, here the dominant firm (the hybrid platform) has two revenue sources and modulates entry of fringe sellers (third-party sellers) through its fee choice as well. We also replace perfect competition with monopolistic competition in order to capture product differentiation and benefits from variety, so consumers with heterogeneous tastes are motivated to join the platform to choose the most suitable option there (and can walk away if nothing is suitable). The monopolistic competition formulation captures the idea that there are many small competing sellers touting their wares on platforms, and is more manageable than oligopoly, for which it is a limit case. Our model embodies discrete choice too, so individuals buy only one unit. This well reflects shopping behavior in online (as well as offline) markets for consumer goods. Specifically, we deploy a logit model, for it delivers intuitive and simple pricing properties under monopolistic competition (analyzed in Anderson et al. (2020)) which allow us to move on to the higher levels of the market interaction in a tractable and intuitive way. The logit is also the foundation for much of the structural empirical industrial organization (IO) literature on the demand side (Reiss and Wolak, 2007; Ackerberg et al., 2007).

The demand for fringe sellers is one component of the demand on the platform. The other is the demand for the platform's product, which we endow with mass in the demand system (via the parameter M) to reflect its substantial equilibrium presence, its prominence, and its strategic advantage. We are therefore engaging a particular mixed market structure.[29] We argue that the mixed oligopoly framework

[29] The seminal papers on this structure, Shimomura and Thisse (2012) and Parenti (2018), assume

well represents a "gatekeeper" trade platform, like Amazon, which is a large player and provides unique access to consumers for millions of small sellers.

The next key feature of our model is the strategic position of the platform in coordinating the actions of the fringe (and the consumers, once we get to the two-sided market analysis). We assume that the platform acts as the first mover. It sets both the seller fee and its own-product price before the fringe entry and pricing choices. Choosing its price thus can be viewed as reflecting a long-run reputational perspective and corresponds to a long-run Stackelberg leader assumption. This market assumption has been analyzed in Etro (2006).

The combination of the logit demand with fringe free entry allows us to access results from the theory of aggregative games with free entry (Anderson et al., 2020). The main simplifying property for our purpose is that the aggregate is tied down by the free-entry condition and it is a sufficient statistic for consumer surplus. In the context of the hybrid model, the aggregate itself is directly determined by the platform's choice of the seller fee. It is the conjunction of all these links that enables our clean and tractable conclusions.

We also point out that the pure platform version of our model forms a useful workhorse model for the literature on platforms involving buyer and seller interaction. Our two-sided market analysis captures that consumers are attracted by product variety and sellers by the consumer volume, although sellers suffer business-sharing with other sellers (so there are own-side negative externalities present too).[30] Different from the literature, we focus on analyzing the impact of own-side negative externalities on a monopoly platform's optimal business model choice, its equilibrium seller fee, resulting variety, and prices consumers face. Another important difference from the literature is that we allow for endogenous cross-group transaction externalities between buyers and sellers by allowing the platform to charge seller fees over transactions.

a representative consumer demand without elaborating on possible micro-foundations of individual discrete choices. Helpman and Niswonger (2020) use a constant elasticity (CES) demand model in such a mixed market structure. The CES component has been the mainstay in international trade literature since Melitz (2003) and is the counterpart to the logit in IO. Indeed, the two models can be viewed as stemming from a common root in discrete choice (Anderson et al., 1992). They both have the Independence of Irrelevant Alternatives (IIA) property, and both beget aggregative oligopoly games.

[30] Belleflamme and Peitz (2018), Belleflamme and Peitz (2019), Halaburda et al. (2018), and Karle et al. (2020) investigate the impact of own-side negative externalities on competition between platforms.

Our main result on the performance short-comings of the hybrid business model stems from showing that better quality (or lower cost) of the platform's product implies that equilibrium consumer welfare surprisingly suffers, and hence performance is better when the platform's product has a smaller market share. This counter-intuitive consumer result is shown by arguing that the aggregate falls (the aggregate being determined from the free-entry condition for the fringe), and this happens when the platform raises the equilibrium seller fee in response to higher quality. So, it remains to argue why this happens. The mechanism is that better quality motivates the platform to seek a higher market share for its product (which comes at the expense of lower fringe entry for any given aggregate). With more earned on its own product, it rebalances upward the profit earned on each fringe product. It does this by raising the seller fee it charges on third-party sellers' revenues. Thus, the equilibrium seller fee goes up, the aggregate goes down, and consumer surplus falls.

