Choosing a Champion: Party Membership and Policy Platform*

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Abstract

We introduce endogenous political parties into the Hotelling-Downs voting framework to model the selection of candidates. First, activists choose which party to join, if at all. Second, party members select a champion for the general election. Third, the electorate median voter determines the (stochastic) general election outcome. Although party members trade off win probabilities candidate location preferences, in equilibrium they vote sincerely, so champions are at party medians. Minimum differentiation is only attained when valence uncertainty vanishes. Otherwise, the electorate median voter is in neither party. Despite asymmetric party and policy positions in equilibrium, electoral successes remain roughly equal.

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1 Introduction

Political parties play a crucial role in selecting candidates for election and marshalling support for the chosen candidate. Yet, the decision to join a party and the ensuing effect on party behavior has not been fully integrated into formal models of the political process. Doing so, as we do in this paper, fundamentally changes the perspective of electoral competition, as the final winner has won *two* elections: selection by the *endogenous party* and then selection by the general electorate. Consequently, the final selection has been pre-screened by the parties, and the general electorate's median preference is diluted, or even hi-jacked. That is, although the preferences of the electoral median voter will influence policy competition between the parties, the two-election process means that the electoral outcome will not mirror her preferences, so that the classical median voter theorem, whereby political competition yields the preferred position of the electoral median voter, does not prevail in this setting. Viewed through this lens, political competition will register the preferences of more extreme types than just the median, and political parties will cluster around more extreme positions.

The foundation of spatial voting theory is the Hotelling-Downs (HD) model (Hotelling, 1929, Downs, 1957) with its median voter outcome for candidates. We introduce parties, with the twin roles of choosing candidates and promoting their prospects. This results in a median voter theorem at the party level but not at the general election level – the candidates chosen for the run-off are at the median "ideal" position of the party memberships, and this despite the fact that party members are strategic in voting for a candidate bearing in mind his/her final election prospects. However, the median voter of the general election is not in either party because s/he is broadly indifferent across the candidates and so it is not worth her joining a party either to promote its candidate or to change the candidate's position marginally. In order to mellow the HD median voter result, we need to allow for uncertainty (at the candidate selection stage) in the final election - otherwise, the closer candidate to the electoral median voter wins with probability one (modulo any valence advantage).[1] In the limit, as this uncertainty goes to zero, equilibrium candidate selections converge to the median electoral preference and parties also shrink to zero support, so confirming the HD result as the outcome when there are no parties. Otherwise, the existence of political parties implies differentiation of candidates.

One innovation of this paper is our model of the electoral process as a three-stage game. First, heterogeneous political activists with differentiated preferences over final outcomes choose which party to join. They rationally anticipate they will have a say in the internal selection of their party's candidate ("champion") for the general election, and that they improve the probability their candidate is elected (through canvassing efforts). They bear in mind that joining a party incurs effort costs and that they will vote over alternative candidates in the party-level election of the party champion[2]. Activists care about the probability (of winning) weighted by the intrinsic desirability of the platform run on. Their preferences are consistent with those modeled by Wittman (1983), here applied to an explicitly spatial context (he has no parties, and attributes such preferences solely to candidates). Second, active members who have signed up for their parties vote over the candidate to offer up as champion. As we show, they select their party's median position, so their votes can be construed as sincere (the party median voter result) even though they are strategic vis-a-vis the final election (they recognize that they need to temper their positions by the desire to win the election). Third, is the final run-off in the (stochastic) general election.

The general election outcome is determined by the median voter's preferences over the two candidates, modulated by party efforts in boosting candidates. The stochastic nature of the general election is captured by a general Contest Success Function (CSF) which is consistent with a number of interpretations including idiosyncratic voter preferences over parties or candidates, uncertain campaigning abilities of candidates, uncertain effectiveness of parties in canvassing, and luck.[3]

[1] A stochastic electoral success function is crucial in allowing parties to deviate from the electoral median voter's position: see Wittman (1983) and Roemer (2001).

[2] We focus on champions here to emphasize, as in the citizen candidate literature, that an individual candidate's policy position is fixed and will not change between the party selection stage and the general election.

[3] We make some technical advances in this area including the use of log-concave densities to describe the election.

The model yields strong predictions for long-run electoral competition, even in the presence of base valence advantages (which represent inherent swings in the population's views of a party's competence). First, we suggest there is no long-run proclivity to deviate much from outcomes in which parties systematically lose more than half the time.[4] This is broadly consistent with the election experience of alternating power between parties. However, each individual election is stochastic because the candidate must be chosen sufficiently early that not all information is known. Second, valence advantages may be completely undone by choosing more extreme positions. Third, parties tend to be the same size despite position and valence differences. These results are driven by the crucial role of the activists' decisions to join a party (or not).

The set-up and outcomes differs from those of classic voting models in several respects. We share with the Hotelling-Downs set-up (Hotelling, 1929, Downs, 1957) the spatial representation of voter taste diversity, but we generate differentiation in platform outcomes. We share with Wittman (1977, 1983, 1989) that candidates have policy preferences, but ours are generated endogenously through parties, while his model, by ignoring parties, fails to capture the idea of choosing an internal champion to contest the opposition's chosen champion.

In recent years there has been progress in moving beyond the view, from the classical literature, of elections as policy contests between exogenous candidates. The citizen candidate literature, initiated by Osborne and Slivinski (1996) and Besley and Coate (1997), observes that candidates must choose to run but they omit the political reality of large modern democracies, that gaining power requires the intermediate step of receiving party support.[5] Two recent papers have focused on the role of primaries in selecting better quality candidates to represent a party. Adams and Merrill (2008) consider a primary for a fixed party with two (internal) candidates. Candidates' uncertain campaigning ability is revealed through the primary, helping a party to select a better

[4]Leading forecasting models of the US presidential election, such as Fair (1978), find little or no role for political policies in predicting election outcomes. However this is misleading since the election is an equilibrium outcome and theory suggests that competing parties should to some degree neutralise each other's policy positions.

[5]Some of the technical issues present in the original papers (like a continuum of equilibria) have been ironed out in Eguia (2007) by introducing stochastic elements to the election.

candidate.[6] Serra (2010) models a similar situation but considers the choice of whether to run a primary or have candidate selection by the party elites.[7]

In a large series of papers, including Schofield (2003, 2004) and Schofield and Miller (2007), Schofield has provided examples of elections in which candidate valence alone is not sufficient to explain policies and outcomes. He concludes there is significant evidence of the importance of activist support in determining policies and electoral outcomes. We provide an explicit individual level model of how this process might work[8].

As a pioneer of the formal modeling of party activism, Aldrich (1983) is an important precursor of our analysis, but there are a number of key differences between his approach and ours. Most significantly his parties are not integrated into a model of political competition (policy or valence) and hence the key concerns of this paper, such as canvassing activity, voting based on forward looking policy selection etc. are not addressed.

The view of parties as representing constituents (voters) appears previously in the work of Roemer (2001, 2004, 2006). Our approach is quite distinct from Roemer, emphasizing individual non-cooperative decisions over participation and voting, while Roemer has an essentially cooperative approach based around Nash Bargaining or ratio conjectural variations. Similar to Roemer (2001), Jackson, Mathevet, and Mattes (2007) consider a two party model which partitions all voters into two parties. However their deterministic elections represent a fundamentally different view of politics and are not meaningfully comparable to literature on stochastic elections .

Many of the terms used in this paper appear in other contexts in the literature. Political parties can refer to parties in a legislator, and this area, focusing on legislative bargaining, has received considerable attention following the seminal work of Baron (1993). The term activist appears in

[6] The problem is difficult technically and while the authors are able to establish existence and bounds results for equilibria point-wise predictions come from numerical simulations.

[7] A key prerequisite for modeling either candidate abilities or party canvassing support is the extension of spatial voting to include valence differences. Advances in this area include Anderson and Glomm (1992), Groseclose (2001) and Schofield (2003, 2004).

[8] To accommodate these empirical findings, Schofield (2007) develops a theory of general elections in which a candidate's valence is not fixed but rather depends on an exogenous valence function.

the literature on voter turn-out in general elections. For example, Herra and Martinelli (2006) develop a theory of potential activists deciding whether to expend the effort to become "leaders" and, in doing so, influence more passive citizens to vote. While their model is of electoral turnout they also provide a theory of non-cooperative group formation through influence activities. In some sense their model provides a micro-foundation for the way in which activists affect party policy and candidate success in our model.[9]

The organization of the paper is as follows. We first describe the key ingredients of the model. Because political outcomes are analyzed as the result of a three-stage process, we describe first (backwards induction) the outcome of the last-stage game, which is simply the electoral median voter's choice. Then comes the choice of champion by each party. This analysis delivers the sincerity property of our party median voter result for political parties without too many extremists. We then add the first stage of party membership decisions to look at the equilibrium of both parties, given the champion choice rule and the way of influencing it via the decision to join. The key predictions of the model are then presented and discussed. Two extensions are considered: an alternative view of activist effort in terms of get-out-the-vote which transpires to be formally equivalent, and the impact of electorate size yielding the prediction that larger electorates exacerbate candidate polarization.

2 Model Ingredients

There are two political parties, which we call Left (L) and Right (R). Each one chooses a platform position, l and r respectively, with $l < r$, on the real line which represents the policy space. Party membership entitles the member to have a say in the position, as well as costing effort, which helps swing the election outcome via persuasion of the median voter. The election outcome is determined in a run-off between the positions chosen.

[9] The Herra and Martinelli model is set on a circle with exogenous party preference (policy etc.). They suggest that it would be interesting to investigate a number of the issues pursued in this paper: group formation on the line with strategic policies and different quality leaders. It should be noted that they are focusing on the election stage while we take a standard model for the election and focus more on the earlier party and policy formation stages.

The game has the following timing structure. First, potential activists simultaneously decide whether to join parties and in doing so commit their activism contribution. Party members then simultaneously vote for a champion in their chosen party. Nature then determines the final (election-day) valence realization, and then the candidate is elected by the median voter. Notice that in equilibrium the number of activists and the position of the rival party are rationally anticipated.[10]

Our model is a game between potential activists. We begin by describing the payoffs of activists over the electoral success of parties and the choice of joining a party. We then describe the final electoral run-off before analyzing policy selection within a party, equilibrium party membership and the overall equilibrium.

