Advertising Content

Simon P. Anderson and Régis Renault*

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Abstract

Empirical evidence suggests that most advertisements contain little direct information. Many do not mention prices. We analyze a monopoly firm's choice of advertising content and the information disclosed to consumers. The firm advertises only product information, price information, or both; and prefers to convey only limited product information if possible. It is socially harmful to force it to provide full information if it has sufficient ability to parse the information imparted, but nor does it help to restrict the information voluntarily provided.

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*Anderson: Department of Economics, University of Virginia, PO Box 400182, Charlottesville VA 22904-4128, USA, sa9w@virginia.edu; Renault: ThEMA, Université de Cergy-Pontoise, 33 Bd. du Port, 95011, Cergy Cedex, FRANCE. regis.renault@eco.u-cergy.fr; Institut Universitaire de France. We gratefully acknowledge travel funding from the CNRS and NSF under grants INT-9815703 and GA10273, and research funding under grant SES-0137001 and from the Bankard Fund at the University of Virginia. The Universities of Perpignan and Toulouse (IDEI) and Melbourne Business School were gracious hosts while some of the research was being undertaken. We thank various seminar participants and conference participants at EARIE 2001 (Dublin), Journée de Microeconomie Appliquée, Nancy, 2001 and ASSA, San Diego, 2004. An early version was presented at the Econometric Society World Congress Meetings in Seattle, 2000.

Consumers are often poorly informed about the price and attributes of products that they buy infrequently. Although consumers may be able to obtain information through their own endeavors, this entails some cost, such as the cost of visiting a store selling the product. Firms may be able to provide information to consumers through advertising before they visit the store. Information advertised may concern price as well as product attributes, and the firm must choose how broadly to inform consumers, if at all. The firm's choice of what information to publicize and the consequent market performance are the main questions we address in this paper. Intriguingly, the market performs optimally only when search costs are high! We find that the firm frequently wants to restrict the information it provides to consumers. However, forcing a firm to disclose information rarely improves performance. Conversely though, restricting information hurts performance.

There is scant evidence in the economics literature on the amount of information contained in ads. In the marketing literature, "content analysis" has spawned a large number of papers comparing content across cultures (e.g., Madden, Caballero, and Matsukubo, 1986), over time (e.g., Bruce L. Stern and Alan Resnik, 1991), across media (e.g., Avery M. Abernethy and George R. Franke, 1996), and across different regulatory regimes (e.g., Abernethy and Franke, 1998). The seminal paper by Resnik and Stern (1977) postulated 14 categories of "information cues," and the typical follow-on study (numbering over 60 to date) has categorized ads by the number of information cues present. The information cues include price, quality, performance, availability, nutrition, warranties, etc. In a survey of other papers, Abernethy and Daniel D. Butler (1992) present the results for an average over 4 studies of US television advertising and show that the mean number of cues was 1.06, with only 27.7% having two or more cues, and 37.5% having no cues. For an average over 7 studies of US magazines, the mean number of cues was slightly higher at 1.59, with only 25.4% having three or more cues, and 15.6% having no cues. Price information was given for only 19% of magazine ads (the figure is not reported for TV ads). For newspaper ads, Abernethy and Butler (1992) find a substantially higher average number of cues, with still only 39.6% having 4 or more cues. Price information was given for 68% of ads.

One explanation for the lack of information in advertisements stems from the theory of advertising as conspicuous "money-burning." Such advertising indirectly informs consumers by signalling high product quality in an adverse selection context. This theme was originally developed by Phillip Nelson (1974) and later formalized by Richard E. Kihlstrom and Michael H. Riordan (1984) and Paul R. Milgrom and John Roberts (1986).[1] Alternatively, the advertising could be persuasive advertising that shifts consumer tastes (see the controversial paper by Avinash K. Dixit and Victor Norman, 1978, and the comments thereon in later issues of the Bell Journal). The present paper proposes an explanation that is rooted in the incentives for a firm to provide directly informative advertising, by which we mean advertising involving credible information about the product (price, availability, characteristics, quality, etc.). The credibility of the message is ensured by legal sanctions on misleading advertising.

Directly informative advertising has been the topic of several previous studies. However, to the best of our knowledge, no previous work has discussed the choice of the *type* of information transmitted in advertisements. In most models (an early example is Gerard R. Butters, 1977), firms are assumed to advertise only the price charged (and therefore also that the product exists in the market at the quoted price). However, much advertising concerns more than just prices; it also (or exclusively) involves informing consumers about product attributes. The choice of content of the advertising message has not been analyzed before. At least in part, this is because most models have assumed the product sold by firms is homogeneous, so that there are no product attributes to communicate anyway.

Even when products are modeled as differentiated, if consumers face no search costs then advertising the existence of a product is all that is necessary since then consumers know prices and characteristics of all products of which they are aware (see Gene M. Grossman and Carl Shapiro, 1984, for the seminal paper in this vein). That is, there can be no role for

[1] Interesting recent extensions include Laurent Linnemer (2002) and Claude Fluet and Paolo Garella (2002). Ignatius Horstmann and Glenn MacDonald (2003) empirically evaluate this body of theory. Asher Wolinsky (1983) looks at how prices can signal quality. See Bagwell (2005) for a fine overview of the economics of advertising.

separate price and attribute advertising because consumers can find these out costlessly once they know that the product is on sale. Hence consumer search costs must be an integral part of any model that purports to look at the two dimensions of advertising messages.[2] These search costs can be viewed as the cost to consumers of going to stores to check out the product: the search good hypothesis is that attributes are observable on inspection (before purchase), as opposed to experience goods that must be bought before attributes are known (wine perhaps). One paper that does analyze firm choice of product advertising is Michael Meurer and Dale O. Stahl (1994). They determine when and whether product attributes are advertised but all consumers are assumed to observe prices.[3]

The objective of this paper is to consider the two dimensions of advertising: price and attributes. We determine the incentives for a firm to provide the two types of information, how much information the firm wants to release, the biases induced by the market system, and whether forced revelation of information improves performance. We assume that incurring the search cost enables a consumer to purchase the good; a consumer may decide not to incur the cost if the expected benefit falls short of the search cost. The visit decision is facilitated by advertising messages that give prior information about the price and/or product. Our starting point is the models of consumer search in Simon P. Anderson and Régis Renault (1999 and 2000), which in turn build on Wolinsky (1986). These models of consumer search in markets for heterogeneous products rule out advertising as a means of transmitting information.

We consider a monopoly firm selling a product for which consumers do not ex-ante know their reservation prices (or "matches"). The firm may choose to advertise price only, match only, both price and match, or not to advertise at all (any match advertising must be truthful in the sense of consistency with the true distribution). The consumer rationally anticipates

[2] Search costs were considered in the Butters paper and the extensions by Mark Stegeman (1989) and Jacques Robert and Dale O. Stahl (1993), but products are homogeneous in these models so there is no role for product advertising.

[3] This may be thought of as a model of an experience good whereby the valuation of the good is only known once it is consumed or product information is disclosed by firms.

the price charged by the firm if it is not advertised. The consumer finds out both the price and her match with the product once she has visited the firm. The consumer must visit the firm to buy the product even if the firm advertises. For low search costs, the firm earns monopoly profits without any advertising since the consumer still anticipates positive surplus at the expected monopoly price: an individual buys if her match exceeds the monopoly price.

If match advertising fully reveals the consumer's reservation price, then the firm never uses match-only advertising. If it did, then the consumer who chooses to visit while knowing her valuation perfectly reveals that she has a high willingness to pay, so that the firm charges a high price. This leads to a hold-up problem that is perfectly anticipated by the consumer, who then chooses not to visit. We then extend the analysis to allow the firm to present partial match information to consumers. The firm optimally uses minimum match advertising, which guarantees a threshold utility to the consumer. This reassures the consumer that visiting is desirable, but, by conveying a minimal amount of information, it does not reveal so much as to precipitate the hold-up problem. Match-only advertising is then restored as an equilibrium strategy, in line with casual observation. Otherwise, the firm discloses price and partial match information and sets a price which is decreasing in the search cost. For intermediate search costs it informs the consumer whether her match exceeds the price. For large search costs, it informs the consumer whether her match exceeds the search cost. In this last case, the firm implements the first-best solution: the consumer buys if and only if her match exceeds the search cost.

If partial match advertising is not feasible (and match information must be fully revealed if at all), then price must be advertised to avoid hold-up. The firm then advertises only price information for intermediate search costs, and only includes match information for high search costs.

