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PRODUCT LINE DESIGN[†]

Abstract

We characterize the product line choice and pricing of a monopolist as the upper envelope of net marginal revenue curves to the individual product demand functions. The equilibrium product varieties to include in a product line are those yielding the highest upper envelope. In a central case (corresponding to a generalized vertical differentiation framework), the equilibrium range of varieties is exactly the same as the first-best socially optimal range. These upper envelope and first-best optimal range findings extend to a symmetric Cournot oligopoly as well.

JEL Classification: L12, L13 and L15
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1 Introduction

One of the most important decisions facing firms is the design of the product line. Yet, typically economic analysis describes single-product firms despite the fact that almost all actual firms offer many products (in the same product class moreover). Our contribution is two-fold. First, we provide a surprisingly simple characterization of the pricing of a monopolist's product line using the elementary tools of marginal revenue curves (which readily extends to allowing marginal cost differences across varieties by deploying marginal profit functions). As we show, the monopolist prices its product line (and segments consumers) in accordance with the upper envelope of marginal revenue curves to the individual product demand functions. This pricing characterization also affords us a characterization of which product designs to include in a product line. Namely, these are the product varieties that yield the highest upper envelope of marginal revenue. It also enables us to analyze the equilibrium as well as welfare effects of introducing a new variety.

Second, we show that, in an important special case, which includes a general Mussa-Rosen (1978) vertical differentiation framework, the equilibrium product range is exactly the same as the first-best socially optimal range. This is because of the striking property that the upper envelope of demands corresponds exactly to the products that constitute the upper envelope of marginal revenues, even though the sets of consumers assigned to each product can be quite different at the two solutions. Furthermore, our main monopoly results extend nicely to a symmetric Cournot oligopoly, thus making our approach more generally applicable.

In their seminal paper on optimal product line design, Mussa and Rosen (1978) consider a monopolist facing a pool of privately informed consumers differing in terms of their willingness-to-pay for quality. In order to exploit this heterogeneity and maximize its profits, the monopolist designs a product line consisting of quality-differentiated versions of a product offered at different prices, and lets consumers self-select. They show that in equilibrium, (i) each consumer is allocated a distinct quality, (ii) quality provision is distorted: all consumers except for the highest type consume inefficiently low qualities.

A large theoretical literature on product line design extends the work of Mussa and Rosen (1978) in various directions. Examples include Moorthy (1984), who considers non-linear preferences and shows that this could induce the monopolist to aggregate distinct consumer segments into one segment; Gabszewicz et al. (1986), who characterize optimal market segmentation depending on the dispersion of consumers' income; Villas-Boas (1998), who introduces distribution channels by analyzing a monopolist manufacturer selling its product line through a retailer; Orhun (2009), who studies optimal product line design when consumers exhibit choice-set-dependent preferences; Liu and Cui (2010), who allow the monopolist to extend its product line depending on whether it sells through a centralized channel or a decentralized channel; and Guo and Zhang (2012), who study optimal product line design when consumers must incur deliberation costs to uncover their valuations for quality.

We focus on the primitives of the classical monopoly product line design problem. As in Mussa and Rosen (1978) and others, we also capture consumer heterogeneity by a single parameter. We, however, take a different approach and assume that the monopolist faces a continuum of consumers with unit demands, while having access to a fixed set of available varieties. This enables us to transform consumer preferences into a general framework of inverse demand curves, thereby enabling the use of marginal revenue curves and a simple graphical representation of equilibrium pricing of product lines and which products to include.

Our analysis builds on Itoh (1983) and Johnson and Myatt (2003, 2006a), and it complements Johnson and Myatt (2014). Itoh (1983) considers a discrete number of products in the Mussa-Rosen framework and analyzes the effects on product prices of introducing a new variety. We provide a simpler derivation of his results while extending them to more general preferences. To do so, we deploy an "upgrades" approach (which was implicit in the work of Itoh), whereby a product variety can be seen as a base product plus a series of upgrades corresponding to each additional higher quality variant in the range. Then, each upgrade can be associated to a price premium. Such an upgrades approach was pioneered by Johnson and Myatt (2003), who emphasized its usefulness in analyzing product line and pricing choices of multi-product firms in monopoly and Cournot duopoly (incumbent-entrant) contexts. Utilizing the same approach, Johnson and Myatt (2006a) uncovered important properties of product line choices in a fairly general n-firm multi-product Cournot oligopoly.[1]

It is noteworthy that Mussa and Rosen (1978) recognized that the monopolist sets the net marginal revenue of each upgrade to zero: "the optimal assignment equates the marginal cost and marginal revenue of increments of quality" (1978, p.311).[2] We implement this insight in our

[1] The upgrades approach is also used in Johnson and Myatt (2006b) in an extension of their main model, where they consider the length and mix of a product line and relate them to consumer-type dispersion.

[2] We thank a referee for this quote and for a very perceptive take on the literature, on which we have drawn heavily.

graphical treatment of the monopolist's solution by noting that setting the net marginal revenue of a quality increment to zero is equivalent to setting the net marginal revenue of adjacent qualities equal. This means that the equilibrium product lines and their prices can be identified from the intersections of the net marginal revenue curves of the variants. One implication is that the choice of product line is characterized from the upper envelope of the net marginal revenue curves: if a variant's net marginal revenue is below the upper envelope, it will not be used.

Johnson and Myatt (2014) clarify some of Itoh's results and extend them to Cournot oligopoly, engaging the set-up in Johnson and Myatt (2003, 2006a). We draw heavily on their insights on the "upgrades" approach and on how to carry over the monopoly results to symmetric oligopoly in our penultimate section. Their analysis is developed in a standard Mussa-Rosen framework and is fully focused on pricing results – in particular that for a wide class of preference distributions, monopoly (and Cournot oligopoly) prices are the same whether or not a variety is part of a product line. We also show that preferences do not need to be multiplicative to get the result that prices are neutral to the presence or absence of a quality.

These papers do not conduct welfare analyses. We deliver a welfare equivalence between monopoly and optimal provision of varieties for a general class of Mussa-Rosen preferences (as used by Itoh, 1983, Johnson and Myatt, 2014, and others). That is, the set of qualities in a monopolist's product line is the same set as chosen by a social planner under the standard preference formulation. To show this result, we show that marginal revenue curves for two qualities cross if and only if the net inverse demands cross. Of course, although there is no distortion in the choice of variants, the quantities offered are distorted.

The framework of Mussa and Rosen (1978) has been used to address many issues beyond product line design; in particular it has initiated a very large literature on non-linear pricing and second degree price discrimination (starting with Maskin and Riley, 1984). Many authors have used and extended their model by considering multiple products, multi-dimensional consumer types or competition (see, for example, Armstrong, 1996, 1999; Armstrong and Vickers, 2001; Rochet and Choné, 1998; and Rochet and Stole, 2002).[3] As noted by Johnson and Myatt (2014), several recent papers express a resurgent research interest in price discrimination (e.g., Anderson and Dana, 2009; Aguirre, Cowan and Vickers, 2010; and Cowan, 2012).