2.1 Party Activists

The ordinary citizens who constitute the members of a party contribute to the electoral success of the party in a number of ways. At the local level, grass-root activity supplies useful information about both the complex problems facing a modern society and their potential solutions. Furthermore in many democracies it is at the grass-roots level that potential candidates are identified – candidate ability/valence is important in real-world politics but is frequently ignored in spatial voting theory. In the short term citizen activism is also an important part of many campaigns. Most visible are campaign contributions, the majority of which in the US are small donations by individual citizens (see Ansolabehere, Figuerido, and Snyder, 2003). Activists also contribute their time directly through door-knocking to inform and influence potential voters. They are also instrumental in "get out the vote" campaigns aimed at getting supporters to polling booths (see section 8 for this extension).

We view potential party activists as those who have relatively strong preferences for one outcome

[10] An alternative game timing would have the activists joining parties and then choosing champions based on the observation of the number (and type) of activists in the rival party. This would mean that the decision to join up would also consider how joining would affect the rival party's champion choice – our formulation does not allow for this effect. Our approach is reasonable insofar as party membership choice is made without conditioning on these variables, but simply on the expectations of rival party's size and position. The overall framework is amenable to considering the (dis)advantages of timing ones primary before / after the rival, although we do not pursue that issue here.

over another, and who have relatively low costs from being activists, so that only a small sub-set of the citizen population will join parties (see Aldrich, 1983, for a further discussion).[11]

We assume that each member incurs an effort cost β to join a party, and incurring the cost improves that party's win probability from $\mathbb{P}_i^{out}$ to $\mathbb{P}_i^{in}, i = L, R$. Think of this as the contribution of a unit of time in canvassing for support. In the sequel, we model how the activists affect the party platform: contributing effort entitles the member to vote over the party's platform position.

Potential activists care only about how their activities affect the political outcome. We do not include any political rents accruing to members of the winning side, although the framework can readily be extended to include this additional motive for joining (and for influencing the candidate position). Therefore, the membership decision is based only on how the expected political outcome is altered by membership, and political outcomes are valued by the linear distance from the potential activist's own preferred political position, x, with distaste rate t. The benefit to a citizen of this activism is the increase in the probability of winning for their preferred party.

Taking as given the position of the Left party, l, and the Right party, r, the incentive from joining the Right party for an individual located at x is to increase the probability of the Right party winning through greater resources as well as shifting the policy of the Right party from r^{out} to r^{in}. The benefit from joining is given by comparing the utility from not being in the party:

$$U^{out} = -t\left|l - x\right|\mathbb{P}_L^{out} - t\left|r^{out} - x\right|\mathbb{P}_R^{out} \tag{1}$$

with that from joining:

$$U^{in} = -t\left|l - x\right|\mathbb{P}_L^{in} - t\left|r^{in} - x\right|\mathbb{P}_R^{in} - \beta. \tag{2}$$

Differencing these gives the benefit from joining as

$$U_R^{in} - U_R^{out} = \left(\mathbb{P}_R^{in} - \mathbb{P}_R^{out}\right)\left[t\left|l - x\right| - t\left|r^{out} - x\right|\right] + \mathbb{P}_R^{in} t\left[\left|r^{out} - x\right| - \left|r^{in} - x\right|\right] - \beta. \tag{3}$$

[11] Thus, if the population comprises citizens with low and high effort costs, the set of potential activists is the set of low-cost types, and we implicitly set the high effort costs so large that no such individual joins up in equilibrium. Our reason here is to be able to generate a relatively small party that does not (necessarily) constitute a large percentage of the populace.

The three terms here are in order: the probability effect of joining, the positional influence, and the cost of joining. The positional influence is the change in the party's policy after joining. We denote positional influence by $\varepsilon = \left|r^{out} - x\right| - \left|r^{in} - x\right|$, which is the factor in the second term in (3) above.[12]

2.2 Contest Success Function

The probability of winning the election is determined by a Contest Success Function (CSF)[13], which depends on the valence and political position of each party. We further assume that the CSF can be expressed simply as a weighted sum of the differences between the parties' valences and their positions relative to the median voter. A party's success probability then depends solely on its net advantage from these two sources.

Accordingly, let $P(z) \in (0, 1)$ denote the probability the right candidate wins, given a net advantage z, and we will later specify how this net advantage depends on policy positions and activist support. We assume $P(\cdot)$ is defined on $\mathbb{R}$, is strictly increasing and twice differentiable. We also impose

ASSUMPTION 1 (Contest Success Function)

(a) $P(z) = 1 - P(-z)$;

(b) $P(\cdot)$ is convex for $z < 0$ and concave for $z > 0$;

(c) $\ln P(\cdot)$ is concave.

The first part of this assumption is symmetry, and means that the choice probabilities are reversed if the candidate net advantages are switched. Hence, $P(0) = 1/2$, which is essentially a normalization insofar as systematic differences are folded into the intrinsic valence (dis)advantage.

[12] We have derived ε from an assumption of individual action. An alternative interpretation consistent with our model is that activists are local leaders exerting social influence over a group of party member. This approach is applied formally to voter turner out in general elections by Herra and Martinelli (2006), building on the earlier models of social influence in crime by Glaeser et al. (1996) and group voting by Uhlaner (1989) and Morton (1991). On the empirical side, Gerber et al. (2008) conducted a field experiment with a single directed mail-out. They found that social pressure had a significant impact on turnout (treatment effects of 5-8%) in the Michigan 2006 primaries. It appears encouragement to vote with social pressure is a practical method of influencing the median position in a vote.

[13] See Skaperdas (1996) for a full discussion of contest success functions.

The second part of the assumption implies that the win probability has a sigmoid shape, with greatest slope at $z = 0$. This slope, denoted $P'(0)$, plays a key role below. The third part is satisfied by most distributions commonly used in the Social Sciences and Statistics (see Bagnoli and Bergstrom, 2005).

The approach permits a number of micro-political foundations for the contest success function. We can rationalize the Contest Success Function above in terms of a median voter's preferences in the following manner. Suppose that the median voter ascribes (utility) values to the two candidates, and chooses the one with the greater realization. This is a standard Discrete Choice Model formulation of the median voter's problem. Write then

$$u_i = k_i - t\,|x_m - x_i| + \mu\xi_i, \qquad i = L, R$$

with $\mu > 0$. Here we endow the median voter with the same "spatial" preferences as those we (later) attribute to the party members, namely a linear distance disutility with distaste rate t. The candidate positions are the locations chosen, x_i, and we set these to $x_L = l$ for the Left party, and $x_R = r$ for the right one. The term k_i includes both any inherent valence attached to a party, $\bar{k}_i$, due for example to competence or integrity, plus "persuasion" through canvassing by party activists.

The term ξ_i is a draw of a standardized random variable (with zero mean and unit standard deviation, and full support). This represents the outcome of unforeseen influences on the median voter that surface between the time the party candidates are chosen and the election. The term μ therefore represents the amount of noise in the system, in the sense of how important are the unanticipated campaigning factors which occur between candidate choice and the final election choice. Since $\mu > 0$, we can therefore take a monotonic transformation of the utilities and write

$$\frac{u_i}{\mu} = \frac{k_i}{\mu} - \frac{t}{\mu}\,|x_m - x_i| + \xi_i, \quad i = L, R$$

so the median voter chooses the option i $(=L, R)$ for which $\frac{u_i}{\mu}$ is greater. This means that the probability that the Right party wins is $\Pr(z > \xi_L - \xi_R)$ where we therefore define z as the Right party's net advantage

$$z = \frac{k_R - k_L}{\mu} - \frac{t}{\mu}\left[|x_m - r| - |x_m - l|\right]. \tag{4}$$

Note that the properties ascribed above to $P(\cdot)$ in Assumption 1 are consistent with assuming that the probability density of the shock difference, $\xi_L - \xi_R$, is symmetric about 0 and quasi-concave (see Anderson, de Palma, and Thisse, 1992, for background). Thus, in particular, our model is a generalization of the binomial logit model widely used in stochastic voting models such as Adams and Merrill (2008) or Schofield and Miller (2007). The logit form arises when the error difference is logistically distributed. This convenient functional form is written simply as

$$P(z) = \frac{1}{1 + \exp(-z)}$$

and has the classic sigmoid shape. Another prominent special case of our model is the classic binomial probit model, which derives from normally distributed errors.

For convenience below, we shall often use the notation $\mathbb{P}_R$ $(= P(z))$ as the Right party's win probability, and $\mathbb{P}_L$ $(= 1 - P(z))$ as the Left party's win probability.

3 Activists' Preferences over Party Positions: Properties

In order to determine the incentives for activists to join parties, we now establish the key properties of activists' preferences over alternative party positions. We then use these properties to show in section 4 how endogenous champion positions are chosen by a fixed party base.

Although the utility an activist derives from a party's policy is well behaved, having a simple additive structure with linear disutility in distance, the single peakedness and single crossing properties which hold for individual policies do not transfer to policy choice by a party. Figure 1 illustrates a canonical case of an individual's utility as a function of r when $l < 0 < x$. The upswing in the tails is due to a combination of utility and probability effects. The direct effect of a policy moving away from an individual in either direction is negative. However as policies become very extreme, relative to the median voter and the activist at x, there is a significant decrease

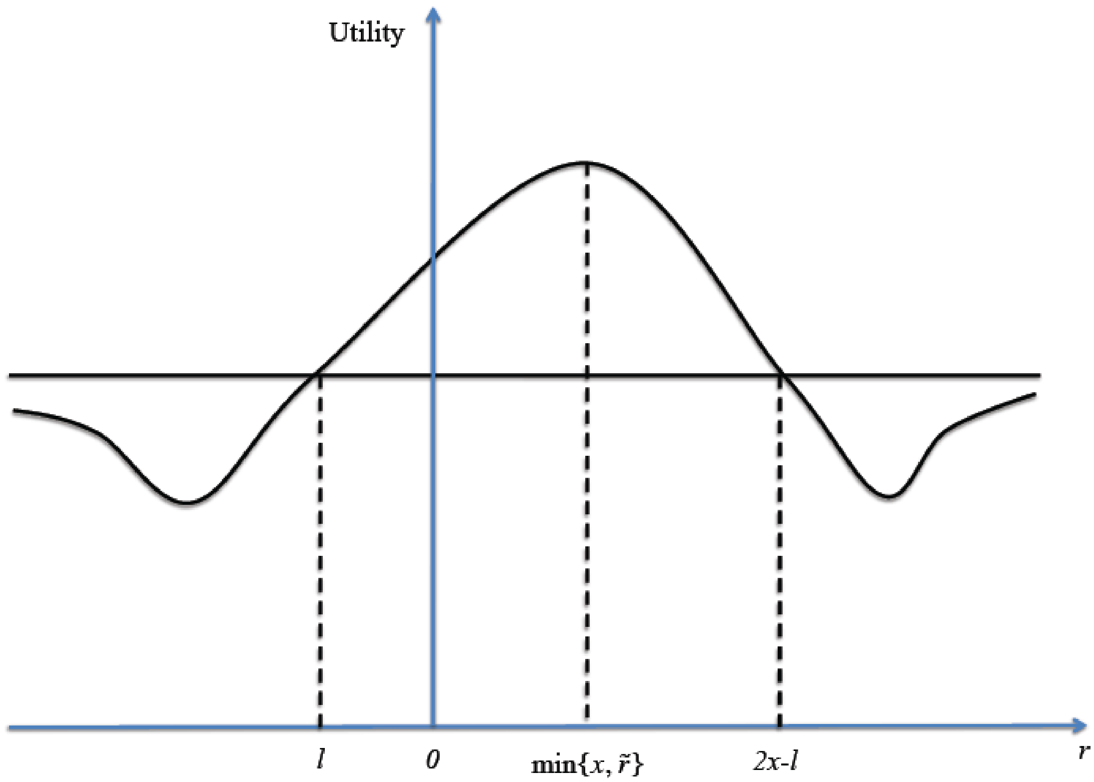

Figure 1: Utility as a Function of Right Party Policy ($l < 0 < x$)

in the probability of that extreme policy winning the election and being implemented. Thus, for our log-concave CSF's, the probability effect dominates in the extreme, and the expected utility increases as the extreme policy becomes more extreme.