We use these results to evaluate information disclosure policies as a possible corrective mechanism. First, a forced disclosure policy for prices is not needed. Second, it would be socially harmful to impose a full disclosure rule for product information if the firm can perfectly parse the information it conveys to the consumer. This is because the firm provides sufficient information to prevent the consumer from wasteful visits when she would end up not buying. Finally, restricting the informational content of ads would be harmful: although restrictions may cause the firm to set a lower price to entice visits, this is more than offset by the direct costs of unfruitful visits.

One of the main results is that the firm will want to convey minimum match information. This raises the issue as to how such information may be feasibly and credibly conveyed. To illustrate how this may be done, we appeal to a characteristics underpinning to the model and indicate that the underlying space of characteristics must be broad enough. We also discuss what happens when the characteristics space is smaller: in particular, it may be that the firm's choice is limited to either full match revelation or else no information at all.

The paper is organized as follows. The model is described in Section 1, which also characterizes the market outcome without advertising and gives other background results. Section 2 derives the threshold match revelation property, and analyzes price and match advertising. Section 3 characterizes equilibrium when only full match product advertising is feasible. Some welfare properties of the solution are discussed along with forced disclosure of information. Section 4 contains a discussion of extensions, and Section 5 concludes.

I. Preliminaries

A. The Model

A single firm sells a single search good which it produces at constant marginal cost, normalized to zero. The firm maximizes expected profit. On the other side of the market, each consumer buys one unit from the firm, at price p, or else does not buy. In order to buy, a consumer must incur a search cost (or visiting cost), c. Each consumer has a reservation price (or "match value") r for the product which is unknown to the firm and to the consumer prior to inspection of the good. It is common knowledge that the distribution of r has support $[a, b]$, with $b > 0$ so that there is some chance a consumer is willing to pay more than marginal cost for the product.[4] Since the consumers and the firm have identical priors, we normalize and to a single consumer. We suppose that the "hazard rate" $f/(1-F)$ is strictly increasing, where f is the density and F is the cumulative distribution.[5] When $b > a$, the consumer is *a priori* uncertain as to how much she values the good. One example is the standard uniform distribution with $a = 0$, $b = 1$ and $f(r) = 1$ for $r \in [0, 1]$. This yields a standard linear expected demand curve with unit intercepts.

Once the consumer is at the store, she buys if $r \geq p$. Beforehand, she decides to visit if her expected surplus exceeds c.[6] Her search costs are either c if she did search, or zero otherwise. The search cost must be incurred if the product is bought or if the consumer visits without buying. If a consumer is indifferent between visiting and not visiting, we assume she visits; if she is indifferent between buying and not, we assume she buys. Once the search cost is incurred, both the match as well as the price charged are fully revealed. The firm may use advertising to inform the consumer before she decides whether or not to visit.

Advertising may inform the consumer of the actual price charged: if not, she must predict it when deciding whether or not to visit. Advertising may also provide additional information on the distribution of r. Even though the firm does not know the consumer's actual value (so it may not disclose it directly), it may provide product information that enables the consumer to update, which she does in a Bayesian manner. The transmission of product information is discussed in more detail in Section III. The Appendix provides a model of characteristics in the vein of Kelvin J. Lancaster (1966) that describes how information may be communicated by describing product characteristics.

In order to concentrate on demand-side incentives, we assume that advertising reaches the consumer at no cost. As a tie-breaking rule, we assume that price information is advertised

[4] This set-up analyzes pure horizontal product differentiation. In the Working Paper version we show how the model can be extended to deal with quality that is not *a priori* known to the consumer. Once the contribution of horizontal product information is understood, it is relatively easy to add on quality.

[5] Equivalently, $1 - F$ is strictly log-concave.

[6] Her expected surplus is the expected maximum of $r - p$ and 0 (because she has the option of not buying). Ads do not directly reduce search costs, but they may reduce the effective cost of search by informing the consumer when it is worthwhile visiting.

only if doing so strictly increases profit, and similarly for match information. We first consider the benchmark cases where no information is provided on some dimension.

B. Some benchmark results

We first describe what happens if the firm provides no information at all. The consumer must then rationally anticipate its price and she searches if her expected surplus exceeds the search cost c. The consumer does not know her reservation price so the firm cannot infer any information about her when she shows up. The probability the consumer buys at price p is then $1 - F(p)$ so that the firm then charges the standard monopoly price p^m which maximizes expected revenue $p[1 - F(p)]$. The solution is either the interior one from the first-order condition, $p^m f(p^m) = 1 - F(p^m)$, or else $p^m = a$ if there is no interior solution.[7]

Anticipating the pricing outcome, the consumer only chooses to search if her expected surplus at price p^m exceeds c. She buys whenever her valuation, r, exceeds the price, p^m, in which case the surplus enjoyed is $r - p^m$: she does not buy otherwise even though she visits. Therefore, the condition for the market to be served is that:

$$(1) \qquad E(\max\{r - p^m, 0\}) = \int_{p^m}^{b} (r - p^m) f(r) dr \equiv c_1 \geq c.$$

If condition (1) is met, the firm does not advertise and consumers rationally anticipate that it charges the monopoly price. If condition (1) is violated and there is no advertising, then the consumer will not search and there will be no market for the good: this case when the market unravels is effectively that noted by Joseph E. Stiglitz (1979).[8] The firm then

[7] The increasing hazard rate assumption ensures the marginal revenue to the demand curve $1-F(p)$ slopes down. Hence the marginal revenue curve crosses marginal cost (assumed zero here) either at an output below one (corresponding to a price above a) or else it is still positive at an output of one.

[8] Search models often assume that the first search is costless. Then, under the assumptions of Peter A. Diamond (1971), the "Diamond paradox" that the market price is the monopoly one sustains even when consumer demand is rectangular and consumers enjoy no surplus. Stiglitz (1979) pointed out that the market would collapse if the initial search cost were larger than the consumer surplus at the monopoly price. This consumer surplus in our model stems from the heterogeneity of consumer valuations.

needs to advertise (either a low price or some match information) to reassure consumers that it is worthwhile they visit.

Several authors have pointed out the role of price advertising to reassure consumers that they will not face the hold-up problem should they invest search costs. This is a central tenet of Hideo Konishi and Michael T. Sandfort (2002), and traces its lineage back through Robert and Stahl (1994) and Stegeman (1991) to Butters (1977). The bigger the search cost, the lower the price that the firm must advertise to get consumers to visit. In our model, the price that would be set simply renders expected consumer surplus zero.[9]

When consumers also have differing reservation prices, one might reasonably expect that the firm could use product information to reassure some consumers they have good matches and hence get them to visit. While our main focus below is on using both product match and price strategies together, we get some useful intuition from looking at the case of advertising product matches only. Indeed, perhaps the most natural first model of match information is to suppose that the firm's match advertising reveals the consumer's reservation price to her: seeing the ad tells her exactly what the product is and then she knows how much she likes it. However, modeling information this way leads to a hold-up situation and so a firm will never want to use match-only advertising.[10] To explain this result, suppose that the consumer expects some price p. Then when the consumer comes to the firm after observing an ad, she must have a match value in excess of $p+c$. The firm could then increase its price to $p+c$ without losing such a customer.[11] Hence, there is no price below b that the consumer will rationally expect the firm to maintain, and so the consumer will never visit. Therefore the firm will never use just product advertising that allows consumers to fully infer their

[9] Konishi and Sandfort (2002) assume a uniform distribution of search costs. Thus the firm faces a trade-off in its price choice at the margin of the consumer who is indifferent between searching and not searching. By contrast, our assumption of a common search cost induces a critical price level at which all consumers are indifferent between searching and not searching. We discuss heterogeneous search costs in Section 4.

[10] An alternative interpretation is that advertising is technologically constrained to be like this. As we show below, this may happen if the product set is narrow.

[11] Similar reasoning underlies the paradox of Diamond (1971). Rajiv Lal and Carmen Matutes (1994) and Birger Wernerfelt (1994) give similar arguments.

matches.

This result poses an empirical problem since much advertising does indeed exclude prices (and so presumably does impart only match information of some sort).[12] We now show that such match-only advertisements may be optimal if the firm can transmit only partial match information. Indeed, suppose that the firm is free to impart any (consistent) information to the consumer. We now argue that this ability may enable it to extend the monopoly profit region for search costs above c_1. To see the underlying idea, consider the demand curve in Figure 1. If the consumer visits and the price is p^m, she will buy only when she is "lucky" and her reservation price exceeds p^m. Her consumer surplus is the area marked c_1 in the Figure (c_1 is defined algebraically in (1)). However, if the consumer is informed whenever she has a valuation below some $\tilde{r} < p^m$, she knows not to visit.[13] Then her surplus if she visits is conditional upon her type having a high valuation, and thus exceeds p^m.