[3] For excellent surveys, we refer readers to Armstrong (2006) and Stole (2007).

2 Model

Consider a single firm (M) operating in a market in which it can offer many varieties of a product for sale. The choice of the number as well as the types of varieties to offer is endogenous. Let $N = \{1, 2, ..., n\}$ denote the set of available varieties. There is a unit mass of consumers. Each will buy at most one unit of the product. If multiple varieties are offered, the consumer chooses the one that yields the greatest utility, provided that this is non-negative (she chooses non-purchase if she does not get a positive surplus). We assume that for each variety the consumers are ordered the same way as regards to their (conditional) willingness-to-pay for the variety. M's problem is to determine the varieties to offer and their prices, under the constraint that consumers self-select. We rule out weakly dominated varieties; this means that varieties that ex post produce no sales are not included in the product line (we could assume an infinitesimal cost of including any new variety in the product line).

We describe each consumer by a unidimensional taste parameter θ, distributed over $[0, 1]$ according to a twice differentiable c.d.f. $F(\cdot)$, and a corresponding willingness-to-pay $u_i(\theta)$ for variety $i \in N$. We order consumers such that $u_i'(\theta) > 0$ for all $i \in N$; i.e., if consumer θ has a higher willingness-to-pay for variety i than consumer θ', then she has a higher willingness-to-pay for all other varieties as well.

Our analysis will typically make use of conditional stand-alone inverse demand functions. This will be described by a continuous function $\tilde{P}_i(q)$, measuring the maximum price M can charge to serve q "top" (highest valuation) consumers conditional on variety i being the only variety offered for sale. With this formulation, $\tilde{P}_i(q)$ is the price that leaves the consumer with a taste parameter $\theta = F^{-1}(1-q)$ indifferent between buying and not. Hence, $\tilde{P}_i(q) = u_i\left(F^{-1}(1-q)\right)$. On the production side, we assume that variety i has a specific constant marginal cost $c_i \geq 0$. By embedding this into $\tilde{P}_i(q)$, we reach the net inverse demand curve

$$P_i(q) = \tilde{P}_i(q) - c_i.$$

In what follows, we will mostly work with $P_i(q)$.

We will impose the following single-crossing condition: for any two varieties i and j, $P_i(q) - P_j(q)$ is monotonic in q. This also implies that $\tilde{P}_i(q) - \tilde{P}_j(q)$ is monotonic in q. Hence, for instance, if $\tilde{P}_i(q)$ crosses $\tilde{P}_j(q)$ from above, and both varieties are commonly priced below the intersection price, then all consumers right of the intersection would buy j over i, and consumers left of the intersection would buy i over j, at least up to some indifferent consumer. Define by $MR_i(q)$ the marginal revenue curve that corresponds to the net inverse demand curve $P_i(q)$:

$$MR_i(q) = P_i(q) + qP_i'(q).$$

With this specification, each MR curve actually measures the corresponding marginal profits. We will refer to MR as net marginal revenue, or sometimes simply as marginal revenue. Assume that each inverse demand curve $P_i(q)$ is strictly (-1)-concave to ensure that the corresponding $MR_i(q)$ is strictly downward sloping.[4] We also assume that any two MR curves can intersect at most once. Finally, we label potential varieties by the convention that a lower index implies a higher $P_i(0)$, so the consumer with the highest willingness-to-pay for all varieties likes 1 best and n the least, etc. Thus the net inverse demand curves are ranked and labeled by their vertical intercepts.

This setup is general enough to cover a wide range of preferences, including Mussa-Rosen and many others. For instance, Itoh (1983) and Johnson and Myatt (2014) use a multiplicative specification by assuming $u_i(\theta) = s_i\theta$, where s_i is naturally interpreted as the quality of variety i as per Mussa and Rosen (1978). With this specification, M faces a (stand-alone) demand of $1-F(p/s_i)$ at a price p. Inverting this for p will then give us the maximum price M can charge to sell q units of variety i: $\tilde{P}_i(q) = s_iF^{-1}(1-q)$. It is important to note that our base specification does not require multiplicative preferences, so we allow for utility functions such as $u_i = v(s_i, \theta)$. The most important restriction we impose is that the ordering of consumers in terms of their willingness-to-pay is identical across all varieties. This is also a standard assumption in second-degree price discrimination models of vertical product differentiation.

2.1 Two varieties: $n = 2$

To set the stage, first suppose that there are only two available varieties of the product. Call these varieties 1 and 2. By our labeling convention, we have $P_1(0) > P_2(0)$. Let q_1 denote the quantity sold of variety 1 only, and q_2 the aggregate quantity sold (of both varieties together). Then, if M charges $\tilde{p}_i$ for variety i, its profits will be

$$\pi = (\tilde{p}_1 - c_1)\,q_1 + (\tilde{p}_2 - c_2)\,(q_2 - q_1). \tag{1}$$

[4] This can be seen as follows. First, $MR_i(q)$ is strictly downward sloping if $P_i''(q)\,q + 2P_i'(q) < 0$. Whenever $P_i''(q) \le 0$ the result easily follows. If $P_i(q)$ is strictly (-1)-concave, then $1/P_i(q)$ is strictly convex, so $P_i''(q)\,P_i(q) - 2\left(P_i'(q)\right)^2 < 0$. Next, note that $MR_i(q)$ is non-negative if $P_i(q) \ge -P_i'(q)\,q$. Thus, when $P_i''(q) > 0$, strict (-1)-concavity of $P_i(q)$ implies $-P_i''(q)\,P_i'(q)\,q - 2\left(P_i'(q)\right)^2 < 0$ in the region where $MR_i(q) \ge 0$, or equivalently $P_i''(q)\,q + 2P_i'(q) < 0$.

We can alternatively rewrite π as

$$\pi = [\tilde{p}_1 - c_1 - (\tilde{p}_2 - c_2)]\, q_1 + (\tilde{p}_2 - c_2)\, q_2. \tag{2}$$