A key result which follows from our analysis of the derived preferences over a party's policy, is that if the party median is not too extreme, then members vote their type, i.e. apparently myopically even though their choice of champion is strategic. That is, they vote for the party position closest to their own preferences even though they internalize the effects of improving the win probability in the general election.

Let $U(r, l, x)$ denote the expected utility of an activist with ideal point x when the Right and

Left parties' policy positions are r and l respectively. Using (2), we can write this as

$$U(r,l,x) = -t|l-x|\mathbb{P}_L - t|r-x|\mathbb{P}_R \quad (5)$$

$$= P(z)\,t(|x-l|-|r-x|) - t|x-l|, \quad (6)$$

which is a continuous function of its arguments.[14] In the second line we have written $\mathbb{P}_R = P(z)$ in order to recall the dependency of Right's win probability on z and we need to bear in mind the dependency of z on r (which is negative when $r > 0$ because the Right party is moving further from the median voter when r rises - see (4)). Analysis of (6) leads to the properties below. The reader may wish to skip the following technical properties (and their proofs), proceeding instead directly to the median voter results in subsequent section (4) .

3.1 Properties of Expected Utility from Party Positions

The first three properties below are general ones. We then specialize (in the main text) to give the relevant preference properties for Right party positions $r > l$ for an activist with ideal point at $x > l$ when the Left candidate locates at $l < 0$. The on-line version gives the full analysis, extending to all activist positions and to all possible positions of r (including left of the Left party). The salient cases of preferences are summarized graphically at the end of the section.

Property 1 *$U(l,l,x) = U(2x-l,l,x)$.*

Proof. By construction, the distance from x to each party's policy is the same in both cases, and so (5) is independent of the probabilities assigned to the alternatives. ■

This is the "mirror" property that the individual is as well off with the parties equidistant on opposite sides as with them both together at the same distance.

Property 2 *If $|r-x| > |l-x|$ then $U(r,l,x) < U(l,l,x)$.*

[14] Recall that the number of activists in each party is given at this point, i.e. "persuasion" is constant.

Proof. From (6), $U(r, l, x) < U(l, l, x)$ as $P(z)\, t(|x-l| - |r-x|) - t|x-l| < -t|x-l|$, or $|r-x| > |l-x|$, as stated. ■

By assumption r is further from x than l and hence gives less utility. Thus a lottery over r and l must give lower utility than having both parties at the shorter distance for sure.

Property 3 *(i) Suppose that $|r-x| < |l-x|$. Then U is* increasing *in r if both $|r|$ and $|r-x|$ fall with r. U is* decreasing *in r if both $|r|$ and $|r-x|$ rise with r. (ii) Suppose that $|r-x| > |l-x|$. Then U is* increasing *in r if $|r|$ rises with r and $|r-x|$ falls with r. U is* decreasing *in r if $|r|$ falls with r and $|r-x|$ rises with r.*

Proof. This follows directly from (6) that $U(r, l, x) = P(z)\, t(|x-l| - |r-x|) - t|x-l|$, where we note that z rises if $|r|$ falls. ■

Part (i) of this property considers local changes in the more preferred party, R. Utility is higher if the preferred party (the closer one) both gets closer and has a higher chance of getting elected (so its distance from the median voter falls). Utility is lower if both changes go the opposite way. Part (ii) of the property covers the case where R is less preferred by dint of its further distance from x. If the conditions of the property do not hold, there is a tension between two opposing effects on utility. These we cover next.

Property 4 *If $x > 0 > l$ then $U(r, l, x)$ is increasing in r for $r \in (l, \min\{x, \tilde{r}\})$ and decreasing in r for $r \in (\min\{x, \tilde{r}\}, 2x - l)$, where, defining $\tilde{z}$ as the value of z when $r = \tilde{r}$, then $\tilde{r}$ is defined implicitly by*

$$\frac{P'(\tilde{z})}{P(\tilde{z})} = \frac{1}{(\tilde{r} - l)} \frac{\mu}{t}. \tag{7}$$

which is independent of x.

Proof. First note that utility is increasing in r for $r \in (l, 0)$ by Property 3 (the preferred party R is getting closer to both x and to the median voter), and it is decreasing in r for $r \in (x, 2x - l)$ by the same Property. Because the utility function is continuous, it remains to show that utility is increasing up to the lesser of x and $\tilde{r}$. Indeed, for all $r \in [0, x]$ expected utility from (6) is $U(r, l, x) = P(z)\, t\,((x-l) - (x-r)) - t\,(x-l)$. Hence,

$$\frac{\partial U(r, l, x)}{\partial r} = P'(z)\, \frac{t^2}{\mu}\,(l-r) + tP(z)$$

Extracting a positive term, we have

$$\frac{\partial U(r, l, x)}{\partial r} = P(z)\,(r-l)\,\frac{t^2}{\mu}\left\{-\frac{P'(z)}{P(z)} + \frac{1}{(r-l)}\frac{\mu}{t}\right\}. \tag{8}$$

The term $\frac{P'(z)}{P(z)}$ is positive and non-decreasing in $r > 0$ (since it is non-increasing in z by log-concavity). The term $\frac{1}{(r-l)}\frac{\mu}{t}$ is positive and strictly decreasing, with range $\mathbb{R}_+$. Thus there is a unique solution. The sign of the bracketed term is strictly positive in the neighborhood of $r = l$ with $r > l$, and is eventually negative as $r \to \infty$. It is negative beyond the turning point, which is $\tilde{r}$ as defined above in (7) (which may be greater than x). ■

Notice that x is contained in the interval $(0, 2x - l)$ and so $\tilde{r}$ plays no role if it exceeds x. Otherwise, because (7) is independent of x, the utility function has the same maximizer, $\tilde{r}$, for all $x \in (\tilde{r}, 2x - l)$. This will imply below that all sufficiently extreme voters will prefer the *same* position, $\tilde{r}$, and this is a key element of the analysis of the next section.

The tension in (8) is as follows. The first term is the (adverse) reduction in the probability of the preferred party (r); the second is the (attractive) closer position. These tensions underpin the quasi-concavity property that utility goes up and then down. The regularity property of log-concave $P(.)$ suffices for this to hold.

In the on-line appendix we prove analogous properties for the case $x \in (l, 0)$. This case follows the same basic method of proof as above (but is more cumbersome because there are two sub-cases depending on whether $2x - l \lesseqgtr 0$). In the on-line appendix, we also prove the following property.

Property 5 *$U(r, l, x)$ is quasi-convex in r for all r such that $|r - x| > |l - x|$, i.e. $r < l$ and $r > 2x - l$.*

In these cases the Right party is less preferred by the activist at x, who can then prefer it to move even further away and so diminish its chances of being elected. The trade-off here is that it is less attractive when it is elected. This idea is manifested in ambivalence about facing extremist rivals: Democrats may relish going up against a Sarah Palin or a Newt Gingrich for the hope of higher chance of getting elected, but may dread the prospect of them actually getting elected. Log-concavity of $P(\cdot)$ ensures that utility is monotonically rising, so that further away is better (the reduction in win probability for the more distasteful position overwhelms the effect of increasing distaste).

Bringing together the various properties, the utility of an activist $x > l$ as a function of the position of the Right party is quasi-concave over the interval from $r = l$ through the mirror point, $r = 2x - l$, and the utility is the same at the end-points of this interval. The preferred point for activist $x > 0 > l$ is $\min\{x, \tilde{r}\}$. This implies that all activists more extreme than $\tilde{r}$ prefer the same position, namely $\tilde{r}$, while all more moderate activists prefer their own locations. In the trade-off between getting elected and getting a good position, the first effect (improving the probability of winning) dominates for extremists, and the second dominates for moderates. For extreme r (either larger or smaller), it is quasi-convex, and has lower utility than on the first interval. The upward slope of the utility function far enough out implies that the preferences are not single peaked or single crossing. Nevertheless, we are able to prove below that there is a unique median voter (internal to the party) equilibrium.[15]

4 Choosing a Champion within the party

When an activist joins a party, in addition to directly affecting the parties chance of winning s/he also affects the party's platform choice. Anticipating the policy influence effect increases the willingness to join. Following the logic of backward induction, our analysis proceeds in reverse order, starting with the determination of policy before proceeding to membership in the next section.

[15] Proposition 4 shows that the case $x < l$, where x is a member of the Right party cannot occur in equilbrium. Thus, for simplicity, the case of $x < l$ is omited from the analysis of this section.

A party with a given base/membership determines its platform democratically, taking the (anticipated) policy and membership of the other party as constant. That is, we find the policy best response function of a party by considering the outcome of majority voting by the party's members on the party's policy. Each party chooses according to open agenda pairwise majority voting by all party constituent voters, resulting in a Condorcet winner, if one exists.

Party members will not necessarily plump for their own ideal positions because they realize that a more central position increases the chance of winning. Furthermore, as we showed in section 3, activist preferences for their party's policy satisfy neither of the standard conditions (single peakedness nor single crossing) used to identify the voting outcomes. Thus in establishing the party level median voter theorems below we have to employ more involved methods.