Advertising that eliminates the lowest valuation types thus extends the range of values of c for which the firm can earn the monopoly profit. For the firm, the best possible scenario (in terms of retaining monopoly profits for the highest possible c) is to be able to tell all types with valuations below p^m that they have low valuations.[14] The ability to communicate threshold information enables the firm to retain the monopoly profit level all the way up to a level of search cost c_2, which is the expected surplus conditional on a match above p^m so

$$c_2 \equiv \int_{p^m}^{b} (r - p^m) \frac{f(r)}{1 - F(p^m)} dr = \frac{c_1}{1 - F(p^m)}.$$

Since the demand when the consumer visits is the original demand truncated at the monopoly price, p^m, this price still maximizes profit once the consumer has sunk the visit cost. Hence it does not need to be advertised because the consumer will be rationally anticipate it.

[12] This is not necessarily inconsistent with some of the other theories of advertising presented in the introduction. But it is troubling that informative advertising would necessarily have to include price information.

[13] Equivalently, she can be told she has a valuation above $\tilde{r}$.

[14] The firm may be worse off if it tells types with valuations above p^m *more accurate* information about their valuations. This is because the types without the very favorable information estimate their matches downward. For example, if the consumer were always told when her match was above $p^m + c$, then whenever she only learnt that her match was above p^m, she would not visit and the firm would forego sales.

The present match-only advertising result contrasts starkly with that of the earlier "hold-up" problem of match-only advertising that perfectly reveals matches. In the latter case, anyone visiting at an expected price of p reveals a willingness to pay of at least $p + c$, and is therefore held up. In the current situation, a visit reveals only that the consumer has a conditional *expected* valuation c above the expected price. The consumer though may find her *actual* valuation is as low as p^m, and risk of losing such consumer types restrains the firm from pricing higher. Note that by providing favorable match information, the firm avoids having to commit to a lower price. As we shall shortly see, for larger search costs the firm will use price and match advertising. Once again it will prefer to provide only partial match information, and the partial information will involve a critical threshold.

II. Price and match advertising

We have just seen that the firm would like (if possible) to send information that tells the consumer if her match is above or below some critical value. We next derive the general threshold match information property, before turning to the firm's optimal price and advertising strategy.

A. Information transmission

We allow for match information to have the most general configuration possible, which we capture formally by letting the consumer receive different signals according to a probability which depends on her tastes and thus, on the true value of r. This formulation is quite general to the extent that any form of informative product information sends each type of consumer a signal as to her actual match, and so fits our specification. This does not mean that different individuals see different ads, but rather that they update differently. The characteristics model presented in the Appendix illustrates how a consumer may interpret an ad differently depending upon her underlying tastes.

The analysis that immediately follows assumes that price is always advertised. This is done without loss of generality because, in an equilibrium where price is not advertised, the consumer anticipates a certain price, so that the firm might as well advertise it. Given our tie breaking rules, whenever the advertised price is the price that would be expected by consumers if it were not advertised, we say that no price is advertised. Similarly, we say that no match is advertised if the signal is not informative for the consumer (i.e. if the distribution of the signal is independent of r). Our task of characterizing the optimal advertising and price strategy is greatly simplified by first showing the following.

Lemma 1 *For any advertised price $p \in [0, b]$, the firm cannot do better than informing the consumer whether or not her reservation price is above some critical threshold $\tilde{r} \in [p, b]$.*

This Lemma is proved in the appendix (as are the subsequent ones). It says that a minimum utility rule cannot be improved upon. The firm advertises a message that informs the consumer whether or not her match utility value exceeds the critical threshold level and provides no further information if her match is above the threshold.

The intuition is that the threshold strategy husbands the information conveyed. Given some advertised price p, think of the threshold $\tilde{r}$ as the lowest consumer valuation for which the signal is good enough to get the consumer to visit. If the firm provides any additional information to those above the threshold, those who get the extra information have better expectations about their match. This means that others (who do not get such favorable news) infer that their match distribution is worse than they would infer from pure threshold information. If they are still willing to visit, then they would visit under the original pure threshold information, so that the latter cannot induce fewer visits. Given some probability of visiting, the firm does best by including those with the highest matches. Replacing low matches by high matches in the "good news set" of those getting favorable information both improves the incentive to visit and increases the probability of buying conditional on visiting.

The firm's optimal strategy therefore induces all those with reservation values above $\tilde{r}$ to visit. The information imparted to those with reservation values below this threshold is irrelevant. When search costs are not too high (for $c \in (c_1, c_2)$ in the solution to the match-only solution presented in Section 1), the firm is actually indifferent between disclosing a threshold at some advertised price and imparting other types of threshold information. It could, for example, disclose a threshold not too far below its price, or provide additional information to those consumers with matches above its price, as long as this information is imprecise enough that all those with matches above price still visit. This slack exists because the visit constraint for those with matches above the threshold price is not binding.

When the visit constraint binds because of high search costs, using an advertising strategy other than a threshold one may hurt the firm. Giving extra information to those with reservation values above the threshold will cause some to downgrade their expectations - the visit constraint is now violated and they would not search unless the firm posted a lower price. However, the firm still has some leeway for imparting more precise information to those below the threshold consistent with such types not visiting.

In the Appendix we describe an example of how the firm may convey threshold match information by means of announcing characteristics, illustrating how threshold information might be communicated in an advertising message. After learning partial information about product characteristics, consumers update their beliefs about the value of the good to them. Some types of consumer conclude that their benefit exceeds a threshold value, while others conclude the opposite. In the next section we return to this discussion of partial match information to stress that it must be credible, given the equilibrium behavior of the firm. We now derive our main results assuming perfect threshold information can be transmitted.

B. Optimal advertising and pricing

Clearly, with threshold match advertising, the probability that the consumer receives the favorable message is $1 - F(\tilde{r})$, which is the probability the consumer has a valuation above $\tilde{r}$. The expected surplus when the consumer receives such a message is

$$\begin{aligned} E(\max\{r-p,0\} \quad | \quad r \geq \tilde{r}) &= \int_{\tilde{r}}^{b} (r-p)\frac{f(r)}{1-F(\tilde{r})}dr \\ &= \frac{\int_{\tilde{r}}^{b} rf(r)dr}{1-F(\tilde{r})} - p. \end{aligned}$$

If the firm is to ever attract the customer, this expectation should be at least as great as the search cost, c. Lemma 1 and the above discussion enable us to write the firm's problem as:

$$\max_{(p,\tilde{r})} p(1 - F(\tilde{r}))$$

subject to the constraints

$$\alpha)\ \tilde{r} \geq p \qquad \text{and} \qquad \beta)\ \frac{\int_{\tilde{r}}^{b} r f(r) dr}{1 - F(\tilde{r})} - p \geq c.$$

From Lemma 1, the first constraint may be imposed with no loss of generality in that any solution satisfying this constraint cannot be improved upon.[15] The second constraint ensures that the consumer visits when she has a match above the threshold.

In the previous section, we showed that the firm can enjoy the monopoly profit for search costs up to c_2. In the current context, this monopoly profit can be achieved by picking $p = \tilde{r} = p^m$. This solution indeed satisfies both constraints for $c \leq c_2$. Because the monopoly price would be correctly anticipated if it were not advertised, there is no need for price advertising in this region.

For $c > c_2$, monopoly pricing can no longer be sustained through match-only advertising. We first show that the second constraint (β) necessarily binds. If neither constraint were binding, then profit could be increased by decreasing $\tilde{r}$: this would increase the probability the customer visits, and she always then buys. Hence, if (β) is not binding, then (α) must be. Substituting $p = \tilde{r}$ in the profit function and maximizing with respect to p then yields the monopoly price. But because c_2 is the expected surplus at the monopoly price, (β) can be rewritten as $c \leq c_2$, which is a contradiction. Hence, (β) must bind for $c > c_2$.[16]

[15] For $c \geq c_2$, where the visit constraint (β) is binding, constraint (α) must be satisfied by an optimal policy. First, if price is at least p^m then (α) is implied by (β). Second, if price is strictly below p^m and (α) does not hold, profit could be increased by raising the price and the threshold while keeping the visit constraint satisfied. However, as discussed in the text, for $c < c_2$ there are solutions to the firm's problem (involving wasteful visits by consumers) yielding the same profit but where constraint (α) does not bind.