Finally, our tractable framework enables us to derive antitrust and tax policy implications. We show that banning the hybrid mode benefits consumers (by increasing variety and lowering prices) if the platform becomes a pure marketplace after the ban, but lowers consumer surplus if the platform becomes a pure reseller after the ban. The latter case arises when the platform product's advantage lies in the higher end of the hybrid business model region. We also illustrate that the hybrid platform prefers to steer consumers toward its product (by increasing its product's perceived value) at the cost of lowering third-party products perceived value when the platform's product has sufficiently high advantage compared to third-party products. Taxing the marketplace revenue of a hybrid platform harms consumers, whereas taxing the platform's revenue from its own-product sales increases consumer surplus.

A Proofs

Proof of Lemma 1: Recall that $A(t)$ is given by the zero-profit condition (8), so the semi-elasticity of the aggregate is

$$\frac{A'(t)}{A(t)} = -\frac{\frac{c}{(1-t)} + \mu}{\mu(1-t)} < 0. \tag{42}$$

Thus, the elasticity of the aggregate with respect to the fee $\epsilon_{A(t)} = t\frac{A'(t)}{A(t)}$ is negative. Moreover, we differentiate the platform's fee revenue with respect to the fee:

$$(tp(t))' = \frac{c}{(1-t)^2} + \mu > 0, \tag{43}$$

and show that a higher tax rate increases seller fee revenue per unit. We next obtain the semi-elasticity of the platform's fee revenue per unit as:

$$\frac{(tp(t))'}{tp(t)} = \frac{\frac{c}{(1-t)^2} + \mu}{t\frac{c}{(1-t)} + t\mu},$$

and derive the per-unit revenue elasticity as

$$\epsilon_{tp(t)} = \frac{\frac{c}{(1-t)^2} + \mu}{\frac{c}{(1-t)} + \mu} > 0. \tag{44}$$

Consider the limit cases. When $t \to 0$, the marginal profit is positive:

$$\begin{aligned}\left(\frac{d\Pi}{dt}\right)_{t\to 0} &= (tp(t))'_{t=0}\frac{(A(0)-1)}{A(0)} \\ &= (c+\mu)\frac{(A(0)-1)}{A(0)} > 0,\end{aligned}$$

where for the first equality we use the profit derivation in (11) and for the second we use the marginal fee revenue per unit, (43). The platform profit derivative is positive by Assumption 2, which implies that $A(0) > 1$. When $t \uparrow \hat{t}$, we have $A(\hat{t}) = 1$ by definition of $\hat{t}$ and using (11), so the marginal profit is negative $\left(\frac{d\Pi}{dt}\right)_{t\uparrow\hat{t}} = \hat{t}p\left(\hat{t}\right)A'\left(\hat{t}\right) < 0$ given that the aggregate is decreasing ($A'(t) < 0$). By continuity of the marginal profit in t there is at least one maximizer t^* between 0 and $\hat{t}$, such that the marginal profit is zero at t^*.

Proof of Lemma 3: By definition, $\hat{t}_h$ is a solution to $A\left(\hat{t}_h\right) = V_A\left(m_A\left(\hat{t}_h\right)\right) + 1$. We write it explicitly by substituting the equation for $A(t)$ (8) and $V_A(t) = M \exp\left(\frac{x_A - m_A(t)}{\mu}\right)$. Thus, $\hat{t}_h$ is a solution to

$$\frac{\mu(1-t)}{K} V(t) = M \exp\left(\frac{x_A - m_A(t)}{\mu}\right) + 1,$$

where $m_A(t) = \mu + t\left(\frac{c}{1-t} + \mu\right)$ is the equilibrium markup of the platform's product (18). Thus, the cutoff commission above which there is no fringe firm, $\hat{t}_h$, is a solution to

$$\frac{\mu(1-t)}{K} \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right) = M \exp\left(\frac{x_A - \mu - \frac{ct}{1-t} - \mu t}{\mu}\right) + 1,$$

which we rewrite as

$$\frac{\mu(1-t)}{K} = M \exp\left(\frac{x_A - x - \mu t}{\mu}\right) + \exp\left(\frac{-v + \frac{c}{1-t} + \mu}{\mu}\right),$$

where $x = v - c$.