In making a policy vote, an activist's desire for her ideal policy and her desire to win interact in different ways for extremists and moderates within the party. Party extremists (outside the party platform) are those who would prefer a more extreme position, ceteris paribus, but they are also the ones who face this tension. For party moderates (inside the party platform), both incentives work in the same direction. That is, moderates always prefer a more interior position which leaves them as moderate. This is because a more interior position will both increase the win probability, and will render their own candidate more attractive when that candidate does win.

As we show, party members' preferred positions are non-decreasing in own distance from the electoral median. This means that the party's median choice will have a cut-off point with all those more extreme wanting no more central a policy, and all those less extreme wanting a (strictly) more central one. As we shall show, in a central case where the membership is not too extreme the party choice is actually the median of the members' ideal positions (i.e., if they were voting sincerely without internalizing the change in their champion's win probability).

The calculus of party policy selection in these subgames is the standard Hotelling-Downs pairwise deterministic majority votes with an equilibrium being any point which is a Condorcet winner, i.e. any point which can beat any other policy point for the given party membership.

Without loss of generality we can focus on the Right party. The Right party's membership is any arbitrary set of activists located to the right of the other party's position. We will let x denote the location of a Right party member and x_m denote the median of the Right party members.

Proposition 1 *If $l < 0$ then for any set of Right party members, if $x_m \geq \tilde{r}$ then $\tilde{r}$ is the unique Condorcet winner for the Right party.*

Proof. By Property 4 all activists to the right of $\tilde{r}$ ($x \geq \tilde{r}$) have as their globally preferred policy $\tilde{r}$ and hence will vote for $\tilde{r}$ in preference to any alternative policy r. Since $x_m \geq \tilde{r}$ this set of people who choose $\tilde{r}$ in preference to all other policies is a majority. This establishes both existence and uniqueness. ■

Notice that the proof of this Proposition does not require that all members are right of l, and so applies to any set of activists. The key characteristic here though is the starting point that the median cannot be left of $\tilde{r}$. We now turn to the case $x_m < \tilde{r}$ which is more involved. In this case we will identify the median location, x_m, as the unique Condorcet winner. We shall assume that $x_m > 0$ so that the Right party does not contain too many left leaning members. Hence the joint restriction $x_m \in (0, \tilde{r})$ means the party is not overpopulated by extremists of either stripe. This property is verified in equilibrium in the following section.

Proposition 2 *If $l < 0$, then for any set of Right party members such that $x > l$ and $x_m \in (0, \tilde{r})$, the unique Condorcet winner for the Right party is $r = x_m$.*

Notice that this last Proposition pertains to any possible positions of the "Right" party (even left of Left), but given that the activists are all right of Left. If the activists were allowed to be left of Left, and there were enough of them, the Condorcet winner could be (for example) at the position corresponding to $\tilde{r}$ but on the left of l. Our restriction on party membership in the Proposition rules this out. In the equilibrium when we consider endogenous memberships, we check that there is no desired deviation of members to the "opposite" party for motives of manipulating their preferred position.

The two propositions of this section are median voter theorems for voting within a given party; equivalent results hold for the Left party. Proposition 2, which turns out to be the more important of the two results for the following analysis, is a median voter theorem not just in the preferred policies but in the original type space, which in this case is the space of voter ideal points. This sincere voting results occurs despite the lack of single-peaked or single crossing preferences when it comes to decisions within the party.

With these results established, we can now turn to equilibrium party locations.

5 Endogenous Parties and Policies

Having determined in the previous section the outcome of voting within a party with fixed membership, we now extend the analysis to a full spatial voting equilibrium. The full equilibrium will determine both the membership and the policy for each party. In this section we will determine an interior symmetric equilibrium under the assumption that no party has an inherent valence advantage.

Real political parties consist of a discrete, integer number of members. However modelling a party as a discrete set introduces two significant technical complications. First, the median of a group of points will be a unique member of the set is the number of elements in the set is odd, while an even size group has a median which straddles the two central members. It is our assertion that the difference between even and odd memberships is not of practical importance to political science. Secondly, although the membership of real parties increases in integer steps, for large parties these steps are sufficiently small to justify the standard assumption of smoothness, facilitating the use of calculus.

For these reasons, and with the goal of producing a workable theory of endogenous political parties, we will employ a continuous approximation that allows us to analyze a party as an interval of activists.

5.1 Joining a Party

Recall that each member incurs an effort cost β to join a party, and incurring the cost improves that party's win probability by raising k_i by one unit. Thus, in considering whether to join a party, an activist needs to consider the affect of both the extra effort on the party's valence and the influence, ε, of voting on the party's policy.

Taking the definition of z from (4) and focusing on the Right party, we take a first order Taylor series approximation as a continuous approximation of the change in probability:[16]

$$\mathbb{P}_R^{in} - \mathbb{P}_R^{out} = \frac{P'(z)}{\mu}(1 + tI\varepsilon) \tag{9}$$

where I is an indicator which equals 1 for a moderate move and -1 for an extremist move.[17] Ceteris paribus, effort (joining) has more impact the smaller is μ (because the uncontrolled aspects have less impact) and the closer is the election (so z is closer to zero)[18].

Combining equations (3) and (9), the first order approximation for the Right party is

$$U_R^{in} - U_R^{out} = \frac{P'(z)}{\mu}(1 + tI\varepsilon)\left[-t\left|r - x\right| + t\left|x - l\right|\right] + P(z)\,t\varepsilon - \beta. \tag{10}$$

The interpretation is as follows. The first term is the increase in the probability that the Right party wins, allowing for the direct effect of activism and the indirect effect of change in policy as perceived by the median voter (see (4)). This is weighted by the utility superiority of the Right party's position over the Left party's; increasing the winning probability of the preferred party "saves" the distance cost to the less preferred party but substitutes it with the distance cost to the more preferred party. The second term is the expected gain in the Right party's position; it is the probability wins times the distance cost improvement that the activist brings about by joining. The final term is the cost of joining the party. By the same logic, the utility for joining the Left

[16] For the Logit case $P'(z) = P(z)(1 - P(z))$.

[17] We showed previously that for $l < x$ that a joining extremist, $x > r$, moves the median ε right; while a joining centrist, $x < r$, moves the party median ε to the left.

[18] Setting the increase in k_i from effort to 1 is a normalization: if effort raised k_i by Δ units and the parameters of the primitive (unnormalized) problem were t_Δ, μ_Δ, and β_Δ, then we can substitutue $t = t_\Delta/\Delta$, $\mu = \mu_\Delta/\Delta$, and $\beta = \beta_\Delta/\Delta$.

party is

$$U_L^{in} - U_L^{out} = \frac{P'(z)}{\mu}(1 + tI\varepsilon)\left[-t|x-l| + t|r-x|\right] + (1 - P(z))\,t\varepsilon - \beta. \tag{11}$$

Note that distance dislike transfer operates in the opposite direction for activists joining the Left party; and the appropriate weighting on the influence term is the probability that the Left party wins, $1 - P(z)$.

5.2 Symmetric Equilibrium

When there is no inherent advantage to either party, equilibrium policies are symmetric, $r = -l$, and parties are the same size, and hence, the net advantage to the Right party is $z = 0$.

Equation (10) gives the utility from joining the Right party. Imposing the assumed symmetry conditions and considering an indifferent extremist $(x > r)$ gives the condition

$$\frac{P'(0)}{\mu}(1 - t\varepsilon)t\,[2r] + P(0)\,t\varepsilon - \beta = 0. \tag{12}$$

Solving this equation yields the candidate equilibrium position as

$$r^* = \lambda\frac{2\beta - t\varepsilon}{1 - t\varepsilon}, \tag{13}$$

where $\lambda = \frac{\mu}{4tP'(0)}$.

The party boundaries are determined by first considering the indifference condition for the marginal moderate in the right party, denoted x_R^m. Taking (10) and substituting $r = -l$ gives

$$\frac{P'(0)}{\mu}(1 + t\varepsilon)t\left[-(r - x_R^m) + (x_R^m - l)\right] + P(0)\,t\varepsilon - \beta = 0, \quad \text{or}$$

$$\frac{P'(0)}{\mu}(1 + t\varepsilon)2tx_R^m + P(0)\,t\varepsilon - \beta = 0 \tag{14}$$

which gives

$$\begin{aligned} x_R^m &= \frac{\mu}{2tP'(0)}\frac{\beta - P(0)\,t\varepsilon}{(1 + t\varepsilon)} \\ &= \frac{\mu}{4tP'(0)}\frac{2\beta - t\varepsilon}{(1 + t\varepsilon)} = \frac{1 - t\varepsilon}{1 + t\varepsilon}r^* \leq r^*. \end{aligned}$$

Assuming that $0 < t\varepsilon < 1$, the moderate boundary of the party lies strictly between 0 and the party's policy. We select as activists those extremists closest to the median. Because the champion selected is at the party median, given the uniform distribution of activists the boundary of the Right party on the extremist side is simply the point equidistant on the other side of r^*, that is

$$\begin{aligned} x_R^e &= r^* + (r^* - x_R^m) \\ &= 2r^* - \frac{1-t\varepsilon}{1+t\varepsilon} r^* = \frac{1+3t\varepsilon}{1+t\varepsilon} r^* \end{aligned}$$

The boundaries for the Left party follow immediately by symmetry. The following Proposition summarizes these results. The proof in the appendix recounts the somewhat involved process of establishing the party median is inside $\tilde{r}$.

Proposition 3 *If activists are uniformly distributed on* $[a, b]$, $\frac{t\varepsilon}{2} < \beta < \frac{1}{2}$ *and* $|a|, b > \frac{\mu}{2tP'(0)}$ $(= 2\tilde{r})$ *then the symmetric equilibrium is* $r^* = -l^* = \lambda \frac{2\beta - t\varepsilon}{1-t\varepsilon}$. *The Right and Left political parties are symmetric intervals either side of the median voter, with* $[x_R^m, x_R^e] = [s^m r^*, s^e r^*]$, *where* $s^m = \frac{1-t\varepsilon}{1+t\varepsilon}$ *and* $s^e = \frac{1+3t\varepsilon}{1+t\varepsilon}$.

This proposition shows that if the cost of activism β is not too high, then we get an equilibrium in which the political parties are of positive size and cleaved; and in which the equilibrium policies of the two parties are separated and symmetric about the position of the median voter in the overall population. This equilibrium also requires that the benefit an activist receives from influencing her own party's policy position, $t\varepsilon$, is not too large relative to the cost of joining a party, β. As $t\varepsilon \uparrow 1$ the moderate boundary of the parties converges to zero, the median voter's position, but so do the equilibrium extremist positions. Thus in the limit, as the policy benefit from joining a party becomes too large, this case of interior type parties characterized here disappears.