With this interpretation, we can think of variety 2 as the base product purchased by all consumers at a price $\tilde{p}_2$, and variety 1 as an upgrade or an add-on purchased by the first q_1 of the consumers by paying an extra $\tilde{p}_2 - \tilde{p}_1$. As described in the Introduction, although implicit in Itoh (1983), this "upgrades" approach to product line determination was first formalized by Johnson and Myatt (2003). Expressed in terms of price mark-ups $p_i = \tilde{p}_i - c_i$, the above profit expression becomes

$$\pi = (p_1 - p_2)\, q_1 + p_2 q_2.$$

The choice of quantities and the resulting market-clearing prices must obey two incentive compatibility constraints, which define the switch-points q_1 and q_2. Namely, q_1 must satisfy the following surplus equality,

$$\tilde{P}_1(q_1) - \tilde{p}_1 = \tilde{P}_2(q_1) - \tilde{p}_2.$$

In other words, q_1 highest consumer types should derive a higher surplus from variety 1 than variety 2, with the particular consumer corresponding to q_1 being indifferent between the two. With our definition of the net inverse demand curve $P_i(q) = \tilde{P}_i(q) - c_i$, we can express this constraint as

$$P_1(q_1) - p_1 = P_2(q_1) - p_2 \tag{3}$$

The second constraint says that the next $q_2 - q_1$ consumers should prefer variety 2 to non-purchase, with the consumer at q_2 being indifferent:

$$\tilde{P}_2(q_2) - \tilde{p}_2 = 0.$$

In other words, it is the participation constraint. Again, in terms of the net inverse demand curve and the price mark-up, this constraint can be written as

$$P_2(q_2) - p_2 = 0. \tag{4}$$

These two constraints enable us to uncover the key structure of the problem and envisage its simple solution. That is, incorporating them into the profit function, we are back to quantities:

$$\pi = (P_1(q_1) - P_2(q_1))\, q_1 + P_2(q_2)\, q_2, \tag{5}$$

Thus, quantities enter the profit function in two additively separate terms, and these two terms have very natural interpretations: The choice of q_2 maximizes the base profits earned on all consumers that are served, and the choice of q_1 maximizes the incremental profits earned on those who purchase the costlier variety. At any interior solution, using the marginal revenue notation, we reach the following two first-order conditions:[5]

$$MR_1(q_1) = MR_2(q_1), \tag{6}$$

$$MR_2(q_2) = 0. \tag{7}$$

This is a remarkably simple and intuitive characterization of a monopolist's pricing problem of its product line. It uses the elementary tools of marginal revenue curves. Graphically, it suffices to draw net demand curves and the corresponding net marginal revenue curves. The total number of consumers served is determined by setting MR_2 equal to zero. The number of consumers that are served variety 1 is then simply determined by the intersection point of the two MR curves. The corresponding price mark-ups to support these quantities are given by the two incentive compatibility constraints. First, given q_2, constraint (4) will tell us the mark-up on variety 2, $p_2 = P_2(q_2)$.[6] The knowledge of p_2 together with q_1 will then give us, via constraint (3), the premium: $p_1 - p_2 = P_1(q_1) - P_2(q_1)$. These are graphically illustrated in Figure 1.

It may so happen that one MR curve is above the other one for all quantities (in the positive quadrant). In such a case, M will sell only the 'superior' variety and set its net marginal revenue equal to zero. In other words, for both varieties to be offered in strictly positive quantities in equilibrium, we need the two MR curves to intersect at some interior q to the left of q_2. Put it differently, M will pick its product line according to the upper envelope of the net marginal revenue curves. This is a strikingly easy and intuitive way to characterize the optimal product mix of a monopolist.

Note that at any interior solution in which both varieties are sold in positive quantities, M serves the same number of consumers that it would have served if it had sold variety 2 only. This is easily seen from the profit function given in (5); q_2 enters only through the term $P_2(q_2)q_2$, which is what M would maximize if it sold variety 2 alone. Also note that at an interior equilibrium with $q_1 < q_2$, the incentive compatibility constraint (3) implies $\tilde{P}_1(q_1) - \tilde{p}_1 > 0$ since $\tilde{P}_2(q_2) = \tilde{p}_2$, so M necessarily leaves a strictly positive rent to the marginal consumer at q_1.

[5] Recall that each MR curve actually measures the associated marginal profits.

[6] Note that this pricing rule does not depend on variety 1.

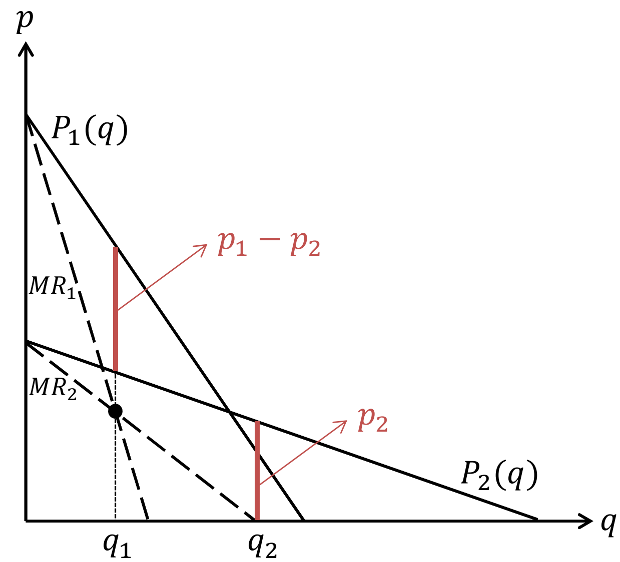

Figure 1
Equilibrium as the intersection of MR curves (n=2)

In solving the profit maximization problem above, we have implicitly assumed that there is an interior solution to $MR_2(q_2) = 0$. In other words, we have assumed incomplete market coverage. This is something we will maintain in the rest of the analysis for each variety. That is, we will assume that $MR_i(1) \leq 0$ for all $i \in N$. Under this assumption, M would never find it optimal to serve the whole market.

2.2 General case: n varieties

The main logic developed for two varieties extends to any number of varieties. Take a general case with $n > 2$. Maintaining the same notational convention, let q_j denote the cumulative quantity of varieties $i = 1, ..., j$. At an interior solution in which all varieties are consumed in strictly positive quantities, M will segment the consumers into n segments and its profits will accordingly be given by

$$\pi = (\tilde{p}_1 - c_1) q_1 + \cdots + (\tilde{p}_n - c_n)(q_n - q_{n-1}).$$

Rewriting this in the alternative premium-form yields

$$\pi = [\tilde{p}_1 - c_1 - (\tilde{p}_2 - c_2)] q_1 + \cdots + (\tilde{p}_n - c_n) q_n.$$

Once again, with this formulation, we can think of variety n as the base product that all served consumers purchase at a price $\tilde{p}_n$. Variety $n-1$ can then be thought of as an add-on purchased at a premium $\tilde{p}_{n-1} - \tilde{p}_n$ by all but the last bracket of consumers, variety $n-2$ as a further add-on purchased at an additional premium $\tilde{p}_{n-2} - \tilde{p}_{n-1}$ by all but the last two brackets, and so on. Expressed in terms of price mark-ups $p_i = \tilde{p}_i - c_i$, the above profit expression becomes

$$\pi = (p_1 - p_2)\, q_1 + (p_2 - p_3)\, q_2 + \cdots + p_n q_n.$$

With n segments of consumers, there will be n switch-points, and hence n incentive compatibility constraints:

$$P_i\,(q_i) - p_i = P_{i+1}\,(q_i) - p_{i+1}, \text{ for } i = 1, ..., n-1,$$

$$P_n\,(q_n) - p_n = 0.$$

Incorporating these into π yields

$$\pi = (P_1\,(q_1) - P_2\,(q_1))\, q_1 + (P_2\,(q_2) - P_3\,(q_2))\, q_2 + \cdots + P_n\,(q_n)\, q_n. \tag{8}$$