To prove the uniqueness of $\hat{t}_h$, first observe that the LHS is greater than the RHS at $t = 0$ according to Assumption 3. Further, the RHS is greater than the LHS at $t = 1$, as the RHS $\to +\infty$ when $t \uparrow 1$ and the RHS goes to zero when $t \uparrow 1$. Lastly, both the LHS and the RHS are convex in t. Thus, we can conclude that a unique $\hat{t}_h \in (0, 1)$ exists.

Proof of Lemma 4 Consider the marginal profit of the platform (19) at extreme values of t:

$$\frac{d\Pi^h}{dt}|_{t\to 0} = (tp(t))'\,|_{t\to 0} \frac{A\,(0) - V_A\,(m_A\,(0)) - 1}{A\,(0)} - \frac{A'\,(0)}{(A\,(0))^2} \mu V_A\,(m_A\,(0))\,.$$

The first term is positive since the per-unit revenue is increasing in the fee: $(tp(t))' > 0$ given $p(t) = \frac{c}{1-t} + \mu$, and we have $A\,(0) - V_A\,(m_A\,(0)) - 1 > 0$ by Assumption 3. The second term is also positive given that the aggregate is decreasing, $A'(t) < 0$, see (8). Thus, the marginal profit of the hybrid platform is positive when $t \to 0$. This implies that the optimal fee is positive.

Now consider the marginal profit at $\hat{t}_h$ where $A\left(\hat{t}_h\right) = V_A\left(m_A\left(\hat{t}_h\right)\right) + 1$:

$$\frac{d\Pi^h}{dt}|_{t\to\hat{t}_h} = \left[\left(V_A\left(m_A\left(\hat{t}_h\right)\right) + 1\right)\hat{t}_h p\left(\hat{t}_h\right) - m_A\left(\hat{t}_h\right) V_A\left(m_A\left(\hat{t}_h\right)\right)\right] \frac{A'\left(\hat{t}_h\right)}{\left(A\left(\hat{t}_h\right)\right)^2},$$

which is negative if, and only if, $\hat{t}_h p(\hat{t}_h) > m_A\left(\hat{t}_h\right) \frac{V_A\left(m_A\left(\hat{t}_h\right)\right)}{V_A\left(m_A\left(\hat{t}_h\right)\right)+1} = m_A\left(\hat{t}_h\right) \frac{V_A\left(m_A\left(\hat{t}_h\right)\right)}{A\left(\hat{t}_h\right)}$, that is, per unit commission profit is above the profit on an in-house unit. This is the case at the bound, $\hat{t}_h$, if and only if

$$\hat{t}_h p(\hat{t}_h) > \mu M \exp\left(\frac{x_A - \mu - \hat{t}_h p(\hat{t}_h)}{\mu}\right)$$

where we used the optimality condition for the platform's markup: $m_A(t) = \mu + tp(t)$ (18). Then, using $p(t) = \frac{c}{1-t} + \mu$, the necessary and sufficient condition to have at least one fringe unit sold by the hybrid platform is:

$$x_A < \mu + \frac{c\hat{t}_h}{(1-\hat{t}^h)} + \hat{t}_h\mu + \mu In\left(\frac{\hat{t}_h}{M} + \frac{c\hat{t}_h}{\mu M(1-\hat{t}_h)}\right) \equiv \tilde{x}_A,$$

since otherwise the platform would never find it profitable to have a fringe product. Note also that $\tilde{x}_A$ is uniquely defined given that $\hat{t}_h$ is unique (by Lemma 3) and the RHS of the latter inequality is a continuous function of $\hat{t}_h$.

Proof of Corollary 1 We firstly show that given $A(t)$ (implicitly with positive fringe present), then the share of the platform's product, $V_A(t)/A(t)$, rises with t (keeping x_A constant). We then show that the equilibrium share of the platform's product, $V_A(t_h^*)/A(t_h^*)$, rises when x_A increases, given that t_h^* increases in x_A from Proposition 1.