[16] Given that the visit constraint is binding, imparting more information may hurt the firm. Suppose that it conveys more favorable information when the consumer's match is somewhat above $\tilde{r}$ and the match is

We only need henceforth to consider solutions at which (β) binds and at which the first constraint (α) is and is not binding, respectively. Since (β) is binding, then

$$p = \frac{\int_{\tilde{r}}^{b} rf(r)dr}{1 - F(\tilde{r})} - c \tag{2}$$

is the price that just yields zero expected surplus for the consumer. Hence, the firm gets all the social surplus, and so wishes to maximize it. When (α) is not binding, this is clearly accomplished by having the consumer visit and buy if and only if $r \geq c$. The firm thus sets $\tilde{r} = c$. More formally, we can substitute (2) into the profit function to give

$$\pi = \int_{\tilde{r}}^{b} rf(r)dr - c[1 - F(\tilde{r})],$$

which can be maximized as a function of the critical match value, $\tilde{r}$. The first derivative is

$$\frac{d\pi}{d\tilde{r}} = [c - \tilde{r}]f(\tilde{r}),$$

so that the solution is $c = \tilde{r}$ for $c \in [a, b]$, and clearly this profit is a quasi-concave function of $\tilde{r}$. The corresponding value of price is then given from the visit constraint as $p = \phi(c)$ where we have defined

$$\phi(z) = \frac{\int_{z}^{b} rf(r)dr}{1 - F(z)} - z \tag{3}$$

Note that $\phi(p)$ is the expected consumer surplus conditional on the match value exceeding price (unconditional consumer surplus divided by the probability of buying). Similarly, $\phi(c)$ is the expected social surplus conditional on the match value exceeding search cost (the social surplus divided by the probability the transaction is socially beneficial).

The solution just described (setting $\tilde{r} = c$ and $p = \phi(c)$) is then the overall solution as long as it satisfies constraint (α): that is, $\phi(c) \leq c$. If this condition does not hold, then (α) is binding. If both constraints bind, the solution is uniquely determined by $\tilde{r} = p$ and

just above $\tilde{r}$. The consumer then infers her match is more likely to be close to $\tilde{r}$. Then there would be fewer visits unless the firm dropped its price.

$c = \phi(p)$.[17] To complete the characterization, we must establish which solution prevails for different values of $c \in (c_2, b]$. To this end we must describe how ϕ behaves over this interval of search costs.

Lemma 2 *The function ϕ is strictly decreasing on $[0, b]$. Furthermore, $\phi(c_2) > c_2$, and $\phi(b) = 0$ so that there exists a unique $c_3 \in (c_2, b)$ satisfying $c_3 = \phi(c_3)$.*

The function ϕ and c_3 are illustrated in Figure 2. Hence both constraints are binding for $c \in (c_2, c_3)$ and only the visit constraint binds for $c > c_3$. The next Proposition summarizes with the full characterization of the equilibrium advertising strategy.

Proposition 1 *Suppose the firm's advertising can convey partial match and price information. The following relations hold between the search cost, c, price, p, and the threshold match advertised, $\tilde{r}$:*

i) there is no advertising for $c \leq c_1$, and $p = p^m$;

ii) only threshold match is advertised for $c \in (c_1, c_2]$, and $p = p^m$;

iii) price and threshold match are advertised for $c \in (c_2, c_3]$, and $\tilde{r} = p = \phi^{-1}(c)$;

iv) price and threshold match are advertised for $c \in (c_3, b]$, and $\tilde{r} = c > p = \phi(c)$;

v) there is no advertising (and no market) for $c > b$.

The market solution is first best optimal when match advertising is unrestricted and $c \geq c_3$.

The firm charges the monopoly price for $c \leq c_2$ (and enjoys the monopoly profit). Above c_2, the equilibrium price is decreasing in c in order to improve the attractiveness of visiting in the face of a higher cost of visiting. Indeed, beyond c_2 the firm needs to improve the deal expected by consumers who visit. This can be done in two ways: either by reducing price and/or by improving the expected match of visitors, which suggests that a higher threshold match be communicated. Nevertheless, threshold values initially fall. They do so in lock-step with prices until c_3, after which prices continue to fall but threshold values rise. Thus

[17] When $p = 0$, $\phi(0) = E(r|r > 0) > 0$, which is the conditional expectation of r given r is positive. For $p = b$, $\phi(b) = 0$. Since we next show that ϕ is strictly decreasing, there exists a unique solution, which is decreasing in c, for $c \in [0, \phi(0)]$.

prices fall monotonically while threshold levels have a dip. For large enough search costs, when the threshold is rising, the firm actually implements the first best social optimum by inducing only those consumers with $r > c$ to visit and buy. The explanation for these patterns (illustrated in Figure 3) is as follows.

First, note that the socially optimal allocation entails consumption by all those types whose valuations exceed the search cost, c. If consumers are told that their valuations exceed c and then are charged a price (below c) that is low enough that they are willing to visit, the optimum may be decentralized through prices and still raise revenue. Surprisingly, the market delivers this outcome for c large enough. Whether or not the firm will want to implement the first-best depends upon whether it can capture the whole social surplus. If it conveys a threshold match value of c, it will get the right people to come and the first-best optimum will be achieved as long as they all buy. This requires that the price charged is below c, but it must also be large enough so as to enable the firm to appropriate all of the consumer surplus. In other words, the price must make the visit constraint bind. Think about this visit constraint evaluated at the social optimum allocation: the net (of search cost) social surplus per consumer is ϕ. This must equal the price for the firm to get all of the surplus. But the price must also not exceed r. Thus the firm decentralizes the optimum for $\phi(c) = p < r = c$, or indeed beyond c_3.

For lower c, the firm cannot get the full surplus out of the market: setting $\tilde{r} = c$ while pricing below $\tilde{r}$ (to ensure all visitors buy) would leave consumers with positive expected surplus. Instead, the firm prefers a price above c. Its profit maximizing strategy depends on whether the visit constraint is binding or not. If it is not, then the firm picks the monopoly price and earns monopoly profit. This yields an expected consumer surplus of c_2 provided that those consumer types willing to pay the monopoly price know this before they visit. So monopoly profit is achieved for $c < c_2$.

At c_2 the visit constraint becomes binding, but the firm is not yet able to achieve the first-best surplus. The reason is that c_2 is less than p_m, which is the price that takes away all of the surplus at c_2[18]. This condition holds as long as the distribution of r is not too degenerate. Indeed, recalling that c_2 is defined as the consumer surplus at the monopoly price divided by the number of consumers buying at the monopoly price, the condition $p_m > c_2$ stipulates simply that the monopoly profit exceeds the standard net consumer surplus under the demand curve. The reader can likely see that this property holds for any concave demand curve (for linear demand, profit is twice the consumer surplus triangle). Anderson and Renault (2003) show that monopoly profit strictly exceeds consumer surplus if demand is not "too convex" - equivalently, in the present context, if $1 - F$ is strictly log-concave (i.e., the demand function exhibits a strictly increasing hazard rate).

Finally, for $c \in [c_2, c_3)$ the firm must charge a price above c to capture all of social surplus. Setting a threshold below price would reduce social surplus and thus the firm's profit because this would induce wasteful visits by consumer types who would end up not buying. Setting a threshold above price would also reduce social surplus by preventing some consumer types whose valuations exceed c from visiting and thus from buying the product. In that range the threshold should therefore equal price and so fall as search costs increase.

III. Constrained information transmission

It was assumed in the analysis above that the firm could use product advertising to impart to consumers any form of match information it wishes that is consistent with the true distribution of tastes (it cannot convince consumers that they all have a high willingness to pay for its product). In our general framework, an ad sends different signals to different consumers, depending on their actual tastes. Although we have mentioned that a firm may be able to advertise the characteristics embodied in its product, we have not yet addressed how the signals such advertising can divulge may be limited.

More specifically, a full description of the message transmission process would have the consumer update her beliefs about her product match from the certifiable product infor-

[18] Equivalently, $c_2 < c_3$, or indeed $\phi(c_2) > c_2$.

mation in the advertising message as well as from the equilibrium advertising and pricing strategy she expects from the firm. The informativeness of signals received by the consumer may therefore be constrained both by the nature of the certifiable information that may be technologically transmitted as well as by credibility restrictions that may lead the firm to adopt different strategies depending on which product it is selling. In this Section we explore the implications of such constraints in more detail. We first use a product characteristics example to show there that the optimal threshold strategy is technically feasible as well as credible since it is part of a Perfect Bayesian Equilibrium. We then discuss an example where the paucity of the product space forces the firm into providing full product information. We end by describing the optimal price and advertising strategy for a firm that is not able to impart partial product information and discuss the desirability of policies that would legally constrain the information the firm can communicate through its ads.

A. Feasible and credible information transmission

We show in the Appendix that a firm may announce a threshold by announcing some of the characteristics that its product embodies. This is equivalent to specifying a subset of products to which its own product belongs. It must be the case however that in equilibrium, all products in that set have an incentive to make the same announcement. That is, we must show how a firm can announce a threshold in a Perfect Bayesian Equilibrium.