Thus, the first-order conditions at any interior equilibrium take the same form:

$$MR_i\,(q_i) = MR_{i+1}\,(q_i)\,, \text{ for } i = 1, ..., n-1,$$

$$MR_n\,(q_n) = 0.$$

Hence, once again, one only needs to draw the net marginal revenue curves and find where they cross to determine the quantities. This process should certainly respect the ordering of the consumer switch-points. To be more precise, if any of the two successive equations in the above system (say, i^{th} and $(i+1)^{st}$) produce an outcome $q_i \geq q_{i+1}$, then this means that variety $i+1$ will not be offered in equilibrium. Graphically, this will be the case when MR_{i+1} is not part of the upper envelope. Hence, it suffices to work with the upper envelope of the net marginal revenue curves and find the resulting switch-points. This will tell us the quantities. Once we have the quantities, we follow a similar recursive solution for the pricing of the product line. The last switch-point will tell us the price to charge for the last variety. The price premiums will then be given by the corresponding incentive compatibility constraints. We summarize these in the following proposition.

Proposition 1 *For a given set of available varieties $N = \{1, 2, ..., n\}$, M will include variety i in its product line if and only if*

$$MR_i\,(q) > \max_{j \neq i} MR_j\,(q) \;\; \textit{for some } q \in \left(0, MR_i^{-1}\,(0)\right).$$

Relabeling the set of varieties included in the product line as $N^* = \{1, 2, ..., n^*\}$, M will choose quantities by

$$MR_{n^*}(q_{n^*}) = 0, \text{ and}$$

$$MR_i(q_i) = MR_{i+1}(q_i), \text{ for } i = 1, ..., n^* - 1.$$

The corresponding net prices will be given by the following recursive system:

$$p_{n^*} = P_{n^*}(q_{n^*}), \text{ and}$$

$$p_i = p_{i+1} + P_i(q_i) - P_{i+1}(q_i), \text{ for } i = 1, ..., n^* - 1.$$

Notice that the first-order condition captured by the marginal revenue equality is interpreted as a switch of a marginal consumer from variety $i+1$ to variety i. The choice of q_i holds constant all other switch-points so that a one unit increase in q_i means a one unit increase in production of variety i and a corresponding one unit decrease in production of variety $i+1$ (in order to keep the marginal consumer q_{i+1} fixed). With this in mind, the intuition for the marginal revenue equality at the switch-point is the following. The value to M of getting a marginal consumer to switch up from variety $i+1$ to variety i is the premium $p_i - p_{i+1}$. But to induce this switch, M must raise the consumer's surplus on variety i by enough. The derivative of the willingness-to-pay difference, $\tilde{P}'_i(q_i) - \tilde{P}'_{i+1}(q_i)$, indicates how much the premium must be reduced, and this premium reduction is suffered on M's demand base, q_i, of consumers buying varieties better than i. Note that $\tilde{P}'_i(q_i) - \tilde{P}'_{i+1}(q_i) = P'_i(q_i) - P'_{i+1}(q_i)$. Pulling this together, the first-order profit derivative is $p_i - p_{i+1} + \left(P'_i(q_i) - P'_{i+1}(q_i)\right) q_i$, which rearranges to the difference in net marginal revenues at q_i.

In Figure 2, we illustrate equilibrium determination graphically for $n = 3$. We highlight two key properties of equilibrium quantities and prices (both pointed out by Itoh, 1983, in his narrower setting). First, the total number of consumers served in equilibrium depends only on $P_{n^*}(q)$. Second, price as well as the quantity sold of an included variety $i \in N^*$ only depends on the varieties that come after it on the product line; in particular, both are independent of varieties $j < i$.

The profits M earns in equilibrium can conveniently be measured by the area under the upper envelope of the net MR curves. This can be seen from equation (8). Treating variety n^* as the base product purchased by all consumers, the area under $MR_{n^*}(q)$ up to q_{n^*} will give the base

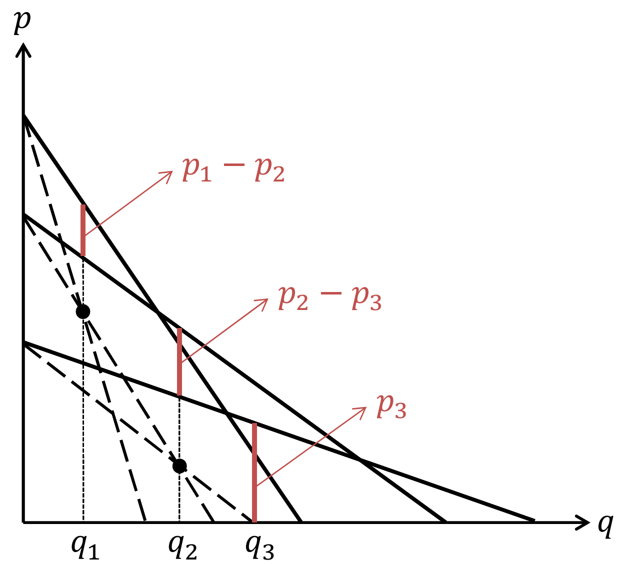

Figure 2
Equilibrium as the intersection of MR curves (n=3)

profits M earns. The incremental profits earned on all but the last bracket of consumers will then be simply the area between $MR_{n^*-1}(q)$ and $MR_{n^*}(q)$ up to where the two intersect. Continuing in this fashion will then get us to the answer. This is illustrated graphically in Figure 3.

2.3 Effects of a new variety

In this subsection, we analyze the effects of introducing a new variety on equilibrium pricing and the quantities produced. Itoh (1983) did a similar exercise for a monopoly in a Mussa-Rosen context, which Johnson and Myatt (2014) later extended for a Cournot oligopoly. Our approach will be based on graphical arguments. This has at least two advantages: First, we are able to replicate most of the results in Itoh (1983) and offer a few new results in a strikingly simpler way. And second, our approach is much more general as we represent each variety by a general stand-alone inverse demand function, which admits Mussa-Rosen model as a special case. In this sense, we offer a generalization of Itoh's results.

Suppose there is a new variety available for production. We know from the above arguments that this new variety will be included in the optimal product line if and only if it strictly expands the upper envelope of the MR curves. While expanding the upper envelope, it may cause M to stop selling one or more of the other varieties. We will focus here on the intermediate case: the new

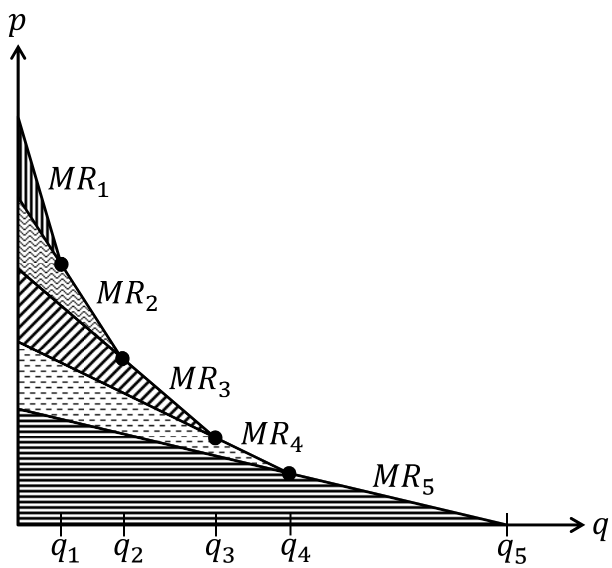

Figure 3
Profits as the area under MR curves (n=5)

variety expands the upper envelope without crowding out any of the existing varieties. We also assume for the remainder that M initially includes each one of the n varieties in its product line.