Recall that $m_A(t) = t\left(\frac{c}{1-t} + \mu\right) + \mu$ from equation (18), and so

$$m_A'(t) = \frac{c}{(1-t)^2} + \mu.$$

Since $V_A(t) = M\exp\left(\frac{x_A - m_A(t)}{\mu}\right)$, then

$$V_A'(t) = -\frac{V_A(t)\, m_A'(t)}{\mu} < 0,$$

which gives us

$$\frac{V_A'(t)}{V_A(t)} = -\frac{\frac{c}{(1-t)^2} + \mu}{\mu} < 0$$

after replacing the equality for $m_A'(t)$ from above.

Moreover, from equation (42), we have

$$\frac{A'(t)}{A(t)} = -\frac{\left(\frac{c}{(1-t)} + \mu\right)}{\mu(1-t)} < 0.$$

So, we show that $\frac{d(V_A(t)/A(t))}{dt} > 0$ since

$$\frac{V_A'(t)}{V_A(t)} - \frac{A'(t)}{A(t)} = -\frac{c}{\mu(1-t)^2} - 1 + \frac{\left(\frac{c}{(1-t)} + \mu\right)}{\mu(1-t)} = -1 + \frac{1}{1-t} = \frac{t}{1-t} > 0.$$

We next show that a higher advantage of the platform's product increases the share of the platform's product at the equilibrium commission, t_h^*. We write

$$\frac{dV_A^*}{dx_A} = \frac{\partial V_A^*}{\partial x_A} + \frac{\partial V_A^*}{\partial m_A}\frac{\partial m_A^*}{\partial t}\frac{dt_h^*}{dx_A}$$

and notice that $\frac{\partial m_A^*}{\partial x_A} = 0$ and is therefore not included above.

So, the expression is

$$\frac{dV_A^*}{dx_A} = \frac{V_A^*}{\mu}\left(1 - m_A'(t_h^*)\frac{dt_h^*}{dx_A}\right),$$

and we want to (analogously to above) determine the sign of

$$\frac{dV_A^*}{dx_A}\frac{1}{V_A^*} - \frac{A'(t_h^*)}{A(t_h^*)}\frac{dt_h^*}{dx_A},$$

where we note that x_A has no direct effect on $A(t_h^*)$ given the result in Lemma 2.

So now we have to determine the sign of

$$\frac{1}{\mu} - \left(\frac{c}{\mu(1-t_h^*)^2} + 1\right)\frac{dt_h^*}{dx_A} + \frac{\left(\frac{c}{(1-t_h^*)} + \mu\right)}{\mu(1-t_h^*)}\frac{dt_h^*}{dx_A}.$$

Here we use the result from the first part of the proof that $V_A(t)/A(t)$ rises with t

(keeping x_A constant) and so the last two terms make the net positive given $\frac{dt_h^*}{dx_A} > 0$ by Proposition 1. We thereby prove the claim that the share of the platform's product, $\frac{V_A(t_h^*)}{A(t_h^*)}$, increases as the platform's product has a higher advantage (x_A goes up).

Proof of Proposition 7 The platform sets its optimal commission by maximizing its profit

$$\Pi^{2sm} = t\left(\frac{c}{1-t} + \mu\right)\frac{\tilde{A}\left(t\right) - 1}{\tilde{A}\left(t\right)}F\left(In\left(\tilde{A}(t)\right)\right),$$

subject to the free-entry condition (32):

$$\tilde{A}(t) = F\left(In\left(\tilde{A}(t)\right)\right)\frac{\mu\left(1-t\right)}{K}V(t).$$

After replacing the equality for $F\left(In\left(\tilde{A}(t)\right)\right)$ from the free-entry condition, the platform's profit can be rewritten as

$$\Pi^{2sm}(t) = t\left(\frac{c}{1-t} + \mu\right)\frac{\tilde{A}\left(t\right) - 1}{\frac{\mu(1-t)}{K}V(t)}.$$

Given that the equilibrium aggregate in the case of a one-sided market is $A(t) = \frac{\mu(1-t)}{K}V(t)$ and using the equilibrium aggregate in the two-sided market, we obtain

$$\Pi(t) = t\left(\frac{c}{1-t} + \mu\right)\left(F\left(In\left(\tilde{A}(t)\right)\right) - \frac{1}{A(t)}\right).$$