The rest of our discussion is in terms of declaring product sets rather than disclosing characteristics. Table 2 gives the match values of consumers of types A, B, and C for the six potential products a through f. The threshold value of 2 is communicated by the firm by announcing $\{a, c\}$ if the product is a or c, $\{b, e\}$ if the product is b or e, $\{d, f\}$ if the product is d or f. Notice that each such disclosure policy amounts to informing one of the consumer types that her match is bad. The others can only infer that they have at least a match of 2. Note too that from the firm's viewpoint, the distribution of matches of those above the threshold is the same whether the product is a or c.

Consider a firm selling product a and wishing to convey to its customers a threshold value of 2. It therefore has to tell consumers A and B that the product is in the group of products yielding for them a match of 2 or 3. By construction there are two such products, a and c. If the other product, c, chose the same announcement, it would also convey a minimal match of 2 to the same consumers A and B. Only these two consumers would choose to visit and the demand for each of the 2 products among visitors would be the same ($r = 2$ with probability .5 and $r = 3$ with the same probability). If it is optimal for product a to use this strategy, then it is optimal for the other product in the set that is disclosed to use this strategy (that is, they all must disclose they are in this set).[19]

We now show with an example, that if the product space is too sparse, the only credible product information that the firm may convey would be full match information.

B. An example with a narrow product set

Consider now a restricted version of the example above. Instead of 6 possible product types, there are now only 3 as given in Table 3.[20] The example retains the property that the reservation price distribution is uniform across both consumer types and firm types.

As above, we can think of information as a firm declaring a sub-set of potential products to which it belongs: characteristics may once more be understood to underlie how this information is portrayed.[21] This example illustrates that if the product space is not rich enough, a firm may not be able to implement the optimal threshold strategy. For instance, here if a firm selling product a wishes to advertise a threshold of 2, it would need to convey to consumer A the information that its product is either a or e while indicating to consumer B that its product is either a or d. This cannot be done if both types of consumer observe the same ad message.

Indeed in this example, it is not possible for the firm to credibly convey ANY partial

[19] A product could technically convey other threshold information. For instance, product a could claim that it is either a or d. However, as we show below, this could not be part of a Perfect Bayesian Equilibrium.

[20] Table 3 may remind the reader of the Condorcet paradox. We thank a referee for the example.

[21] For example, using Table 1, if only products a, d, and e are possible, then announcing characteristic 5 tells the consumer that the product must be d or e.

information for almost all c. To see this, suppose that there were an equilibrium at which two different products credibly convey the same information. Without loss of generality, suppose a firm selling either a or d announced $\{a, d\}$ in equilibrium. We now show that for almost all values of c, $c \neq \frac{1}{2}$ and $c \neq 1$, it is always profitable to deviate to providing full match information for a firm selling one of the products a or d.

First, it is clear that if a firm selling either product sells only to one consumer type in equilibrium, it can do strictly better by fully revealing its product and selling to whichever consumer type has a valuation of 3 at a price of $3 - c$. Now assume a firm with product a sells to all three consumer types in equilibrium. Then in order for consumer type C to visit and buy product a, the price, and thus profit, must be at most $\min\{1, \frac{3}{2} - c\}$, where the critical visit price is obtained by noting that if C buys a then she also buys d. If the firm reveals that its products is a, then it may earn a profit of $\frac{4-2c}{3}$, by selling to consumer types A and B at a price of $2-c$; this is better than the profit in the assumed equilibrium if $c \neq \frac{1}{2}$.

Finally, assume that in equilibrium, product a is sold to two consumer types: then it must be types A and B. Furthermore we have already shown that product d is sold to at least two consumer types so that either C or A buy it. If A buys d, then in order for A to visit and buy both products the equilibrium price is at most $\min\{1, 2 - c\}$ so that profit is at most $\min\{\frac{2}{3}, \frac{4-2c}{3}\}$; full match disclosure and selling to consumer type A alone would yield a profit of $1 - \frac{c}{3}$, which is larger except if $c = 1$. Now suppose to the contrary that A does not buy product d so that C does, while product a is still sold to two types, A and B. Since C visits expecting to buy product d alone, the price is at most $2 - 2c$: however, by disclosing full match information, a firm with product a could still sell to types A and B at a price of $2 - c$. Hence, in any Perfect Bayesian Equilibrium, either the firm conveys no product information or it conveys full product information.

C. Full-match advertising

The previous example showed that it may only be feasible to impart full match information (or else nothing at all). We now consider the case when advertising of matches necessarily reveals full match values to the consumer. This is also useful to analyze because it corresponds to a full information disclosure policy that we discuss below.

Suppose then that an advertisement contains both price information and fully-revealing match information so that the consumer learns everything when she sees an advertisement. In that case, she will buy if the advertised price is not above the revealed match value net of the search cost. The demand facing the firm when it advertises matches and price p is therefore $1 - F(p + c)$, the inverse demand curve being thus shifted down by c from its original position. Clearly then the (producer) price, p^f, that maximizes profit against this demand curve is below p^m.[22] Profits are likewise lower. However, the *full price* paid by the consumer, $p^f + c$, is higher than p^m.[23]

If search costs are zero, the firm's profit is the standard monopoly profit whether it advertises price only or price and match. For low c such that (1) holds, i.e., $c \leq c_1$, the firm still earns the full monopoly profit under price-only advertising (advertising a price p^m) while its profit is strictly decreasing in c under price-and-match advertising. By continuity price-only advertising still strictly dominates price and match advertising for c slightly larger than c_1. Since (1) does not hold for such values of c, price-only advertising is the optimal strategy because the market could not be served without advertising. At the other extreme, if $c > b$, there is no price at which the market could be served regardless of the advertising strategy because no consumer's valuation is higher than the search cost. For c slightly below b, price only advertising is not profitable, because even if a zero price were advertised, there would still be no demand since the expected surplus of a consumer coming to the firm is strictly less than b. However, with price-and-match advertising, the firm could sell to consumers with high matches at a (small) positive price. Hence, for a high enough $c < b$, price-and-match advertising maximizes profit. We now show that the two profits as functions of c cross only

[22] >From the first-order condition for a profit maximum of $p(1 - F(p + c))$, p^f is the (implicit) solution to $p^f = \frac{(1-F(p^f+c))}{f(p^f+c)}$, where the LHS is a decreasing function of p^f and c under the monotone hazard rate assumption. It follows directly that p^f is less than p^m.

[23] This can be derived directly from the first-order condition for price and again using the monotone hazard rate condition, similar to the previous footnote. A proof is in Lemma 4 below.

once, meaning that the optimal strategy for the firm runs from no advertising to price-only advertising and then to price-and-match advertising as search costs increase. We prove the crossing property by showing that profit is concave in c under price-only advertising but convex under price-and-match advertising.

Lemma 3 *Profit under price-only advertising is decreasing and concave in price.*

Lemma 4 *Profit under price–and-full-match advertising is decreasing and convex in price.*

>From the above arguments, the profit function for price-only starts out above that for price-and-match and ends up below. These functions being continuous, they must cross at least once. At the earliest crossing point, the price-only profit crosses the price-and-match profit from above, so that it is steeper. The concavity and convexity properties ensure that they do not cross again at larger values of c. Hence there is a unique crossing point, denoted $\hat{c}$. The following proposition summarizes.

Proposition 2 *When match advertising must fully reveal consumer valuations:*

i) there is no advertising for $c \leq c_1$*;*

ii) only price is advertised for $c \in (c_1, \hat{c})$*;*

iii) price and match are advertised for $c \in (\hat{c}, b)$*;*

iv) there is no advertising for $c \geq b$*.*

There is an interesting comparison in the predictions from our two propositions about the information content of advertising. If advertising must be fully revealing, then the firm is more likely to use price information and less likely to use product information in the following sense. A firm that can use perfect threshold information does so for $c > c_1$, but it only starts using price information for $c > c_2$. Conversely, a firm that must use fully revealing match information starts using price advertising at c_1 but only introduces product information later, for $c > \hat{c}$. Thus product information is used later when match information must be fully revealing, and price information is used earlier.

The results from the two different versions of advertising information also provide some insights into the desirability of policies that require firms to disclose particular information in ads, or else restrict them from doing so.[24] Suppose that firms are able to provide perfect threshold information and consider the desirability of forcing firms to provide full match information.[25] For visit costs above c_1, the firm provides threshold. Forcing the firm to advertise full match information[26] is inferior because fewer trades are consummated under full match information (since the full price exceeds p^m) and all trades consummated under threshold match information ought to be consummated since they entail reservation prices higher than the search cost. Thus, whenever the firm is able to convey perfect threshold match information, it is never optimal to force it into full match advertising.