Consider a scenario in which the MR curve associated with the new variety expands the upper envelope in the proximity where MR_j and MR_{j+1} intersect. We will conveniently call the new variety as "variety $j.5$" (j and a half). Using the properties of equilibrium characterization described in Proposition 1, we can summarize the effects of introducing variety $j.5$ into the product line as follows:

(i) Quantities $q_1, ..., q_{j-1}, q_{j+1}, ..., q_n$ will stay unaffected since these are determined by local MR curves that do not involve $MR_{j.5}$. Thus, the number of consumers that are sold varieties $1, ..., j-1, j+2, ..., n$ will also stay unaffected. The quantities sold of varieties j and $j+1$ ($q_{j.5} - q_j$ and $q_{j+1} - q_{j.5}$ respectively), on the other hand, will go down.

(ii) Given that $p_n = P_n(q_n)$ and that $q_{j+1}, ..., q_n$ are unaffected, the prices $p_{j+1}, ..., p_n$ will also stay unaffected.

(iii) The new variety will be sold at a net price $p_{j.5} = p_{j+1} + P_{j.5}(q_{j.5}) - P_{j+1}(q_{j.5}) > p_{j+1}$. The net price of variety j can go in any direction (this depends on the curvature of the demand curves – see below for more discussion).

(iv) Suppose p_j changes by an amount Δ. Then $p_1, ..., p_{j-1}$ will change in the same direction, also by Δ. This follows from the incentive compatibility constraints $P_i(q_i) - p_i = P_{i+1}(q_i) - p_{i+1}$; since q_i and thus $P_i(q_i) - P_{i+1}(q_i)$ stay unaffected for $i = 1, ..., j-1$, it follows that $p_i - p_{i+1}$ must also stay the same.

These effects are illustrated in Figure 4 where M starts with three existing varieties, and the new variety comes in between varieties 2 and 3. As is clear from the figure, in terms of quantities, the new variety causes a change only in q_2. The price of variety 3 stays the same. However, the introduction of variety 2.5 changes the incentive conditions, and as a result the prices of varieties 1 and 2 change (by the same amount). The net change in profits is indicated by the shaded area.

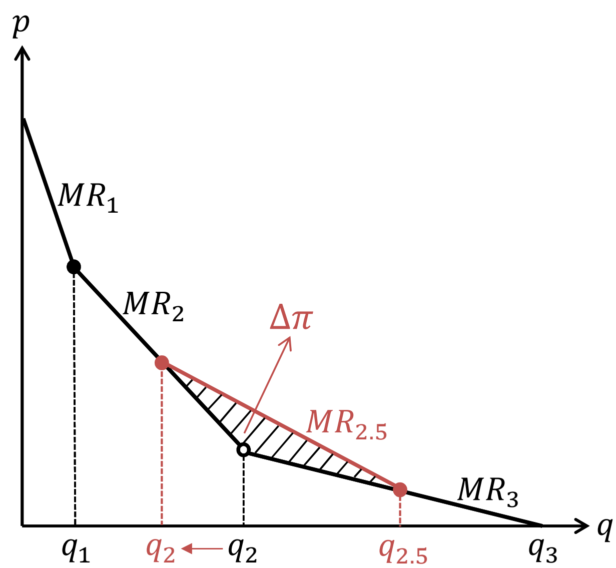

Figure 4
Effects of a new variety

As we mentioned earlier, results (i)-(iv) were established by Itoh (1983) within a standard Mussa-Rosen framework. With our interpretation of $P_i(q)$ as the net inverse demand curve, the Mussa-Rosen specification is equivalent to assuming $P_i(q) = -c_i + s_i F^{-1}(1-q)$, which, as we show below, generates a specific family of demand curves (see (9) below). The effects we summarized in (i) through (iv) above are valid for any system of inverse demand curves so long as they satisfy a minimal number of technical conditions to guarantee existence and uniqueness. In this sense, our approach provides a generalization of Itoh's results by using the upper envelope of MR curves. This provides a nice graphical representation of the profits earned as the area under the upper envelope of MR curves. In the next section, we analyze a different dimension – the comparison of equilibrium and first-best optimal product lines – to derive new welfare results for the general class of Mussa-Rosen preferences.

We conclude this section with a remark on the price effects of introducing a new variety into the product line. Itoh (1983) shows within the standard Mussa-Rosen framework that if the inverse hazard rate $\frac{1-F(\theta)}{f(\theta)}$ is linear in θ, then addition of a new variety will have no effect on the prices of the existing varieties. That is, in the terminology of (iv) above, $\Delta = 0$. As later highlighted by Johnson and Myatt (2014), this is a remarkable finding because it means that if $\frac{1-F(\theta)}{f(\theta)}$ is linear in θ, then M charges the optimal stand-alone profit-maximizing price for each variety, regardless of how many other varieties there are in the product line. Moreover, Itoh (1983) also shows that addition of a new variety causes prices to go up (by the same amount) for lower-indexed varieties if this ratio is convex, and to go down if the ratio is concave.

Saying $\frac{1-F(\theta)}{f(\theta)}$ is linear in the standard Mussa-Rosen context is equivalent to saying in our context that the stand-alone (direct) demand curve for each variety is ρ-linear (Caplin and Nalebuff, 1991).[7] A demand curve is ρ-linear if D_i^ρ is linear in p, or simply if it is in the form $D_i(p) = (\alpha_i - \beta_i p)^{1/\rho}$ for some $\alpha_i > 0$ and $\beta_i \rho < 0$.[8] It then follows that the associated inverse demand curve must take one of the following two forms:

$$\tilde{P}_i(q) = \alpha_i - \beta_i q^\rho, \text{ with } \beta_i \rho < 0,$$

$$\tilde{P}_i(q) = \alpha_i - \beta_i \ln q$$

The latter form arises when D_i^ρ is linear in the limit as $\rho \to 0$, referred to as a log-linear demand curve.

To see an example, take the first form above. Suppressing c_i in α_i, net marginal revenue can be expressed as

$$\begin{aligned} MR_i(q) &= \alpha_i - \beta_i(\rho+1) q^\rho \\ &= (\rho+1) P_i(q) - \rho\alpha_i. \end{aligned}$$

[7] ρ-linear demand curves have some strong (and useful) properties. They imply constant pass-through rates for changes in marginal costs under monopoly as well as Cournot oligopoly (Bulow and Pfleiderer, 1983). They also imply that consumer surplus is a fixed fraction of profits, and thus of aggregate welfare (Anderson and Renault, 2003).