By adding and subtracting 1 in the equation inside the latter parentheses, we rewrite the platform's profit as that of a one-sided market, $\Pi(t)$, minus a loss term, $Z(t) = t\left(\frac{c}{(1-t)} + \mu\right)\left(1 - F\left(In\left(\tilde{A}(t)\right)\right)\right)$:

$$\begin{aligned}\Pi^{2sm}(t) &= t\left(\frac{c}{(1-t)} + \mu\right)\frac{A(t) - 1}{A(t)} - Z(t).\\ &= \Pi(t) - Z(t).\end{aligned}$$

Observe that $Z(t)$ captures the platform's loss due to those consumers who choose not to visit the platform, which has a measure of $1 - F\left(In\left(\tilde{A}(t)\right)\right)$. We next show that the loss from non-visitors increases in t:

$$Z'(t) = \left(\frac{c}{(1-t)^2} + \mu\right)\left(1 - F\left(In\left(\tilde{A}(t)\right)\right)\right) - t\left(\frac{c}{(1-t)} + \mu\right) f\left(In\left(\tilde{A}(t)\right)\right)\frac{\tilde{A}'(t)}{\tilde{A}(t)} > 0,$$

given that the equilibrium aggregate decreases in the platform's commission ($\tilde{A}'(t) < 0$). If we start at t, such that $\Pi(t) = 0$, then $\Pi^{2sm}(t) < 0$, since $Z'(t) > 0$. Hence, we show that the platform chooses a lower commission in the two-sided market.

Proof of Proposition 8 Recall that the platform's profit is

$$\Pi_h = \left[m_A V_A + t\left(\frac{c}{(1-t)} + \mu\right)\left(\tilde{A}(t) - V_A - 1\right)\right]\frac{K}{\mu\,(1-t)\,V(t)},$$

where $V_A = M\exp\left((x_A - m_A)/\mu\right)$, and so depends only on m_A. We want to calculate how the marginal profit of raising the fee changes in the advantage of the platform's product at equilibrium prices, that is, the sign of $\frac{\partial^2 \Pi_h^{2sm*}}{\partial t \partial x_A}$. Observe that raising x_A changes the platform's profit only via raising V_A. We therefore first calculate the marginal profit of raising V_A:

$$\frac{\partial \Pi_h^{2sm}}{\partial V_A} = \left(m_A - t\left(\frac{c}{1-t} + \mu\right)\right)\frac{K}{\mu(1-t)V(t)}, \tag{45}$$

which is the platform's margin over the opportunity cost of selling an own product multiplied by the fraction of consumers visiting the platform divided by the equilibrium aggregate. We next calculate how this changes in the fee:

$$\begin{aligned}\frac{\partial^2 \Pi_h^{2sm}}{\partial V_A \partial t} &= -\frac{K}{[\mu(1-t)V(t)]^2}\mu\left[(1-t)V'(t) - V(t)\right]\left(m_A - t\left(\frac{c}{1-t} + \mu\right)\right) \\ &\quad - \frac{K}{\mu(1-t)V(t)}\left(\frac{c}{(1-t)^2} + \mu\right). \end{aligned} \tag{46}$$

Note that $V'(t) = -V(t)\frac{c}{\mu(1-t)^2}$, since $V(t) = \exp\left(\frac{v - \frac{c}{1-t} - \mu}{\mu}\right)$. Substituting $V'(t)$ and the optimal markup $m_A(t) = t\left(\frac{c}{1-t} + \mu\right) + \mu$ in (46), we calculate how the marginal profit of raising $V_A(t)$ changes in the fee at $m_A(t)$:

$$\frac{\partial^2 \Pi_h^{2sm*}}{\partial V_A \partial t} = \frac{Kt}{(1-t)^2 V(t)}. \tag{47}$$

Finally, using $V_A(t) = M \exp\left(\left(x_A - m_A\right)/\mu\right)$, we calculate how the marginal profit of raising the fee changes in x_A in equilibrium:

$$\frac{\partial^2 \Pi_h^{2sm*}}{\partial x_A \partial t} = \frac{\partial^2 \Pi_h^{2sm*}}{\partial V_A \partial t}\frac{\partial V_A}{\partial v_A} = \frac{KtV_A(t)}{\mu(1-t)^2 V(t)}. \tag{48}$$