Whenever the firm chooses not to advertise a price, consumers rationally expect the price it charges and it always has the option of advertising another price. If it is forced to advertise a price, it will merely advertise the expected price and so no welfare gains can be achieved. Hence, in this set-up, it is never optimal to force the firm to reveal price information.

Finally, we investigate the desirability of *restricting* the information that the firm may provide through the ads. It is never optimal to require the firm *not* to advertise its price. When the firm finds it profitable to advertise a price, it commits to a lower one than would be otherwise anticipated so stopping price advertising would never enhance efficiency. Now

[24] For example, prescription drug ads must list side-effects and contra-indications (problems a user may experience if she also takes other drugs). Because the prospective user must see a physician to get the prescription, she will later get this information anyway. An example of an ad limitation is the European case of restrictions on using comparative advertising.

[25] If only full match information is feasible, it may be optimal to force the firm to provide it when it would choose to provide only price information. At $\hat{c}$, the firm is indifferent between advertising price only or advertising price and match information. Consumer surplus is zero under the first option and positive under the second, and so social surplus is higher if the firm can be coerced to add match information. For c just below $\hat{c}$ the firm prefers to advertise price only though welfare is higher forcing it to add full match information.

[26] The firm would then also necessarily advertise price since otherwise the market will collapse.

consider match information. If the firm can advertise perfect threshold match information, then consumer surplus is zero if it does so. But consumer surplus is also zero if it advertises only price (since in both cases it binds consumers down to their visit constraint). Thus, if the firm chooses to advertise match information, it is socially beneficial since it captures the whole social surplus. If the firm is only able to provide full match information, then if it chooses price-and-match it earns a higher profit than under price only. Furthermore, consumer surplus under price-and-match is positive while it is zero under price-only. Hence an outcome where the firm provides only price information is clearly dominated.

IV. Discussion

We have assumed that ads are costless, and reach all consumers. Most of the key insights come from the demand side and our main results are not based on costlessness. There are several ways to introduce costs into the model. First, the basic ad could be costly so that the firm has a decision whether to advertise or not. Adding such costs multiplies the number of cases to consider, but would not seem to add substantially to the insights. Second, the cost could depend on the reach of the ad, meaning the number of potential consumers who see it. We address this issue separately in the next sub-section. Third, the cost of the ad could depend on the degree (or type) of information it imparts. If it is costly to disclose price, the firm strictly prefers to not disclose it for the parameter values where we found match-only advertising. However, if it is quite costly to communicate (partial) match-only information, the firm might prefer a price-only advertising strategy to imparting complex match information.

There are some other issues involved with price advertising. First, it may not be legal, insofar as resale price maintenance is not legal, although manufacturers often post the statement "suggested retail price" in car ads. Second, it may be very difficult for a firm selling many products (like a hypermarket, or a dentist with many different services) to advertise all of its prices. Third, a national ad may cover several regional sub-markets, in which the firm may want to price according to local market conditions and not commit to a uniform price for all regions. It would be worth investigating equilibrium advertising strategies in a model where a manufacturer deals with product advertising and the local retailers add more specifics if they so desire.

A. Advertising reach

Much of the previous literature on informative advertising has been pre-occupied with the quantity of advertising sent. The number of consumers reached depends on the number of ads sent. Assume now that consumers are uninformed unless they receive an ad message, and a broader reach entails a higher expenditure. As in the traditional analysis of informative advertising (such as Butters, 1977, or Grossman and Shapiro, 1984), the equilibrium reach depends on the profitability of communicating with an additional consumer, while the optimum reach depends on the additional social surplus generated. When these two magnitudes differ, equilibrium and optimum reach diverge.[27]

Our previous analysis enables us to compare the endogenous equilibrium reach with the optimum one. Equilibrium reach is determined by equating profit per consumer with the cost from reaching an extra consumer, whereas the optimum equates social surplus per consumer to the cost. The equilibrium sequencing of advertising content is exactly as when advertising is costless, and, since we already found that per consumer profits decline with search costs, it is now apparent that equilibrium reach declines with the search cost.

In equilibrium, neither content nor price is advertised for $c \leq c_1$, and the firm needs only advertise where the product can be bought. Only the minimum match threshold is advertised for $c_1 \leq c \leq c_2$. For $c \leq c_2$, the firm under-advertises insofar as the monopoly profit understates social surplus. That is, the second-best reach - constrained by the monopolist's price and content choice - would be higher than the equilibrium, and the first-best would be

[27] Following A. Michael Spence (1976), the equilibrium reach will exceed the optimum if and only if the business stealing effect exceeds the non-appropriation of consumer surplus effect. With a monopoly, advertising can never be excessive (barring advertising externalities) because there is no business stealing.

higher still. For $c_2 \leq c \leq c_3$, the equilibrium content is price and threshold match. Reach is still insufficient relative to the first-best optimum because profit understates the first-best surplus. However, in this range the equilibrium reach is second-best optimal since profit captures all the social surplus.

The last segment is the most striking: for $c_3 \leq c \leq b$, the full social optimum is attained in reach as well as content. The content choice is optimal because it elicits the optimal set of visitors from those receiving the ads, and the reach decision is optimal because profit and social surplus coincide, consumer surplus being zero. Thus the first-best optimum is attained in both the content and reach dimensions.

B. Heterogeneous search costs

We have assumed that consumers are ex-ante identical. Their valuations are drawn from the same distribution and they share identical visit costs. If instead there is a finite number of positive search costs, the hold-up argument presented in Section 1 still holds so advertising full match information alone can never be optimal. The firm still would want to use threshold information if there are no restrictions on the degree of information that may be communicated. There may now be multiple thresholds, but the number of thresholds should not exceed the number of search cost values. Indeed, following the intuition for Lemma 1, there is no point to providing more information than a threshold match value to those with the highest search cost. Similarly, it may be optimal to merely tell those with the second highest search cost that they exceed some other minimal match value below the first one (those who find out that their valuations lie between the two critical matches then choose to visit if they have the lower search cost) but there is no point to telling them more, etc. The monopoly solution may involve fewer thresholds than there are cost types when search costs are close together (bunching) or when high search costs are so high (or the number of consumers is so small) that the firm gives up on selling to those types. (This is analogous to models of second degree price discrimination where it can be optimal to give up serving some groups to save on the informational rent appropriated by other groups of consumers.)

We expect similar results if priors about the match value differ across consumers. For instance, a consumer's reservation price could comprise an unknown component that is only revealed to her through search or ads plus some other component that differs across consumers but for which each consumer knows her realization. Clearly, it will be easier to attract those consumers with better ex-ante expectations about their matches, so that the firm may wish to disclose lower threshold values for such consumers. Then the firm's incentives to reveal information would depend on the heterogeneity in matches known by consumers relative to the heterogeneity of the unknown component.

The potential use of multiple thresholds suggests that ex-ante heterogeneity among consumers induces the firm to use more informative advertising messages. Accounting for such heterogeneity would provide a richer model of advertising content.

C. Oligopoly

The monopoly analysis provides some pointers for oligopoly. As long as advertising is costless, each firm should always advertise its price. Doing so avoids a hold-up problem whereby consumers would expect it to charge more than it would advertise. The issue is then whether a firm wishes to disclose product information and, if so, how much.

In Meurer and Stahl (1994), prices are observed, and products are experience goods (so that search cannot reveal any additional information before buying). With no advertising, price equals marginal cost because products are ex-ante identical. Product advertising is a public good for firms because both firms benefit from the increased product differentiation: a firm that advertises its own product uses its market power to charge a higher price, thus allowing its competitor to do the same. In their framework a firm may choose not to advertise because advertising has a cost. Our analysis suggests that if the Meurer and Stahl framework is recast in terms of search goods with costly visits by consumers, then a firm may volunteer no product information even if there is no cost.

Suppose now that prices may *not* be advertised (as we suggested above, this situation may arise for various reasons). Anderson and Renault (2000) show in a simple search model that firms earn more when consumers have more product information before searching: consumers search less so there is less competition for searching consumers, and thus higher prices. That suggests that firms may have a strategic incentive to transmit product information.

The above discussion assumes some product differentiation. If instead firms sell the same product (and consumers know this), the unique equilibrium outcome (retaining the zero advertising costs assumption) will be marginal cost pricing. As in the standard Bertrand analysis, any higher price will be undercut. If a firm does not advertise, consumers expect it to price at the lesser of the monopoly price or c above the price advertised by the other firm. Neither firm (expecting zero profit) has any incentive to provide product information, so that there are wasteful visits. If the search cost exceeds expected consumer surplus at marginal cost, nothing is sold so the market collapses (whereas it is optimal that all those with $r > c$ buy).