5.3 Switching Parties

One possibility is that activists might join the other party and manipulate it to increase the probability it loses. We show this equilibrium is robust against such deviant behavior (feigning to be in a rival party in order to sabotage it). Robustness to sabotage also applies to the asymmetric equilibrium analyzed below. The following Proposition, proved in the Appendix, shows that this cannot happen in equilibrium.

Proposition 4 ***(No Sabotage)*** *If $z = 0$ and there are two parties, one either side of 0 (the median voter) then no activist wants to join the party further from its location.*

There are plenty of examples of dirty tricks and negative campaigning in elections.[19] There is little evidence of the alternative Trojan Horse strategy, of activists joining rival parties in order to legitimately change the rival's policy. The preceding proposition shows that although feasible, joining a rival party is not an optimal choice for an activist in equilibrium.

6 Equilibrium properties: the politics of extremism

Turning now to comparative statics on equilibrium policy and parties, we can by symmetry restrict attention to the right party, with parallel results holding for the left party. In equilibrium the moderate and extremist boundaries of the right party are multiples of the right party's policy, with the moderate party boundary at $s^m r^*$ and the extreme party boundary at $s^e r^*$. Hence the party radius is $(s^m - s^e)\, r^*$. From Proposition 3, the scaling coefficients in both cases depend only on t and ε. Hence changes (caused by factors other than t and ε) in the moderate and extremist boundaries and the size of the party will vary proportionately with changes in the party's policy.

The comparative statics properties then follow immediately from (13) (noting that s^m and s^e are independent of β, $P'(0)$ and μ).

Proposition 5 *For an equilibrium satisfying the conditions of Proposition 3, a higher β or μ or a lower $P'(0)$ lead parties to increase in size, become more extreme, and choose more extreme candidates.*

[19] The swift boat incident of the 2004 presidential campaign is a notable example, Rich (2004).

Those factors which decrease the attractiveness of activism at the individual level, either through increasing the cost, β, or by decreasing its effectiveness at the margin, (higher μ or lower $P'(0)$) lead to larger and more extreme parties with more extreme policies.

The mechanism is driven by the effect on party extremists because it is they who are the key party members by dint of their extreme elasticity for party membership. Take for example a higher cost of activism. Following such an increase, in order to retain the extremists in the party, they must be compensated by a policy change they like. That is a more extreme position, meaning that the median of the party becomes more extreme. The striking result is that higher participation costs actually increase party membership. This is an equilibrium effect: because the rival party is also more extreme, it is a more daunting prospect should it win the election. This effect causes more activists to join up to counter the rival's extremism.

A lower value of $P'(0)$ corresponds to less responsiveness in swaying the median voter through effort. This means that again the benefits of joining a party are diminished, and the key extremist players need to be "compensated" through a more attractive (i.e., extreme) position. With the opposing party also being drawn away from the center for the same reason, more activist extremists join up to counter it.

A higher μ means more uncertainty in the final election's outcome. The power of activist effort is less effective, which reduces the desire to join a party. This must then be offset for a more extreme position to keep the extremists on board. The limit result as μ gets small uncovers the links between our model and the classic Hotelling-Downs set-up.

Proposition 6 *The limit result for equilibrium candidate positions as electoral uncertainty, μ, goes to zero generates the Hotelling-Downs result of minimum candidate differentiation and party sizes vanish.*

The proof follows directly from (13). As the electoral noise vanishes, a candidate can only win the election if it is closer to the median than its rival (modulo persuasion differences, which anyway net out under symmetry). Now, if the candidates are "back-to-back" at the center of the market at the population median voter position, then there is no incentive to join a party in order to reposition it, because moving it away from the median voter will lose the election. Hence party sizes also collapse in the limit along with the electoral uncertainty.

Through this lens, the Hotelling-Downs minimum differentiation outcome is seen as the result of there being no accounting for the role of party activists in that set-up. Instead, once we introduce, as here, uncertainty over election-day returns along with political parties, with a two-stage role of choosing champions for the run-off, we generate dispersed positions with parties that internally elect their candidates at the median of their parties' constituencies. This leaves a cleave between the parties where the population median voter plays no direct role in candidate selection because the median voter does not join a party. Instead, the median voter's preferences only enter the calculus of candidate selection through the final stage of the electoral contest.

A final comparative static result traces the effects of changing ε. This we defer to Section 8, where we micro-found ε as a function of the density of activists in the population.

7 Parties with Canvassing and Endogenous Policies

We assumed above that parties have no intrinsic advantage. We now show that equilibrium parties adjust if there is a starting asymmetry in the median voter's perception of the parties' abilities (a valence advantage) to neutralize the advantage. The key motor (where we get traction for the results) is in the "free-entry" conditions for marginal types. These yield some strong characterization results, which are to be interpreted as describing a long-run situation where all variables have equilibrated to free entry of activists. The results are neutrality results: there is no systematic difference in party size; they stay the same over time (as valences vary) and the election probability, evaluated at the time of candidate selection, is one half. Hence there is no systematic bias toward one side or the other. Clearly, these party results cannot arise in other formulations that do not model endogenous parties. The 50-50 election probability result holds in any deterministic model virtually by construction (e.g., Besley and Coate, 1997, or Hotelling-Downs). Here the affect of the valence difference is cancelled out in the stochastic election through the endogenous party membership and the ensuing position choice process.

We now prove a sequence of equilibrium properties for asymmetric valences. A valence difference here means that the starting value (i.e., before including the effects of activists' efforts) of the attractiveness of the parties is different. Hence $\bar{k}_R > \bar{k}_L$ in (4) - where the overbars denote the starting values - means that the Right party has a valence advantage. This advantage represents, for example, a perception by the median voter that the Right party has superior skill in foreign policy. Once activists have joined parties and exerted effort in persuasion, the valence advantages become k_R and k_L, and the party positions are chosen as l and r, so that the net advantage of the Right party becomes z as per (4).

Proposition 7 *The equilibrium probabilities of winning are equal: $z = 0$.*

Proof. At any interior equilibrium, the joining condition for the marginal extremist in each party holds with equality, that is taking $I = -1$ in (10) and (11) gives

$$\frac{P'(z)}{\mu}(1 - t\varepsilon)t(r - l) + P(z)t\varepsilon - \beta = 0. \tag{15}$$

and

$$\frac{P'(z)}{\mu}(1 - t\varepsilon)t(r - l) + [1 - P(z)]t\varepsilon - \beta = 0 \tag{16}$$

Therefore the Left party's win probability, $[1 - P(z)]$ equals the Right party's win probability, $P(z)$. This is true by Assumption 1 only for $z = 0$. ∎

This is a very strong characterization result. It says that the basic 50:50 outcome holds in this model despite any intrinsic valence advantage possessed by either candidate or party. This is due to the endogenous party size. It should be borne in mind that the 50% win probability ought to be evaluated at the time the candidate/policy is chosen. Clearly on election day these probabilities have resolved to 0 and 1. *To put it another way, there is no long run tendency for one party to be larger than the other.* This result does not require specific functional forms for the CSF.

Proposition 8 *Parties are of equal size under a valence advantage.*

Proof. At an interior equilibrium the joining condition for the marginal centrist in each party holds with equality, that is (from (15) and (16)):

$$\begin{aligned} \frac{P'(z)}{\mu}(1+t\varepsilon)\left[t(\bar{x}_R-l)-t(r-\bar{x}_R)\right]+P(z)\,t\varepsilon-\beta &= 0 && (17)\\ \frac{P'(z)}{\mu}(1+t\varepsilon)\left[-t(\bar{x}_L-l)+t(r-\bar{x}_L)\right]+(1-P(z))\,t\varepsilon-\beta &= 0 && (18)\end{aligned}$$

Using Proposition 7, the last two terms on the left hand side of both equations are the same. Hence the first term is the same in each equation. Simplifying then yields $r+l-2x_L=-r-l+2x_R$, and so

$$r-x_R=x_L-l.$$

This means the inside radii are the same. Since there is an equal number of centrists as extremists in each party then under a uniform distribution of activists, parties are the same size. ■

This result says that in any election, opposing parties (in a two-party system) should be of similar magnitudes. They could of course differ across elections (as parameters change) and indeed as different elections involve different valences, although we shall see below an invariance result which suggests they can be the same sizes across elections too, if the only difference across elections is valence differences.

Corollary 1 *The Right party's valence advantage is exactly offset by asymmetric party positions with a relatively more extreme position for the Right party in equilibrium.*

Proof. From Proposition 7, $z=0$, so the left party and right parties are equally likely to win. By Proposition 8, the parties are of equal size, and thus valence and position are the only source of difference between the two parties. Valence is higher for the Right party by assumption; thus, for the probability of winning to be the same for both parties, the Right party must be further right from the median voter than the Left party is left from the median voter. ■

The next result delivers the neutrality property for inter-party position differences and for party sizes. The first is determined by the indifference property for the extremists; the second comes from the indifference conditions for centrists.

Proposition 9 *The equilibrium inter-party policy difference, $r-l$, is independent of the valence difference; as too are party sizes.*

Proof. By Proposition 7, $z=0$, and so the extremist indifference conditions (15) and (16) become

$$\frac{P'(0)}{\mu}(1-t\varepsilon)t(r-l)+P(0)t\varepsilon-\beta=0. \tag{19}$$

Hence the equilibrium position difference, $r-l$, is independent of any valence difference.

Given $z=0$, the equilibrium conditions for the marginal centrists (17) and (18) become:

$$\begin{aligned}\frac{P'(0)}{\mu}(1+t\varepsilon)\left[t\left(\bar{x}_R-l\right)-t\left(r-\bar{x}_R\right)\right]+P(0)t\varepsilon-\beta &= 0;\\ \frac{P'(0)}{\mu}(1+t\varepsilon)\left[-t\left(\bar{x}_L-l\right)+t\left(r-\bar{x}_L\right)\right]+P(0)t\varepsilon-\beta &= 0,\end{aligned}$$

which add up to $\frac{P'(0)}{\mu}(1+t\varepsilon)t\left(\bar{x}_R-\bar{x}_L\right)+P(0)t\varepsilon-\beta=0$, and with (19), ($\frac{P'(0)}{\mu}(1-t\varepsilon)t(r-l)+P(0)t\varepsilon-\beta=0$), this means that

$$(1+t\varepsilon)\left(\bar{x}_R-\bar{x}_L\right)=(1-t\varepsilon)(r-l).$$

But $r-l$ is independent of valence differences, and so therefore is $\bar{x}_R-\bar{x}_L$. This implies the party bases are the same size. ■

This result indicates that valence differences change party positions, but not absolute differences in party positions.[20] Whatever one party gains is played out in equilibrium and exactly offset through positional changes.