[8] When $u_i = s_i\theta - p_i$, the marginal consumer will be $\tilde{\theta} = p/s_i$ and thus $D_i(p) = 1 - F(\tilde{\theta})$. Hence, we get the linear Mills ratio for ρ-linear demands: $-D_i(p)/D_i'(p)$ is linear. The property that the *inverse* demand curve slope elasticity is constant for this class also directly follows, termed the constant curvature property by Johnson and Myatt (2014).

In equilibrium, the last variety in the line will have $MR_{n^*}(q_{n^*}) = 0$, implying that $p_{n^*} = P_{n^*}(q_{n^*}) = \frac{\rho}{\rho+1}\alpha_{n^*}$. The other prices are determined by the optimality condition $MR_i(q_i) = MR_{i+1}(q_i)$, which implies by the incentive compatibility constraint that $p_i - p_{i+1} = \frac{\rho}{\rho+1}(\alpha_i - \alpha_{i+1})$. Now suppose there is a new variety offered, variety $j.5$. Then, in the new equilibrium, we must have

$$p_j - p_{j.5} = \frac{\rho}{\rho+1}(\alpha_j - \alpha_{j.5}),$$
$$p_{j.5} - p_{j+1} = \frac{\rho}{\rho+1}(\alpha_{j.5} - \alpha_{j+1}).$$

Adding these up, we see that $p_j - p_{j+1} = \frac{\rho}{\rho+1}(\alpha_j - \alpha_{j+1})$, which is the same as it was before variety $j.5$ was introduced. Hence, when the stand-alone demand curves are ρ-linear, addition of a new variety does not change any of the existing prices.[9]

Itoh (1983) result implies that addition of a new variety will improve consumer surplus when demand curves are ρ-concave. Because prices do not go up, those consumers who stay with their initial choices are equally well off and those who switch to the new variety must be strictly better off (because otherwise they would not have switched). Since profits must be higher if the firm has chosen to introduce the new variety, aggregate welfare must improve as well. In other words, if a monopolist finds it profitable to offer a new variety into its product line, then it is also welfare-improving. In the next section, we consider quite a different welfare link between optimum and equilibrium, by looking at the first-best optimum product line. The first-best optimum involves quite a different allocation of consumers to varieties than the equilibrium with its monopoly pricing. Nonetheless, we show an equivalence result for the whole class of general Mussa-Rosen preferences (and therefore for the Itoh, 1983, and Johnson-Myatt, 2014, models).

3 Socially optimal product line

Given our specification that each $P_i(q)$ in fact measures the net inverse demand, the socially optimal matching of consumers to varieties can easily be traced using the upper envelope of the demand curves. Since this will typically differ from the way M will segment the market, the equilibrium outcome will be associated with consumption inefficiencies. For instance, while it is socially optimal that all consumers left of the intersection of $P_1(q)$ and $P_2(q)$ consume variety 1, only those left of

[9] We should note here that in a more general Mussa-Rosen framework with $u_i = s_i v(\theta)$, the inverse hazard rate $\frac{1-F(\theta)}{f(\theta)}$ will not be a sufficient statistic anymore because whether the prices will stay the same or not will also depend on the curvature of $v(\cdot)$. The concept of ρ-linearity, on the other hand, will still be valid since it is a property of the final demand curve, not of the underlying preferences.

the intersection of $MR_1(q)$ and $MR_2(q)$ consume it in equilibrium. As we move down along the product line (from variety 1 to 2, variety 2 to 3, and so on), the amount of information rents M has to leave to high value consumers diminishes, implying that consumption efficiencies will also diminish. However, they will never disappear.

As for the optimal product line design, social optimum requires that a variety should be included if it has the highest social value for a non-empty set of consumers. To make an assessment of M's product line choice, one then needs to compare the upper envelope of net inverse demands with the upper envelope of the corresponding net marginal revenue curves. We will here focus on a particular class of net inverse demand curves:

$$P_i(q) = \alpha_i - \beta_i \eta(q), \tag{9}$$

where each variety $i \in N$ is described by a pair $(\alpha_i, \beta_i) >> 0$, and a function that depends on quantity where $\eta'(q) > 0$. Without any loss of generality, set $\eta(0) = 0$. By our labeling convention, $P_1(0) > P_2(0) > \cdots > P_n(0)$, so $\alpha_1 > \alpha_2 > \cdots > \alpha_n$. In this specification, marginal cost of production c_i is accounted for in α_i.

This class of inverse demand curves is widely used in economics. For example, $\eta(q) = q$ corresponds to the classical linear demand curve. Assuming $\eta(q) = q^\rho$ means that the (direct) demand curve is ρ-linear. It also represents a general class of Mussa-Rosen preferences. Take, for instance, $u_i = s_i v(\theta)$ where $v'(\theta) > 0$. This translates into a net inverse demand curve $P_i(q) = -c_i + s_i v\left(F^{-1}(1-q)\right)$, which can be rewritten as $P_i(q) = s_i v(1) - c_i - s_i\left[v(1) - v\left(F^{-1}(1-q)\right)\right]$. With this formulation,

$$\alpha_i = s_i v(1) - c_i > 0,$$

$$\beta_i = s_i > 0, \text{ and}$$

$$\eta(q) = v(1) - v\left(F^{-1}(1-q)\right).$$

Note that $\eta(q)$ satisfies $\eta(0) = 0$ and $\eta'(q) > 0$. Hence, in this formulation, $\eta(q)$ summarizes both $v(\cdot)$ and $F(\cdot)$. In particular, the general Mussa-Rosen model (as used by Itoh, 1983, Johnson and Myatt, 2014, and others) implies the net inverse demand form (9).

As we show in the next lemma, this class of inverse demand curves also possesses an important property.

Lemma 1 *Assume* $P_i(q) = \alpha_i - \beta_i \eta(q)$, $\forall i \in N$, *where* α_i *and* β_i *are positive constants,* $\eta(0) = 0$ *and* $\eta'(q) > 0$. *For any two varieties* $i, j \in N$, *if* $P_i(q) = P_j(q) = \phi > 0$ *at some* $q > 0$, *then* $MR_i(\tilde{q}) = MR_j(\tilde{q}) = \phi$ *for some* $\tilde{q} \in (0, q)$. *Similarly, if* $MR_i(\tilde{q}) = MR_j(\tilde{q}) = \phi > 0$ *at some* $\tilde{q} > 0$, *then* $P_i(q) = P_j(q) = \phi$ *for some* $q > \tilde{q}$.

In other words, this lemma says that two net inverse demand curves cross each other if and only if their corresponding net marginal revenue curves also cross. Moreover, it says that both of these crossings occur at the same height (i.e., attain same values evaluated at the crossing points). This has an important implication for this class of demands, and therefore for product line selection under Mussa-Rosen demands. If a particular variety is part of the upper envelope of the marginal revenue curves, then it is also part of the upper envelope of the inverse demand curves (and vice versa).