V. Conclusions

Many advertisements inform the consumer about product characteristics. Others give price information with very little product information, and some provide both. The economics literature has not discussed the content of advertisements, and has instead looked at whether too many or too few potential customers are informed in equilibrium. We have analyzed the incentives for a firm to provide various types of information. There is no incentive to provide *precise* information on product characteristics only, since doing so leads to a hold-up problem: consumers would rationally expect the firm to charge such a high price that no consumer would incur the prior search cost.

The firm prefers to provide threshold match information which merely informs the consumer if her utility is above some value. A price is advertised along with the threshold only if the search cost is large enough. Thus for intermediate search costs the firm may use product-only advertising, in line with casual observation.

Threshold information advertising prevents the consumer from wasteful visits where she would incur the search cost and end up not buying the good. For large enough search costs the firm actually implements the first best optimum in which the consumer buys if and only if trade results in a surplus which exceeds search costs. A forced disclosure policy is only desirable when the firm provides no information or when it cannot parse the information it provides. Such a policy may even be harmful in the case where the firm chooses to provide partial match information.

An important research direction is to look at restrictions on the type of horizontal match information that may be imparted through an ad. Feasible information transmission may be based on disclosing the characteristics embodied in products. The ability to divulge intermediate match information (i.e., short of the full match advertising that follows from fully revealing all characteristics) depends on two factors. Technological feasibility requires that some characteristics be shared across different possible products, so that advertising a particular sub-set of characteristics leaves some doubt for the consumer as to her precise valuation. Message credibility requires that any advertising choice ought to be an equilibrium choice for all product types that are purported to be making that choice (i.e., the product types should be pooling in a Perfect Bayesian Equilibrium). We have suggested through examples that the product space needs to be sufficiently broad for threshold match information to be possible, while a sparse product space might render only full match advertising possible. A broader analysis of information revelation in advertising would provide a more complete picture of the market failures at play in the provision of informative advertising.

A Appendix. Characteristics model.

Suppose there are three consumer types, A, B, and C. There are six possible products, as determined by the alternative combinations of six constituent characteristics that they embody. Assume that any one of these products is equally likely and similarly for the consumer types, and suppose that consumer A cares only about characteristics 1 and 2, consumer B about 3 and 4, and consumer C about 5 and 6. Each consumer has a base value of 1 for the product plus one for each characteristic it embodies that she cares about. Each potential product embodies 3 characteristics as shown in Table 1. The products (a through f) are the row entries and the columns are the characteristics. An $\times$ denotes that the product has the characteristic.

For each product there is one consumer who has a maximal match (hence the block diagonal entries), one consumer with an intermediate liking (given by putting in the first of the 2 characteristics a consumer cares about), and one consumer who has the least valuation of 1. Table 1 and the probability distributions over products and consumer types are common knowledge. This example corresponds to a uniform distribution of tastes (like a linear demand in that the willingness to pay generates three points on a line - non-linear demands are generated from different consumer valuations for the different characteristics).

Suppose that the firm selling product a were to advertise characteristic 1, then the consumer knows that the product sold is one of a, b, c, or e. A consumer of type A learns that her match is at least 2, and no further information. whereas a consumer of type B or C learns that her match is 1 with probability 1/2, 2 with probability 1/4 or 3 with probability 1/4. [28]

The example above illustrates that it is feasible for firms to communicate different match information to different individuals, even though they all receive the same message. We now indicate how the firm may convey threshold matches of 1, 2, or 3. A match value 1 is associated with no desired characteristics, and so this minimum match value is communicated by announcing nothing. A threshold of 3 is achieved by a firm selling product a or b by announcing characteristics 1 and 2, etc. Only a seller of a or b can claim these characteristics. A threshold of 2 is attained by a firm with product a or c by announcing characteristics 1 and 3. Again, only these two products can truthfully claim these two characteristics. (Products b and e would claim 1 and 5; and products d and f would claim 3 and 5).[29]

[28] These probabilities would be correct inferences in equilibrium only if the four products had the same equilibrium advertising strategy, which may not be the case. We return to this issue in Section III.

[29] This example readily extends to more types. With n types, there are $n!$ potential products. The exclusion principle (keep out all those who must have bad matches) still holds.

Appendix. Proof of Lemmas

Proof of Lemma 1.

In order to make the analysis as general as possible we define an information transmission mechanism (henceforth ITM) as follows. An ITM induces a probability measure over the joint space of valuations and signals sent via advertising and enables the consumer to infer something about her valuation from the interpretation of the signal received. Hence an ITM is a probability space $([a,b] \times S,\, ß([a,b]) \times H, P)$ where $ß([a,b])$ denotes the σ-field of Borel sets in $[a,b]$, S is a set of signals, H is a σ-field of subsets of S and P is a probability measure over $[a,b] \times S$ that satisfies $P(r \leq \bar{r}) = F(\bar{r})$ for all $\bar{r} \in [a,b]$. For each ITM and for a price p let us define the good news set to be the largest set $D_P \in H$ such that for all $s \in D_P$,

$$E[\max\{r - p, 0\} \mid s, P] \geq c,$$

where the argument P indicates that the expectation is taken using probability P. The Lemma is proved by proving the following claim.

Claim 1 *Consider an ITM* $([a,b] \times S,\, ß([a,b]) \times H, P)$. *For any price* p*, there exists another ITM with signal set* S' *and probability* P' *such that for some* $s^* \in S'$ *and some* $\tilde{r}$*,*

(i) $P'(s = s^* \mid r \geq \tilde{r}) = 1$ *and* $P'(s = s^* \mid r < \tilde{r}) = 0$*;*

(ii) the probability of buying is at least as large as in the initial ITM.

Proof. If $P(s \in D_P) = 0$, the claim trivially holds. We now assume $P(s \in D_P) > 0$. Let $s' \in \{\bar{s}, s^*\}$ and define P' as follows. Let $\tilde{r}$ be the unique solution to $P(s \in D_P) = 1 - F(\tilde{r})$ and let $P'(r \leq \bar{r}) = F(\bar{r})$ for all $\bar{r} \in [a,b]$, $P'(s = s^* \mid r) = 1$ if $r \geq \tilde{r}$ and $P'(s = \bar{s} \mid r) = 1$ if $r < \tilde{r}$.

By construction, $([a,b] \times S, ß([a,b]) \times H', P')$ is an ITM that satisfies (i), where H' is comprised of all subsets of S'. We first show that for all $\bar{r} \in [a,b]$,

$$P'(r > \bar{r} \mid s^*) \geq P(r > \bar{r} \mid s \in D_P). \tag{4}$$

We have $P'(s = s^*) = 1 - F(\tilde{r}) = P(s \in D_P)$. We also have

$$P'(s = s^* \mid r > \bar{r}) = \frac{1 - F(\max\{\bar{r}, \tilde{r}\})}{1 - F(\bar{r})} \geq \min\left\{1, \frac{P(s \in D_P)}{1 - F(\bar{r})}\right\} \geq P(s \in D_P \mid r > \bar{r}).$$

Now, $P'(r > \bar{r} \mid s^*) = \frac{P'(s=s^*|r>\bar{r})[1-F(\bar{r})]}{P'(s=s^*)} \geq \frac{P(s \in D_P|r>\bar{r})[1-F(\bar{r})]}{P(s \in D_P)} = P(r > \bar{r} \mid s \in D_P)$

To prove condition (ii) let us define the conditional cumulative distribution functions $G'(. \mid s^*)$ and $G(. \mid D_P)$ on $[a, b]$ by $G'(\bar{r} \mid s^*) = 1 - P'(r > \bar{r} \mid s^*)$ and $G(\bar{r} \mid D_P) = 1 - P(r > \bar{r} \mid D_P)$ for all $\bar{r} \in [a, b]$. From (4),

$$G'(\bar{r} \mid s^*) \leq G(\bar{r} \mid D_P) \tag{5}$$

for all $\bar{r} \in [a, b]$. Using integration by parts we have

$$E[\max\{r - p, 0\} \mid s^*, P'] = \int_p^b (r - p) dG'(r \mid s^*) = \int_p^b [1 - G'(r \mid s^*)] dr. \tag{6}$$

and

$$E[\max\{r - p, 0\} \mid D_P, P] = \int_p^b (r - p) dG(r \mid D_P) = \int_p^b [1 - G(r \mid D_P)] dr. \tag{7}$$

>From the definition of D_P we have $E[\max\{r - p, 0\} \mid D_P, P] \geq \inf_{s \in D_p} E[\max\{r - p, 0\} \mid s, P] \geq c$, and thus, using (5), (6) and (7) we have $E[\max\{r - p, 0\} \mid s^*, P'] \geq c$.