Proposition 9 is perhaps the strongest of the consequences of the free entry assumption for party membership. It also reflects the fact that everything in the model has been achieved without

[20] The Right party cannot be dragged left of the Median Voter because then the probability derivatives go the other way. We are implicitly assuming interior solutions for the analysis in the text.

employing a pure consumption value from party membership. One of the most likely scenarios for pure consumption affects to play a significant role is in a charismatic leader motivating higher than normal party membership.

8 Voter Participation

An alternative view of political activism is that it mobilizes supporters through 'get out the vote'. Indeed there is substantial evidence of activists literally helping voters to get to polling stations on election day. Furthermore, campaign activities may be as much about shifting the median voter by changing the population of voters (in most democracies voting is not compulsory) as it is about convincing the median of a fixed population to prefer one party more. Imai (2005) analyzed field experiment data on mobilization in the 1998 election and found a 5% increase in turnout from mobilization activities.[21] Similar positive effects of get-out-the-vote on turnout were found in Gerber, Green and Larimer (2008). In this section we suppose that activism shifts the position of the median voter rather than changing the valence of the party. We then show a formal equivalence between these modeling approaches, enabling us to interpret the previous results equivalently in terms of getting out the vote activities.

Suppose that an additional activist for the Right party succeeds in getting out more Right voters and hence shifts the position of the median voter to the right by some small amount δ (the analogous move the opposite direction applies to the Left party). Letting the initial, un-normalized position of the median voter be m, the net advantage of the Right party from this new median location, z', is now calculated relative to position $m + \delta$

$$z' = \frac{k_R - k_L}{\mu} - \frac{t}{\mu} \left[|m + \delta - r| - |m + \delta - l| \right].$$

[21] Imai (2005) analyzes data from Gerber and Green (2000). Propensity score matching is used to correct for imperfect randomization which significantly distorted the results of the original study.

Now if δ is relatively small so that $l < m < m + \delta < r$ then we can rewrite this as

$$\begin{aligned} z' &= \frac{k_R - k_L}{\mu} - \frac{t}{\mu}\left[|m-r| - \delta - |m-l| - \delta\right] \\ &= \frac{k_R - k_L}{\mu} - \frac{t}{\mu}\left[|m-r| - |m-l|\right] + 2\frac{t}{\mu}\delta. \end{aligned}$$

Thus shifting the median voter by δ is equivalent to increasing k_R by $2t\delta$.[22] Previously the direct impact of party membership on the electoral outcome (i.e. through valence) was normalized to 1. If we replace this normalization in equations (12) and (14) it is straightforward to reinterpret the symmetric equilibrium results within a 'get out the vote' setting. The equilibrium policies become

$$r^* = -l^* = \frac{\mu}{4t^2 P'(0)} \frac{2\beta - t\varepsilon}{2\delta - \varepsilon},$$

and the equilibrium party sizes are likewise accordingly adjusted.

The upshot of this section's analysis is that all the qualitative results of the base model carry over to the reinterpretation of effort effects as impacting voter turn-out rather than persuasion of the median voter. The results above confirm the formal equivalence between these settings.

9 Electorate size and electoral outcomes

Although one would expect more resources to help a party, the benefit of those extra resources realistically depends on the size of the electorate a party is trying to influence. Most simply, as an electorate grows larger it takes more effort to influence the outcome. One might imagine that a larger electorate is associated to a larger pool of potential activists, and, if both increase proportionately, there would be no change in the equilibrium outcome. However, such brief inference ignores the crucial role of the influence motive for joining parties, and this gets smaller the greater the number of activists. In this section we trace the upshot of this comparative static result,

[22] This section highlights the difference between our model and that of Osborne and Slivinsky (1996), which is applied to group formation in Osbourne and Tourky (2008). Interpreting Osborne and Tourky within our framework, they assume only activists vote in the final election, so that choosing to become an activist changes the population of voters by exactly one member, shifting the identity of the median voter accordingly. In their model group cohesion is driven by economies of scale in advocating a position and the authors take great care in analyzing integer effects. Their assumptions are best suited to voting within a committee or legislature since factional activism is tied with voting whereas our approach is better suited to larger elections.

generating predictions for candidate positions and party sizes as a function of electoral size. These outcomes are driven by the smaller positional influence attributable to joining a party, and, as we shall see, this causes equilibrium candidate positions to become more radical.

In this section we remove the relevant normalizations implicit in the earlier sections. Let now N denote the size of the electorate (country) and n the size of the potential political activist sector in the electorate. Think of scaling both up together, in order to look at a pure size effect, so $\alpha \equiv n/N < 1$, the relative size of the activist sector to the total electorate, will be treated as a constant.

We introduce electorate scale by dividing the effort contributions, k_i, by the size of the electorate, N. This reflects that canvassing efforts need to be spread over more voters. Or, equivalently, in terms of the re-interpretation of the previous section, a given amount of effort directed towards voter turn-out on the party's side causes a smaller percentage increase in voters the larger is the electorate. The policy influence of an activist is also scale-dependent in that under majority voting within a party, an individual's impact on the median position is smaller the greater the density of activists for each preferred position within the party. Indeed, think of the positional influence of a pivotal individual within a party. How much she can move the party's position depends on the "gap" between individuals around the equilibrium position. Because we are assuming that the distribution of activists' preferences is uniform, a marginal voter within the party can move the champion's position one notch (corresponding to one voter) in her preferred direction. Therefore, for an interval of unit length, the positional influence ε becomes $1/n$, which is the distance between individuals' preferred points, and we use this measure for the change in champion position induced by a new joining party member. The key insight here is that an increase in overall scale reduces the positional influence, and this changes the desire to join a party, which in turn changes the party size and median, so changing the selection of champion point.

Based on the best response policy functions determined in Section 5, we now determine the full spatial voting equilibrium in political party membership and party policies. We consider a symmetric equilibrium stemming from no intrinsic valence difference. Hence, the net equilibrium advantage to the Right party is $z = 0$, equilibrium policies are symmetric, $r = -l$, and parties are the same size.

Substituting the changes $\varepsilon = 1/n$ and dividing k_i by N in equation (14) gives the utility from joining the Right party. Imposing the symmetry conditions and considering the marginal extremist $(x > r)$ gives the condition

$$\frac{P'(0)}{\mu}(\frac{1}{N} - \frac{t}{n})t\,[2r] + P(0)\,\frac{t}{n} - \beta = 0 \tag{20}$$

as the one that determines the equilibrium right party candidate's position, r^*.

Proposition 10 *If activists are uniformly distributed on* $[a, b]$, $\frac{t}{2n} < \beta < \frac{1}{2}$ *and* $|a|, b > \frac{\mu}{2tP'(0)}$ *then a larger electorate or a larger fraction of activists implies more extreme party candidates.*

Proof. The conditions for Proposition 3 hold by assumption. From (20) and setting $\frac{\mu}{4tP'(0)} = \lambda$ gives

$$r^* = \lambda\frac{2\beta - \frac{t}{n}}{(\frac{1}{N} - \frac{t}{n})} = \lambda\frac{2\beta N - \frac{t}{\alpha}}{(N - \frac{t}{\alpha})} = \lambda\frac{2\beta\alpha N - t}{(\alpha N - t)}.$$

Differentiating gives the result that $sgn\frac{\partial r^*}{\partial N} = sgn\frac{\partial r^*}{\partial \alpha} = sgn(1 - 2\beta) > 0$. ■

Hence increasing the size of the electorate leads to more extreme policy positions in equilibrium. The driving force here is that the positional influence motive for joining a party is reduced because it is diluted by there being a thicker density of party constituents, so an incremental party member moves the median less. Having less influence reduces the desire to join a party. Then, because equilibrium candidate selection is driven by the effects on extremists, to reinstate their joining up means increasing their benefits from the candidate position itself. Hence the party champion position must become more extreme.

An increase in the fraction of activists has a similar effect because more activists reduce the positional impact for each individual party member reducing the net benefit of membership. As we saw above, a reduction in the benefits of party membership produces more extreme policies as extremists will require more interparty differentiation in order to stay in the party.

One has to be careful inferring that a simple cross country regression would indicate that larger countries will have more extreme politics since there will be other idiosyncratic effects at work in the politics of individual countries. Nevertheless, the result is quite provocative, and stems from the joint impact of reduced incentive to join political parties and the overarching political clout of party extremists in determining the agenda.

10 Conclusions

Political parties are a crucial determinant of election outcomes. Party activists not only influence outcomes through voter persuasion and getting out the vote, but they also determine the party platform. In the typical two-step democracy of primary elections followed by runoffs, first a champion is chosen, and then the champions battle for supremacy. Explicitly modeling platform competition recognizing this sequential election process radically alters the perspective on the efficiency of the political process.

Perhaps the strongest result in spatial voting theory is the Median Voter Theorem (which is in some ways analogous to the "Adam Smith" Fundamental Theorems of Welfare Economics that establish the efficiency of a market economy). The Theorem establishes an efficiency result on political performance, albeit with caveats.[23] In a two-step political process, parties do not get to the Median Voter Theorem result for the simple reason that the Median Voter has no reason to join a party. Since the Median Voter is broadly indifferent between alternatives too left and too right means it is not worth spending the time investment to join a party and actually influence the win probability of one party or the other (and so centrists are not activists). This suggests that the party structure induces inefficiencies absent when parties are not explicitly considered. Nonetheless, parties do still respond to the preferences of the Median Voter insofar as doing so

[23] For instance, one caveat is that the MVT does not account for the *intensity* of voter preferences, which follows because each voter has a single vote, irrespective of willingness-to-pay over outcomes.

improves the probability of the party candidate being elected, and the candidates chosen resolve the tension between getting elected and representing the preferences of the party membership. This process leads to a modified Median Voter Theorem: candidates take the (sincere) platform of the median voter of each political party. This though can lead to candidates, and hence outcomes, quite far from the preferences of the population voter median preference.

Because standard spatial voting models do not account for parties, they cannot answer questions about party size, how parties affect outcomes, nor indeed how party variables are correlated with election outcomes, etc. Our model produces a number of novel predictions which appear to correspond with empirical evidence – especially concerning the more extreme positions taken by candidates with a valence advantage relative to their valence-disadvantaged challengers who tend to be more moderate (Sullivan and Uslaner, 1978, p. 551).