Lemma 1 immediately leads us to the following result:

Proposition 2 *When each variety* $i \in N$ *has an inverse demand function in the form* $P_i(q) = \alpha_i - \beta_i \eta(q)$, *as implied by the general Mussa-Rosen framework, the equilibrium product line M chooses is exactly the same as the first-best socially optimal product line.*

Thus, when the preferences fit the generalized Mussa-Rosen specification, M voluntarily offers the first-best socially optimal range of varieties and there is no need for any social intervention in this respect. This situation is graphically illustrated in Figure 5 below. Note that Proposition 2 is not to mean that consumers will purchase their first-best varieties in the market equilibrium. Since M is a profit maximizer and will therefore be tempted to optimally segment the market, it will have to leave some informational surplus to high-valuation consumers. Therefore, not all consumers will end up consuming the varieties that are socially best for them.

4 Cournot oligopoly

The framework we have built for monopoly above extends very easily to a symmetric Cournot oligopoly.[10] Suppose there are m identical firms. Similar to the monopoly notation, we will define quantities cumulatively and will denote by q_i^j the total number of consumers firm j serves with

[10] Johnson and Myatt (2014) extend Itoh's (1983) results to Cournot oligopoly, and emphasize that equilibrium prices are often close to prices in stand-alone single product markets.

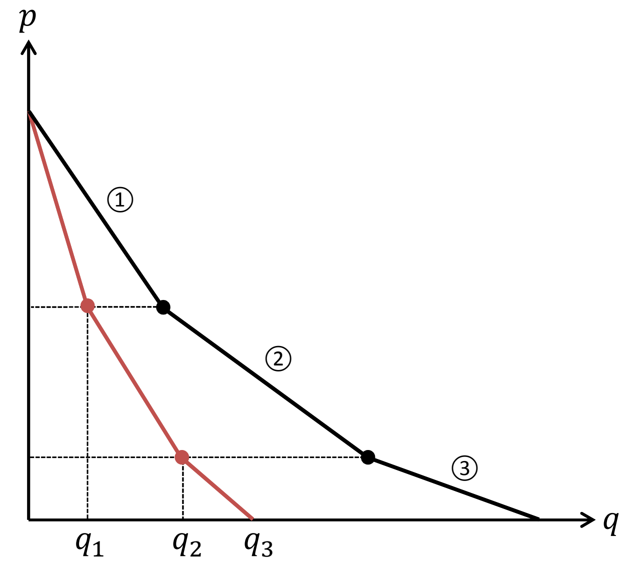

Figure 5
Upper envelope of demand curves vs. MR curves

varieties $1, ..., i$. We now introduce a new notation Q_i, which aggregates q_i^j across firms; i.e., $Q_i = \Sigma_{j=1}^m q_i^j$. Let us take each firm's behavior as given and analyze firm k in isolation. For given price mark-ups $(p_1, ..., p_n)$, we can express firm k's profits as

$$\pi_k = p_1 q_1^k + \cdots + p_n \left(q_n^k - q_{n-1}^k \right),$$

which can be rewritten in the alternative premium-form as

$$\pi_k = (p_1 - p_2)\, q_1^k + \cdots + p_n q_n^k.$$

The incentive compatibility constraints are now given by:

$$P_i(Q_i) - p_i = P_{i+1}(Q_i) - p_{i+1}, \text{ for } i = 1, ..., n-1,$$

$$P_n(Q_n) - p_n = 0.$$

Incorporating these into the above profit function, we can express profits as a function of quantities only:

$$\pi_k = (P_1(Q_1) - P_2(Q_1))\, q_1^k + (P_2(Q_2) - P_3(Q_2))\, q_2^k + \cdots + P_n(Q_n)\, q_n^k.$$

The first-order conditions at any interior equilibrium will satisfy (with n actively produced varieties):

$$P_i(Q_i) + q_i^k P_i'(Q_i) = P_{i+1}(Q_i) + q_i^k P_{i+1}'(Q_i), \text{ for } i = 1, ..., n-1,$$

$$P_n\left(Q_n\right)+q_n^k P_n'\left(Q_n\right)=0.$$

In a symmetric equilibrium, $q_i^k = Q_i/m$. Hence, letting

$$SMR_i\left(Q;m\right)=P_i\left(Q\right)+QP_i'\left(Q\right)/m,$$

denote the symmetric Cournot residual marginal revenue, the first-order conditions will read as

$$SMR_i\left(Q_i;m\right)=SMR_{i+1}\left(Q_i;m\right), \text{ for } i=1,...,n-1,$$

$$SMR_n\left(Q_n;m\right)=0.$$

We now argue that any variety in the upper envelope of the SMR curves will be produced in equilibrium. Our argument will be based on the property that if some variety (or varieties) that is in the upper envelope were not produced, then there would be extra profit for any firm producing it. To see this, suppose that varieties 1 and 2 are produced, with switchpoint Q_1, but that variety 1.5 has a higher SMR at Q_1 (the argument applies for any variety). We then have without variety 1.5 produced that

$$\pi_k=\left(P_1\left(Q_1\right)-P_2\left(Q_1\right)\right)q_1^k+\left(P_2\left(Q_2\right)-P_3\left(Q_2\right)\right)q_2^k+\cdots+P_n\left(Q_n\right)q_n^k.$$

Suppose now that firm k produces Δq_1^k units less of variety 1 and substitutes with Δq_1^k units of variety 1.5, which is "above" the other (in terms of the SMR at Q_1). So then (because Q_1 is now the total amount produced of varieties 1 and 1.5, and q_1^k becomes the total amount of varieties 1 and 1.5 produced by firm k):

$$\begin{aligned}\pi_k=&\left(P_1\left(Q_1-\Delta q_1^k\right)-P_{1.5}\left(Q_1-\Delta q_1^k\right)\right)\left(q_1^k-\Delta q_1^k\right)+\left(P_{1.5}\left(Q_1\right)-P_2\left(Q_1\right)\right)q_1^k\\&+\left(P_2\left(Q_2\right)-P_3\left(Q_2\right)\right)q_2^k+\cdots+P_n\left(Q_n\right)q_n^k.\end{aligned}$$

Hence, the change in profit is the difference

$$\begin{aligned}\Delta\pi_k=&\left\{\left(P_1\left(Q_1-\Delta q_1^k\right)-P_{1.5}\left(Q_1-\Delta q_1^k\right)\right)\left(q_1^k-\Delta q_1^k\right)+\left(P_{1.5}\left(Q_1\right)-P_2\left(Q_1\right)\right)q_1^k\right\}\\&-\left\{\left(P_1\left(Q_1\right)-P_2\left(Q_1\right)\right)q_1^k\right\}\\=&\left\{\left(P_1\left(Q_1-\Delta q_1^k\right)-P_{1.5}\left(Q_1-\Delta q_1^k\right)\right)\left(q_1^k-\Delta q_1^k\right)+P_{1.5}\left(Q_1\right)q_1^k\right\}-P_1\left(Q_1\right)q_1^k.\end{aligned}$$