Hence, we show that the marginal profit of raising the fee increases in the advantage of the platform's product at the equilibrium price: $\frac{\partial^2 \Pi_h^{2sm*}}{\partial x_A \partial t} > 0$. Note that we could re-write the latter by using the equilibrium aggregate (32):

$$\frac{\partial^2 \Pi_h^{2sm*}}{\partial x_A \partial t} = \frac{tV_A(t)}{(1-t)\tilde{A}(t)} F\left(In\left(\tilde{A}(t)\right)\right), \tag{49}$$

which is the same expression in the one-sided market (24) (with a different equilibrium aggregate) times the fraction of buyers visiting the platform.

B A sufficient condition for uniqueness

A sufficient condition for uniqueness can be obtained by re-writing the solution to the optimal fee condition, equation (11), as

$$\frac{\mu\left(1-t\right)}{K}\exp\left(\frac{v - \frac{c}{(1-t)} - \mu}{\mu}\right) = \frac{\mu\left(\frac{c}{(1-t)} + \mu\left(1-t\right)\right) + t\left(\frac{c}{(1-t)} + \mu\right)\left(\frac{c}{(1-t)} + \mu\right)}{\mu\left(\frac{c}{(1-t)} + \mu\left(1-t\right)\right)}.$$

The LHS decreases in t. If the RHS increases, we get a unique solution. We define $\hat{\mu} = \mu/c$ and rewrite the RHS in terms of $\hat{\mu}$

$$RHS = \frac{\hat{\mu}\left(\frac{1}{(1-t)} + \hat{\mu}\left(1-t\right)\right) + t\left(\frac{1}{(1-t)} + \hat{\mu}\right)\left(\frac{1}{(1-t)} + \hat{\mu}\right)}{\hat{\mu}\left(\frac{1}{(1-t)} + \hat{\mu}\left(1-t\right)\right)}.$$

We then normalize it via multiplying and dividing it by $\hat{\mu}^2$:

$$RHS = \frac{\left(\frac{1}{\hat{\mu}(1-t)} + (1-t)\right) + t\left(\frac{1}{\hat{\mu}(1-t)} + 1\right)\left(\frac{1}{\hat{\mu}(1-t)} + 1\right)}{\left(\frac{1}{\hat{\mu}(1-t)} + (1-t)\right)},$$

which can be further simplified to

$$RHS = 1 + \frac{t\left(\frac{1}{\hat{\mu}(1-t)} + 1\right)\left(\frac{1}{\hat{\mu}(1-t)} + 1\right)}{\left(\frac{1}{\hat{\mu}(1-t)} + (1-t)\right)}.$$

If $\frac{\left(\frac{1}{\hat{\mu}(1-t)}+1\right)\left(\frac{1}{\hat{\mu}(1-t)}+1\right)}{\left(\frac{1}{\hat{\mu}(1-t)}+(1-t)\right)}$ is increasing in t, then it suffices for the RHS to be increasing. Then we rewrite it (with $k \equiv \frac{1}{\hat{\mu}} = \frac{c}{\mu}$ and $x \equiv \frac{1}{(1-t)} > 1$) as it suffices that the following increases in x:

$$\frac{(kx+1)^2}{(kx+x^{-1})},$$

which true both for $k \geq 1$ and for k small enough. Let us look at the relation whether $\frac{(kx+1)^2}{(kx+x^{-1})}$ is increasing in x for $x > 1$ and $k > 0$. It is true for $k \geq 1/9$, and that otherwise there is first a local maximum at some level of $x > 1$ and a local minimum at a larger x.