This shows that the consumer visits if she observes s^*, so that the probability of visiting is at least as large as in the initial ITM. If $\widetilde{r} \geq p$, the probability of buying conditional on visiting is 1, which implies (ii). If $\widetilde{r} < p$, the $\widetilde{r}$ could be increased to p and the probability of buying would be $1 - F(p)$ which is an upper bound given price p. Once again (ii) holds. ■

Proof of Lemma 2.

Using integration by parts, for $z \geq a$, the derivative may be written as

$$\phi'(z) = \frac{f(z)}{1 - F(z)} \frac{\int_z^b (1 - F(r)) dr}{1 - F(z)} - 1.$$

Rewrite the integral term as $\int_z^b \frac{1-F(r)}{f(r)} f(r) dr$; since we assume that $\frac{f}{1-F}$ is strictly increasing, its reciprocal is strictly decreasing and the integrand is smaller than $\frac{1-F(z)}{f(z)} f(r)$, so the term $\int_z^b (1 - F(r)) dr$ is less than $\frac{1-F(z)}{f(z)} \int_z^b f(r) dr$, which means that the derivative is strictly negative. For $z < a$ the derivative is simply -1.

Proof. We now prove that $\phi(c_2) > c_2$. From Anderson and Renault (2003), we know that since $1 - F$ is strictly log-concave then monopoly profit exceeds consumer surplus, i.e., $p^m[1 - F(p^m)] > \int_{p^m}^{b}(r - p^m)f(r)dr$. Equivalently, $p^m > c_2$. Now, since ϕ is strictly decreasing, $\phi(p^m) < \phi(c_2)$. But $\phi(p^m) = c_2$ and so $c_2 < \phi(c_2)$ as desired. ■

Proof of Lemma 3.

For price-only advertising, the profit function is $\pi = \hat{p}[1 - F(\hat{p})]$, where $\hat{p}$ is the price that sets consumer to zero. Hence $\int_{\hat{p}}^{b}(r - \hat{p})f(r)dr = c$, and

$$\frac{d\hat{p}}{dc} = \frac{-1}{1 - F(\hat{p})}.$$

Hence

$$\begin{aligned}\frac{d\pi}{dc} &= [1 - F(\hat{p}) - (\hat{p})f(\hat{p})]\frac{-1}{1 - F(\hat{p})} \\ &= -1 - (\hat{p})\frac{f(\hat{p})}{1 - F(\hat{p})}.\end{aligned}$$

The second term on the right-hand side of the last equation is increasing in $\hat{p}$ from the increasing hazard rate assumption. Since $\hat{p}$ is decreasing in c, profit is concave in c.

Proof of Lemma 4.

Under price-and-full-match advertising, the profit function is $\pi = p[1 - F(p + c)]$. The first order condition that determines p^f yields $p^f - \frac{1-F}{f} = 0$, where the argument of F and f is $(p^f + c)$ and has been suppressed. Applying the implicit function theorem gives

$$\frac{dp^f}{dc} = -\frac{f^2 - [1 - F]f'}{2f^2 - [1 - F]f'}.$$

Both the numerator and the denominator are positive under the increasing hazard rate assumption, so that $\frac{dp^f}{dc} \in (-1, 0)$. With this property in mind, we can evaluate the derivative of the profit function, using the envelope theorem, is

$$\begin{aligned}\frac{d\pi}{dc} &= -p^f f(p^f + c) \\ &= -\left(1 - F(p^f + c)\right).\end{aligned}$$

The last step follows from the first order condition. Now, since $p^f + c$ is increasing in c, $d\pi/dc$ is increasing in c so that profit is convex.

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Tables

Table 1. Characteristics representation of products.

	A		B		C	
	1	2	3	4	5	6
a	×	×	×			
b	×	×			×	
c	×		×	×		
d			×	×	×	
e	×				×	×
f			×		×	×

Table 2. Match values for the products.

	a	b	c	d	e	f
A	3	3	2	1	2	1
B	2	1	3	3	1	2
C	1	2	1	2	3	3

Table 3. Match values for restricted example.

	a	d	e
A	3	1	2
B	2	3	1
C	1	2	3

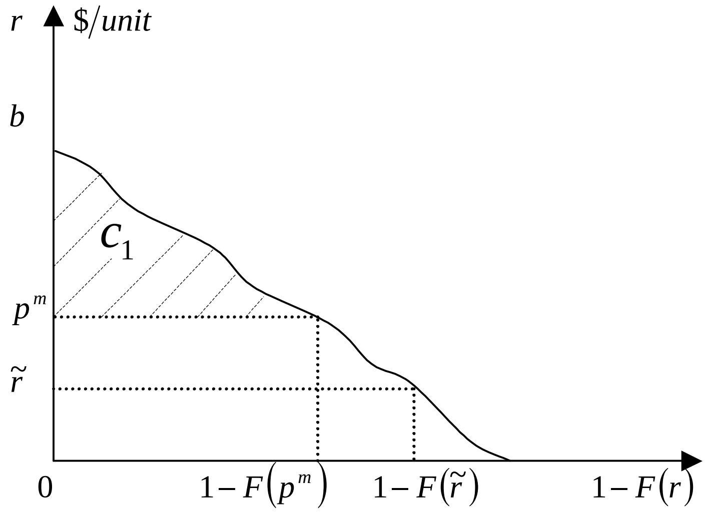

Figure 1

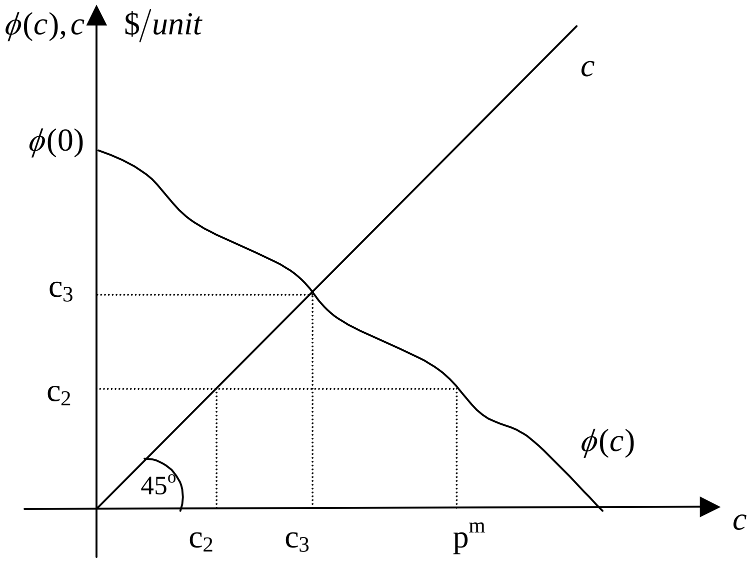

Figure 2

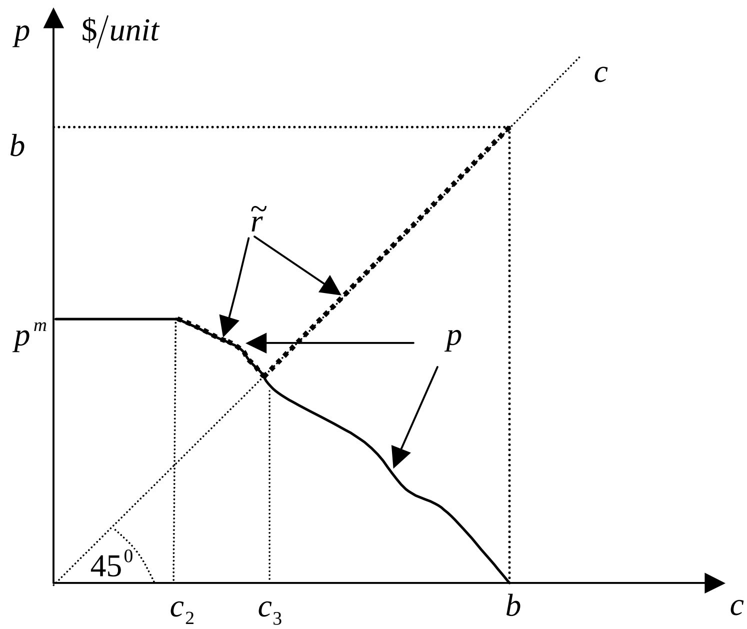

Figure 3

Titles of the figures:

Figure 1: How partial match information raises profit.

Figure 2: Critical values of search cost and the function $\phi(\bullet)$.

Figure 3: Equilibrium price, p, and threshold value, $\tilde{r}$.