We can rewrite this as

$$\begin{aligned}\frac{\Delta\pi_k}{\Delta q_1^k}=&\frac{P_1\left(Q_1-\Delta q_1^k\right)-P_1\left(Q_1\right)}{\Delta q_1^k}q_1^k-P_1\left(Q_1-\Delta q_1^k\right)-\frac{P_{1.5}\left(Q_1-\Delta q_1^k\right)-P_{1.5}\left(Q_1\right)}{\Delta q_1^k}q_1^k\\&+P_{1.5}\left(Q_1-\Delta q_1^k\right).\end{aligned}$$

In the limit as $\Delta q_1^k \to 0$, this becomes

$$\begin{aligned}\pi_k' &= -P_1'(Q_1)\,q_1^k - P_1(Q_1) + P_{1.5}'(Q_1)\,q_1^k + P_{1.5}(Q_1)\\ &= SMR_{1.5}(Q_1;m) - SMR_1(Q_1;m),\end{aligned}$$

where the last line follows from evaluating at the prior candidate symmetric equilibrium, $q_1^k = Q_1/m$. Thus, profit is higher if $SMR_{1.5}(Q_1;m) > SMR_1(Q_1;m)$, or if the putatively excluded variety is produced. A similar argument shows that any variety for which the symmetric Cournot residual marginal revenue lies below the upper envelope of the SMR curves will not be produced by any firm because it will not be profitable even if the rivals produce none of it.

Hence, once again, the equilibrium consumer allocation, prices, and product line will be determined by the upper envelope of the Cournot (net) marginal revenue curves: the aggregate quantity of each variety (of which each firm will produce an equal share) will be given as before by the intersection points of the relevant MR curves, with the corresponding prices determined by the incentive compatibility conditions. These are precisely the same steps we followed to obtain the monopoly outcome, just adjusted for the number of firms. Note that when $m = 1$, we get back to the full monopoly configuration.

Moreover, when the inverse demand curves take the form $P_i(q) = \alpha_i - \beta_i \eta(q)$, the equilibrium product line chosen in an oligopolistic market will be exactly the same as the socially optimal product line (see the proof of Lemma 1 in the Appendix, which shows that the SMR functions intersect at the same height as do the demand functions for an m-firm Cournot oligopoly). To summarize:

Proposition 3 *For a given set of available varieties $N = \{1, 2, ..., n\}$, m symmetric Cournot oligopolists will include variety i in their product lines if and only if*

$$SMR_i(Q;m) > \max_{j \neq i} SMR_j(Q;m) \text{ for some } Q \in \left(0, SMR_i^{-1}(0)\right),$$

where $SMR_i(Q;m) = P_i(Q) + QP_i'(Q)/m$ and quantities and prices will be given by analogy to Proposition 1. When each variety $i \in N$ has an inverse demand function in the form $P_i(q) = \alpha_i - \beta_i \eta(q)$, as implied by the general Mussa-Rosen framework, the equilibrium product line under symmetric Cournot oligopoly is exactly the same as the first-best socially optimal product line.

Hence, equilibrium determination and comparison to the socially optimal product line are parallel to the monopoly case. All we need to do is to adjust the net marginal revenue function for the m-firm Cournot oligopoly environment, and base all of the analysis on the new Cournot SMR curves. In this sense, the toolbox we have developed for the monopoly configuration is quite strong.

Indeed, in parallel to Johnson and Myatt (2014), the effects of introducing a new variety can also be tracked similarly. Consider a new variety (variety $j.5$) with a Cournot marginal revenue curve in the proximity where $SMR_j(Q;m)$ and $SMR_{j+1}(Q;m)$ intersect. Then:

(i) Quantities $Q_1, ..., Q_{j-1}, Q_{j+1}, ..., Q_n$ will stay unaffected; only Q_j will change.

(ii) Prices $p_{j+1}, ..., p_n$ will stay unaffected.

(iii) Suppose p_j changes by an amount Δ. Then $p_1, ..., p_{j-1}$ will also change by Δ in the same direction. Moreover, $\Delta = 0$ when each $P_i(Q)$ is ρ-linear.

5 Conclusion

In this paper, we analyze a monopolist's choice of its product line. Even though this is generally a complex problem, we are able to reach very clean results. In particular, we show that the monopolist's product line choice problem reduces to including those varieties that are part of the upper envelope of the net marginal revenue curves. The equilibrium quantities of the included varieties are then determined by finding where the associated marginal revenue curves cross. We also show that, for an important class of preferences, the monopolist offers only those product designs that are (first-best) socially desirable. However, since the monopolist will optimally segment the market to maximize its profits, there will be distortions in consumption, so consumers will not always get the variety that is best for them. We also show that these results smoothly extend to a symmetric Cournot oligopoly framework.

Appendix: Proofs

Proof of Lemma 1. Suppose $P_i(q) = P_j(q) = \phi$ for some q. Then at such q

$$\alpha_i - \beta_i \eta(q) = \alpha_j - \beta_j \eta(q),$$

$$\eta(q) = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Hence,

$$\phi = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Marginal revenue curve $MR_i\left(q\right)$ is given by

$$MR_i\left(q\right) = \frac{d\left(q\left(\alpha_i - \beta_i \eta\left(q\right)\right)\right)}{dq} = \alpha_i - \beta_i\left(\eta\left(q\right) + q\eta'\left(q\right)\right).$$

If $MR_i\left(\tilde{q}\right) = MR_j\left(\tilde{q}\right)$ for some $\tilde{q}$, then it must be that

$$\eta\left(\tilde{q}\right) + \tilde{q}\eta'\left(\tilde{q}\right) = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

This implies that at any such crossing,

$$MR_i\left(\tilde{q}\right) = MR_j\left(\tilde{q}\right) = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j} = \phi.$$

Thus marginal revenue curves always cross at the same height as the demand curves. Note that the same arguments also apply to a Cournot oligopoly with m firms, where we define

$$\begin{aligned} SMR_i\left(Q;m\right) &= P_i\left(Q\right) + QP_i'\left(Q\right)/m \\ &= \alpha_i - \beta_i\left(\eta\left(Q\right) + Q\eta'\left(Q\right)/m\right). \end{aligned}$$

If $SMR_i(\tilde{Q}, m) = SMR_j(\tilde{Q}, m)$ for some $\tilde{Q}$, then it must be that

$$\eta(\tilde{Q}) + \frac{\tilde{Q}\eta'(\tilde{Q})}{m} = \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j},$$

which implies

$$SMR_i(\tilde{Q}, m) = SMR_j(\tilde{Q}, m) = \alpha_i - \beta_i \frac{\alpha_i - \alpha_j}{\beta_i - \beta_j}.$$

Hence, marginal revenue and demand curves intersect at the same height for an m-firm oligopoly setup as well. ■

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