Our treatment of valence is summarized as an effect on the median voter's behavior which is not tied to the observable characteristics of candidates. One area for future research would be to consider candidate selection from amongst a small set of high ability candidates focusing on how valence characteristics are traded-off against other characteristics, depending on the conditions of a particular election. Finally, the party pre-election choice of a champion is an imperfect sketch of the process of primary elections of candidates. But the ideas of the model can be used and extended to capture some key elements of a larger process. In particular, we can imagine a number of possible candidates of different initial valence and different political position. Supporters line up behind them, with perhaps some share of rents conditional on success ("consumption benefits" from being in parties). Then the party chooses the perceived best candidate, after some uncertainty about candidate performance caliber is revealed. Finally, the champions run off, all residual uncertainty is realized, and the election outcome is determined.

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A Proofs

A.1 Proof of Proposition 2: Median Voter

Proof. By assumption, there is at least one party member whose position is at x_m, and, since by assumption $0 <$

$x_m < \tilde{r}$, then this individual's most preferred position for the Right party is at $r = x_m$, by Property 4 from the previous section.

First, any $r' \leq l$ is dominated by $r = 0$, since a policy left of l leads to a strictly higher probability of an outcome further from x ($\geq x_m$). Second, by Property 4 the utility of $x \geq x_m$

is increasing in r over the range (l, x_m).Thus combining these two observations: any deviation of the Right party position from x_m to any position $r' \in (-\infty, x_m)$ will not be supported by any type $x \geq x_m$ and so cannot overturn $r = x_m$.

Now consider any deviation to a position $r' > x_m$. In order to beat x_m this must be supported by at least some types from "both sides of the aisle," meaning that it must involve at least some type with position below x_m and some type with position above x_m in order to gain majority support against $r = x_m$.

Denote the putative type from below the median and supporting a deviation as x_B, and the putative type from above the median and supporting a deviation as x_A. We will prove by contradiction there does not exists a policy r' which will beat $r = x_m$ in a pairwise choice. That is, it is not possible to have both $r' \underset{x_B}{\succ} x_m$ and $r' \underset{x_A}{\succ} x_m$. There are two cases to consider for deviations: $x_A \geq r'$ and $x_A < r'$. In what follows, we let z' denote the value of z when $r = r'$ and let z_m denote the value of z when $r = x$.

Case (i): $x_A \geq r'$ $(> x_m \geq x_B)$. For the left activist at x_B to prefer r' over x_m implies that

$$\begin{aligned} -(r' - x_B)P(z') - (x_B - l)(1 - P(z')) &> -(x_m - x_B)P(z_m) - (x_B - l)(1 - P(z_m)) \Leftrightarrow \\ P(z')\left(2x_B - l - r'\right) &> P(z_m)(2x_B - l - x_m). \end{aligned} \tag{21}$$

Similarly, for the right activist, x_A, preferring r' over x_m implies that

$$\begin{aligned} -(x_A - r')P(z') - (x_A - l)(1 - P(z')) &> -(x_A - x_m)P(z_m) - (x_A - l)(1 - P(z_m) \Leftrightarrow \\ P(z')\left(r' - l\right) &> P(z_m)\left(x_m - l\right). \end{aligned} \tag{22}$$

Summing inequalities (21) and (22) gives

$$P(z')\left(2x_B - 2l\right) > \left(P(z_m)\left(2x_B - 2l\right).\right.$$

Now, by assumption, $(x_B - l) > 0$, thus canceling from both sides gives

$$P(z') > P(z_m).$$

This is a contradiction since $r' > x_m$. Hence $r = x_m$ beats all $r' \leq x_A$ for arbitrary $x_A < x_m$.

Case (ii): $r' > x_A$. This implies the following inequalities. For the left activist, x_B, preferring r' over x_m implies again (21). For the right activist, x_A, preferring r' over x_m implies that

$$\begin{aligned} -(r' - x_A)P(z') - (x_A - l)(1 - P(z')) &> -(x_A - x_m)P(z_m) - (x_A - l)(1 - P(z_m)) \Leftrightarrow \\ P(z')\left(2x_A - l - r'\right) &> P(z_m)\left(x_m - l\right). \end{aligned}$$

Summing the two inequalities gives

$$P(z')\left(2x_A + 2x_B - 2l - 2r'\right) > P(z_m)\left(2x_B - 2l\right). \tag{23}$$

Now by assumption $r' > x_m$ therefore $P(z') < P(z_m)$. But this implies that for the last inequality (23) to hold it must be that $2x_A + 2x_B - 2l - 2r' > 2x_B - 2l$ or $x_A > r'$, which contradicts our initial assumption for this case that $r' > x_A$. It follows that x_m cannot be overturned.

Virtually the same arguments show that the Condorcet winner is uniquely at $r = x_m$. Namely, we wish to show that it is not possible to have both $r' \underset{x_B}{\succsim} x_m$ and $r' \underset{x_A}{\succsim} x_m$. In case (i) we have, with the appropriate inequalities now, that $P(z') \geq P(z_m)$, which is again a contradiction since $r' > x_m$. In case (ii), the necessary condition is again $x_A > r'$, which contradicts the starting hypothesis. ■

A.2 Proof of Proposition 3: Symmetric Equilibrium

Proof. The condition $\frac{t\varepsilon}{2} < \beta$ guarantees $\infty > \frac{2\beta - t\varepsilon}{1 - t\varepsilon} > 0$ and hence r^* as given by (13) is of the correct sign and is well defined. The assumption on the values of $|a|$ and b ensures that the parties are contained in the support of the activist distribution.

Equation (12) is only appropriate if proposition 2 holds, which in turn requires $r^* \leq \tilde{r}$. The following proof by contradiction establishes $r^* \leq \tilde{r}$ when $\beta < \frac{1}{2}$.

Assume r^* is part of a symmetric equilibrium and instead that $r^* > \tilde{r}$. By definition the net advantage to the right party in the symmetric equilibrium, denoted z^*, is $z^* = 0$ and, ceteris paribus, the net advantage if the Right party policy is $\tilde{r}$ is $\tilde{z} > 0$. The condition implicitly defining

$\tilde{r}$ is given by equation (7), imposing the symmetry conditions which gives

$$\frac{P'(\tilde{z})}{P(\tilde{z})} = \frac{1}{(\tilde{r} + r^*)} \frac{\mu}{t}. \tag{24}$$

Solving for $\tilde{r}$ yields

$$\tilde{r} = \frac{P(\tilde{z})}{P'(\tilde{z})} \frac{\mu}{t} - r^*.$$

Now, by assumption, $\tilde{r} < r^*$ so

$$\begin{aligned} r^* &> \tilde{r} = \frac{P(\tilde{z})}{P'(\tilde{z})} \frac{\mu}{t} - r^* \\ &\Leftrightarrow \frac{P(\tilde{z})}{P'(\tilde{z})} \frac{\mu}{t} < 2r^*. \end{aligned}$$

Substituting in the equilibrium value of $r^* = \frac{\mu P(0)}{2tP'(0)} \frac{2\beta - t\varepsilon}{1 - t\varepsilon}$ and canceling the common terms gives

$$\frac{P(\tilde{z})}{P'(\tilde{z})} < \frac{P(0)}{P'(0)} \frac{2\beta - t\varepsilon}{1 - t\varepsilon}. \tag{25}$$

By the assumption of the Proposition, $\beta < 1/2$, which implies

$$\frac{2\beta - t\varepsilon}{1 - t\varepsilon} < 1.$$

Combining with (25) gives

$$\frac{P(\tilde{z})}{P'(\tilde{z})} < \frac{P(0)}{P'(0)}.$$

However, by the log-concavity of P, P/P' is increasing in z and $\tilde{z} > z^* = 0$ therefore

$$\frac{P(\tilde{z})}{P'(\tilde{z})} > \frac{P(0)}{P'(0)}$$

which is a contradiction and hence $\tilde{r} < r^*$ is not true. ■

A.3 Proof of Proposition 4: No Sabotage

Proof. Without loss of generality we restrict attention to activists on the right of the median voter ($x > 0$). Consider a right party activist and a right party moderate. Both such activists are closer to r than to l, and so prefer to exert effort for the right party rather than the left party. So now consider the pure manipulation incentives to join parties (net of the valence contribution).

Consider first a right party extremist, $x > r^*$. As an extremist, we know that this activist wishes to move the right party further to the right, and the marginal benefit to doing so is[24]

$$\frac{\partial U(r,l,x)}{\partial r} = tP(z) - t(r-l)\frac{P'(z)}{\mu}. \tag{26}$$

The first term is the expected reduction in distance to the right party, while the second, conflicting force, is the reduction in the chance the right party is elected, valued at the cost of the distance difference as probability is transferred from the right party to the left party.

Such an extremist might instead wish to move the Left party either further left or further right. Suppose first the move was right. This makes L more likely to be elected (and R less). Also, L is closer. The utility benefit is thus

$$\frac{\partial U(r,l,x)}{\partial l} = t(1-P(z)) - t(r-l)\frac{P'(z)}{\mu}. \tag{27}$$

Now, $z = 0$ implies $P(z) = 1/2$, hence the manipulation benefit from joining L is exactly the same as joining R. However, as noted above, the effort benefit is higher for joining R (since r is closer) so that the activist will strictly prefer joining R and moving r right to joining L and moving l right. Notice that the benefit derivatives have the same terms – in both cases the probability R is elected falls and this involves a "distance transfer" of $r - l$; and in both cases the party joined is moved closer (which is weighted by the electoral win probability).

Exactly the same comparison applies to moving R in (decreasing r) and moving L out (decreasing l); they give the same utility increment for $P(z) = 0$, as is readily verified by writing out the relevant expressions. Therefore, since an extremist wants to increase r, joining L to decrease l is dominated.[25]

Next consider that a right moderate (in the interval $(0, r)$), at $x' > (l + r)/2$. The utility gain from moving R in (decreasing r) is given by

$$-\frac{\partial U(r,l,x)}{\partial r} = tP(z) + \frac{P'(z)}{\mu}t(2x' - r - l). \tag{28}$$

[24] This follows by differentiating $U(r,l,x) = -t(x-r)P(z) - t(x-l)(1-P(z))$.

[25] From equation (27), a decrease in l will yield a change in utility of -$t(1-P(z)) + t(r-l)\frac{P'(z)}{\mu}$; however, this is necessarily negative, and hence not countenanced, for $z = 0$, since $\partial U/\partial r$ as given by (26) is positive.

where both terms are positive – the party moves closer to the moderate activists, its win probability is increased, and R is the closer party.