We have $\frac{(kx+1)^2}{(kx+x^{-1})}$ weakly increasing if, and only if,

$$\begin{aligned} 2k\left(kx+x^{-1}\right) - (kx+1)\left(k-x^{-2}\right) &\geq 0 \\ 2k\left(kx^3+x\right) - (kx+1)\left(kx^2-1\right) &\geq 0 \\ k^2x^3 + 3kx - kx^2 + 1 &\geq 0. \end{aligned}$$

Note it is positive at $x = 1$ (at $t = 0$) and has a positive derivative at $x = 1$ for $k > 0$, $3k^2 + k > 0$. The roots (which give turning points of the function) are

$$\begin{aligned} x &= \frac{2k \pm \sqrt{4k^2 - 4.9k^3}}{6k^2} \\ &= \frac{1 \pm \sqrt{1-9k}}{3k}, \end{aligned}$$

so if $k > 1/9$ there are no real roots and the function is always increasing. If $k = 1/9$, the function is constant in x. Let us look at the roots for $k < 1/9$. First, we show that the lower root always exceeds 1. That is,

$$\begin{aligned} 1-\sqrt{1-9k} &> 3k \\ 1-3k &> \sqrt{1-9k} \\ 1+9k^2-6k &> 1-9k \\ 9k^2 &> -3k. \end{aligned}$$

So, we conclude there is a local maximum first, then a local minimum on the feasible domain. The minimum is at $x=\frac{1+\sqrt{1-9k}}{3k}$. If we plug this back into the condition to see whether $k^2x^3+3kx-kx^2+1\geq 0$, we obtain

$$\begin{aligned} k^2\left(\frac{1+\sqrt{1-9k}}{3k}\right)^3+1+\sqrt{1-9k}-k\left(\frac{1+\sqrt{1-9k}}{3k}\right)^2+1 &\geq 0 \\ \frac{1}{k}\left(\frac{1+\sqrt{1-9k}}{3}\right)^3+\sqrt{1-9k}-\frac{1}{k}\left(\frac{1+\sqrt{1-9k}}{3}\right)^2+2 &\geq 0. \end{aligned}$$

Let $y\equiv\sqrt{1-9k}$ for $0<k<1/9$, so $0<y<1$ and $k=\frac{1-y^2}{9}$, and rewrite the latter:

$$\left(\frac{1+y}{3}\right)^3+y\frac{1-y^2}{9}-\left(\frac{1+y}{3}\right)^2+2\frac{1-y^2}{9}\geq 0,$$

which is the case if, and only if,

$$\frac{(1+y)^2}{3}+y(1-y)-(1+y)+2(1-y)\geq 0 \text{ or if and only if } y^2+2y-2\leq 0.$$

Since $f(y)=y^2+2y-2$ is a continuous and increasing function for $0<y<1$, $\lim_{y\to 1}f(y)>0$ and its positive root is $y=0.732$, for any $0<y\leq 0.732$ the inequality holds, that is, for any $k\geq 0.0516$, we have $k^2x^3+3kx-kx^2+1\geq 0$, and so the RHS of equation (11) is increasing, which implies a unique seller fee for the platform's problem. Hence, a sufficient condition for profit quasiconcavity is $k=\frac{c}{\mu}\geq 0.0516$.

C Log-convex search cost distribution

Suppose search costs are distributed with a log-convex cumulative distribution function $F(\cdot)$. Figure 10 illustrates the subgame equilibrium of the participation of buyers

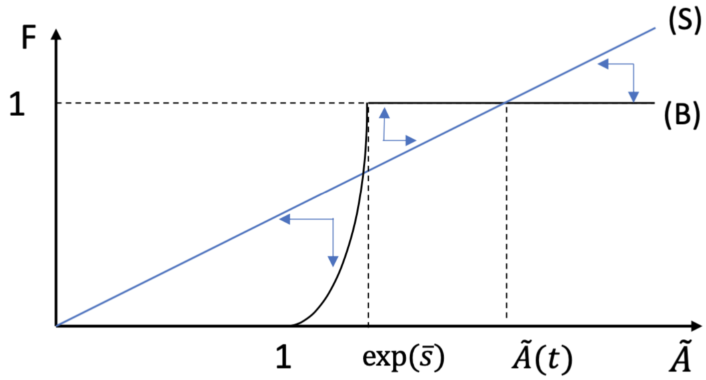

Figure 10: Subgame equilibrium participation of buyers (B) and sellers (S) when the platform sets commission t and consumers' search costs are distributed with a log-convex cdf $F(\cdot)$.

(B) and sellers (S) given the platform's commission. In that case the stable equilibrium is the largest intersection point between curves (B) and (S) in the figure. This corresponds to when all consumers participate ($F(\cdot) = 1$) and effectively the market becomes one-sided.

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