Joining party L and moving it towards the median voter (increasing l) gives

$$\frac{\partial U(r,l,x)}{\partial l} = t\,(1 - P(z)) - \frac{P'(z)}{\mu} t(2x' - r - l). \tag{29}$$

This implies that utility increment from (29) is below that in from moving the R party in, equation (28) (because the first term in each case is the same when $z = 0$ and the second term has opposite signs in the two cases – probability is transferred the "wrong" direction when L is more likely to win).

If the right moderate joins the L party and moves it away from the median voter, the utility change is the negative of that in (29). However, comparing with (28) again shows (28) is larger. This is because the distance effect works in the wrong direction when joining L, although the probability transfer effect works the same way (by decreasing the likelihood that L wins). Hence the activist does not want to deviate into the rival party and manipulate the within party vote. ■

B Online Appendix

B.1 Further Properties of Preferences over Party Positions

We collect here the proofs of the assertions in the text that describe the utility function. The first result is the extension of Property (4) from the main text.

Property 6 *If $x > l$ and $0 > l$ then $U(r, l, x)$ is quasi-concave in r for $r \in (l, 2x - l)$.*

Proof. We consider two cases, $x \geq 0$ and $x < 0$. The case $x \geq 0$ is analyzed in the text under Property (4); the remainder of the proof follows.

Case 1 $x < 0$

First note that utility is increasing in r for $r \in (l, x)$ by Property 3, and it is decreasing in r for $r \in (0, 2x - l)$ (if this region exists) by the same Property. It remains to show utility is quasi concave on the middle interval, $r \in (x, \min\{0, 2x - l\})$. Here expected utility is, from (6) $U_R(r, l, x) = -P(z)\, t\left((r - x) - (x - l)\right) - t(x - l)$. Hence,

$$\frac{\partial U(r, l, x)}{\partial r} = P'(z) \frac{dz}{dr} t(2x - r - l) - tP(z) \tag{30}$$

$$= P'(z) \frac{t^2}{\mu} \left[(2x - l - r)\right] - tP(z) \tag{31}$$

Where the second line follows from $\frac{dz}{dr} = \frac{t}{\mu}$ for $r < 0$ (see (4)). Because $r < 2x - l$ the terms are of opposite signs and we can investigate the sign of the overall derivative as follows. Extracting a common positive factor gives

$$\frac{\partial U(r, l, x)}{\partial r} = P(z)(2x - l - r) \frac{t^2}{\mu} \left\{ \frac{P'(z)}{P(z)} - \frac{1}{(2x - l - r)} \frac{\mu}{t} \right\}.$$

Thus the sign of the derivative over the interval $r \in (x, \min\{0, 2x - l\})$ is given by the sign of the difference in curly braces. By log-concavity P'/P is decreasing in z, and z is increasing in r for $r < 0$, thus P'/P is decreasing in r and strictly positive and finite over the interval. The hyperbola is always positive $(2x - l > r)$, increasing in r and ranges from $1/(x - l)$ to infinity. Thus the two curves either cross once (with the hyperbola coming from below) or not at all (with the hyperbola above). Thus the derivative is either always negative or positive and then negative with its unique turning point at $\hat{r}$.

Note this implies the expected utility function is quasi concave in r on $(l, 2x-l)$ and either has a unique maximum at x or $\hat{r}$, whichever is bigger, i.e. at $\max\{x, \hat{r}\}$. ■

Property (5) *$U(r,l,x)$ is quasi-convex in r for all r* such that $|r-x| > |l-x|$, i.e. $r < x - |l-x|$ or $r > x + |l-x|$; *and U attains two local minima on $\mathbb{R}$.*

Proof of Property (5). $U(r,l,x) = -P(z)\,t\,|r-x| - (1-P(z))\,t\,|x-l| = -P(z)\,t\,(|r-x| - |x-l|) - t\,|x-l|$ therefore since $\frac{\partial z}{\partial r} = -\frac{t}{\mu}\frac{\partial |r|}{\partial r}$ under the normalization $x_m = 0$:

$$
\begin{aligned}
\frac{\partial U(r,l,x)}{\partial r} &= -P'(z)\,t\frac{\partial z}{\partial r}\left[|r-x| - |l-x|\right] - P(z)\,t\frac{\partial}{\partial r}|r-x| && (32)\\
&= P'(z)\,\frac{t^2}{\mu}\frac{\partial |r|}{\partial r}\left[|r-x| - |l-x|\right] - P(z)\,t\frac{\partial}{\partial r}|r-x| && (33)
\end{aligned}
$$

If $r \neq 0$ and $r \neq x$ we have

$$
\begin{aligned}
\frac{\partial U(r,l,x)}{\partial r} &= P'(z)\,\frac{t^2}{\mu} sgn(r)\left[|r-x| - |l-x|\right] - sgn(r-x)tP(z) && (34)\\
&= sgn(r)P(z)\left[|r-x| - |l-x|\right]\frac{t^2}{\mu}\left\{\frac{P'(z)}{P(z)} - sgn(r)sgn(r-x)\frac{1}{\left[|r-x| - |l-x|\right]}\frac{\mu}{t}\right\} && (35)
\end{aligned}
$$

To show the relevant function is quasi-convex, we need to show it is either monotonic, or that its derivative changes sign from negative to positive. Hence we focus on the sign of the derivative

$$
sgn\left(\frac{\partial U(r,l,x)}{\partial r}\right) = sgn\left(sgn(r)\left\{\frac{P'(z)}{P(z)} - sgn(r)sgn(r-x)\frac{1}{\left[|r-x| - |l-x|\right]}\frac{\mu}{t}\right\}\right) \qquad (36)
$$

Now, consider the behavior of the function in brackets. The first term is positive and non-increasing in z by log-concavity, and z is increasing (respectively decreasing) in r for $r < 0$ (respectively $r > 0$). Thus P'/P is non-increasing in r for $r < 0$ and non-decreasing for $r > 0$. By assumption the hyperbola in r, $\frac{1}{[|r-x|-|l-x|]}$, is always positive and is decreasing in $|r-x|$ from infinity to zero on the feasible range, $|r-x| > |l-x|$. Thus we need to look at four cases depending on the $sgn(r)$ and $sgn(r-x)$. The structure of the proof will be the same in all four cases: the sign of the derivative $\partial U/\partial r$ will be determine by the difference between a non-decrasing function of P'/P in r, and the hyperbola (which will always be decreasing in r). As a result the difference (and hence the derivative) will either be always negative or will change exactly once from negative to positive. The cases are:

		$sgn(r-x)$	
		-1	$+1$
$sgn(r)$	-1	(a)	(b)
	$+1$	(c)	(d)

For (a) and (b) $r<0$ therefore $-P'/P$ is non-decreasing in r.

(a) $r<0$ **and** $r>x$

$$sgn\left(\frac{\partial U(r,l,x)}{\partial r}\right) = sgn\left(-\left\{\frac{P'(z)}{P(z)}+\frac{1}{[r-x-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{37}$$
$$= sgn\left(\left\{\left(-\frac{P'(z)}{P(z)}\right)-\frac{1}{[r-x-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{38}$$

Both P'/P and the hyperbola are always positive over the feasible region hence the derivative is always negative and hence quasi-convex.

(b) $r<0$ **and** $r<x$

$$sgn\left(\frac{\partial U(r,l,x)}{\partial r}\right) = sgn\left(-\left\{\frac{P'(z)}{P(z)}-\frac{1}{[|r-x|-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{39}$$
$$= sgn\left(-\left\{\frac{P'(z)}{P(z)}-\frac{1}{[(x-r)-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{40}$$
$$= sgn\left(-\left\{\frac{P'(z)}{P(z)}-\frac{1}{[(x-|l-x|)-r]}\frac{\mu}{t}\right\}\right) \tag{41}$$

P'/P is non-increasing in r since $r<0$. The denominator $(x-|l-x|)-r$ is decreasing in r therefore the hyperbola is increasing in r. Assuming $(x-|l-x|)<0$, as $r\uparrow(x-|l-x|)$ the hyperbola $\rightarrow\infty$, while P'/P is finite at $r=(x-|l-x|)$. Thus for r below but close to $(x-|l-x|)$ the diference of the two terms in parenthesis must be strictly negative. As $r\rightarrow-\infty$ the value of P'/P becomes no smaller than at $r=(x-|l-x|)$ while the hyperbola $\rightarrow 0$. Thus the two curves must cross and the derivative must change sign exactly once. In particular, the difference of the two terms in parenthesis must switch from positive to negative exactly once. Thus the sign of the derivative switches from negative to positive, hence quasi-convexity; and we call the local associated minimum $r^b_{\min}(x,l)$.

If $(x-|l-x|)>0$ then the change in sign might occur outside the feasible region implying monotonicity and hence again quasi-convexity.

For (c) and (d) $r>0$ therefore P'/P is non-decreasing in r.

(c) $r>0$ **and** $r>x$

$$sgn\left(\frac{\partial U(r,l,x)}{\partial r}\right) = sgn\left(\left\{\frac{P'(z)}{P(z)} - \frac{1}{[|r-x|-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{42}$$

$$= sgn\left(\left\{\frac{P'(z)}{P(z)} - \frac{1}{[r-x-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{43}$$

The hyperbola is decreasing in r, therefore the difference of the two bracketed terms either changes once in sign from negative to positive or is always negative. Hence quasi-convexity. By a similar arguement to (b) if $x+|l-x|>0$ there must be a unique local min $r^c_{\min}(x,l)$ otherwise the function may be monotonic over the feasible region.

(d) $r>0$ **and** $r<x$

$$sgn\left(\frac{\partial U(r,l,x)}{\partial r}\right) = sgn\left(\left\{\frac{P'(z)}{P(z)} + \frac{1}{[|r-x|-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{44}$$

$$= sgn\left(\left\{\frac{P'(z)}{P(z)} + \frac{1}{[(x-r)-|l-x|]}\frac{\mu}{t}\right\}\right) \tag{45}$$

$$= sgn\left(\left\{\frac{P'(z)}{P(z)} - \frac{1}{[(r-x)+|l-x|]}\frac{\mu}{t}\right\}\right) \tag{46}$$

Since P'/P is non-decreasing and the hyperbola is decreasing in r we can apply a similar arguement to (c) to establish the derivative either changes once in sign from negative to positive, at the local minimum $r^d_{\min}(x,l)$, or is always negative. Hence quasi-convexity. ■