Comparative Advertising: disclosing horizontal match information

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Abstract

Improved consumer information about horizontal aspects of products of similar quality leads to better consumer matching but also higher prices, so consumer surplus can go up or down, while profits rise. With enough quality asymmetry though, the higher quality (and hence larger) firm's price falls with more information, so both effects benefit consumers. This is when comparative advertising is used, against a large firm by a small one. Comparative advertising, as it imparts more information, therefore helps consumers. While it also improves profitability of the small firm, overall welfare goes down because of the large loss to the attacked firm.

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1 Introduction

Until the late 1990s, mentioning a competitor's brand in an ad was illegal in many EU countries. This situation was ended by a 1997 EU directive that made "comparative advertising" legal subject to the restriction that it should not be misleading. This brought the European approach closer to that of the FTC in the US. In other countries, comparative advertising remains illegal, or little used (see Donthu, 1998, for a cross-country comparison). The rationale for a favorable attitude towards "comparative advertising" on the part of competition authorities is that it improves the consumers' information about available products and prices (see Barigozzi and Peitz, 2005, for details and a wealth of examples and discussion). This raises a number of questions for the economic analysis of informative advertising. What is the scope of a practice that involves disclosing information that the product's supplier would choose not to reveal? Is the benefit to consumers from improved information mitigated by a welfare loss for competitors who are (presumably) hurt by comparative advertising about their products? Are consumers hurt by higher prices because product differentiation rises due to comparative advertising about product attributes?

The FTC's position is admirably clear: "Comparative advertising, when truthful and non-deceptive, is a source of important information to consumers and assists them in making rational purchase decisions."[1] This view underlies our modeling approach. The FTC also expects performance benefits: "Comparative advertising encourages product improvement and innovation, and can lead to lower prices in the marketplace." To a very large extent we corroborate these conclusions.

Here we consider a game between rival firms and their incentives to provide information. Consumers do not know the characteristics of a firm's product unless they are revealed through advertising, although consumers have (correct) priors about their evaluations. Firms

[1] The STATEMENT OF POLICY REGARDING COMPARATIVE ADVERTISING by the Federal Trade Commission, of August 13, 1979, is to be found at http://www.ftc.gov/bcp/policystmt/ad-compare.htm

are fully aware of each other's product attributes. If comparative advertising is illegal, then firms can only inform consumers about their own goods. Comparative advertising allows a firm to also inform consumers about rival product attributes that the other firm might not wish to communicate. We also address the welfare economics of comparative advertising.

Evaluating the impact of comparative advertising requires identifying when it changes the information available to consumers. That is, there must be some information that firms would not disclose if restricted to direct advertising, and that will be brought out if comparative advertising is allowed. In much of the literature on informative advertising, it is the cost of advertising that limits the information transmission by firms (see the seminal papers of Butters, 1977, Grossman and Shapiro, 1984, and the review coverage in Bagwell, 2007). Anderson and Renault (2006) show that a monopoly firm might limit information about its product attributes even if advertising has no cost. This result is a starting point for the present paper because it identifies situations where a firm is hurt by information disclosure about its own product, so that there might be some incentives for competitors to provide that information through comparative ads.

The paper contributes to the economics of asymmetric oligopolistic competition by first indicating how quality-cost advantages feed into equilibrium prices and sales. Second, it provides results on the impact of increased product information on market outcomes: while more information tends to increase profits, and welfare when qualities are similar, welfare can be harmed with more information due to price distortions when qualities are quite different. Third, it provides some predictions on when comparative advertising might be used – by smaller firms with cost or quality disadvantages – and welfare implications.

There is curiously little economics literature on comparative advertising, although in marketing there is quite a lot of documentation of the phenomenon and discussion of its effectiveness (see Grewal et al., 1997, for a comprehensive survey).[2] Barigozzi and Peitz

[2] A recent paper by Thompson and Hamilton (2006) shows subjects four different ads, with different

(2005) give a survey and some background modeling of alternative approaches.

Barigozzi, Garella, and Peitz (2006) take a signalling approach. An entrant with uncertain quality confronts an incumbent whose quality is known. The entrant chooses between "generic" advertising, which is standard money-burning to signal quality (as in Nelson, 1974), and "comparative" advertising, which involves a claim comparing the two firms' qualities. Firms may have favorable or unfavorable information about the entrant's quality but do not observe it perfectly. If comparative advertising is used, the incumbent may litigate in the hope of obtaining damages if the court, which observes quality perfectly, finds that it is low. Comparative advertising may credibly signal favorable information about entrant quality when a firm with such information expects it unlikely a court will find its quality is low.

Aluf and Shy (2001) model comparative advertising as shifting the transport cost to the *rival's* product in a Hotelling-type model of product differentiation. While this is an interesting angle in its own right, the modeling approach does not capture the informative aspect of comparison advertising and is not micro-founded in information revelation. Instead, it seems more like a model of (negative) "persuasive" advertising. In a Hotelling-Downs model of political competition, Harrington and Hess (1996) model negative political advertisements as moving the rival's perceived location away from the center, and positive advertisements as moving one's own perceived location towards the center (and hence increasing the chances of being elected). They show that a candidate with a quality (valence) disadvantage will use more negative advertising, in line with observed facts. Insofar as comparative advertising in our model is information about a rival that the rival would rather not see revealed, this result has an interesting parallel to our findings that a quality disadvantage is necessary for comparative advertising.

We consider the disclosure of horizontally differentiated attributes (valued differently by different consumers), assuming that product qualities are known. Here, product qualities are

comparative and analytical cues and judges ad effectiveness by surveying participants' impressions.

a device to indicate large or small firms in terms of their equilibrium market shares, and we shall refer to the firms as strong (to be thought of as one with a quality advantage) or weak (quality disadvantaged). If market sizes are very different (product qualities are sufficiently different), the equilibrium to the disclosure game has only the weak firm disclosing horizontal attributes and the strong one not. If comparative advertising is allowed, then the weak firm will disclose the horizontal attributes of both products (and so it is truly comparative).

To see how the model works, it is first useful to describe some background results which nonetheless hold independent interest for the economics of product differentiation and information. First, under firm symmetry, or close to it, more match information makes for more product differentiation and therefore raises prices (as expected). As we show, consumers may be better or worse off, as their improved ability to select the better match may be swamped by the price hike. Nonetheless, firms are better off. Results are surprisingly different when products are asymmetric, a case rarely treated by the literature. With zero horizontal match information, Bertrand competition leads the weak firm to price at cost while the strong one takes all the market by pricing at its quality advantage. Assume for the sequel this quality advantage is large. If the match information for just one firm is known (the weak say), the strong firm has to actually price *lower* to retain the whole market since it must attract the consumer who likes the weak firm most. With full information, the strong firm must set an *even lower* price if it is to retain (almost) all customers, since it must now attract the customer who likes it least and likes its rival most.[3] This means that not only are prices lower when there is more information (more product differentiation) but also the strong firm's profits are lower the more information there is. However, the weak firm retains a foothold under full information, so it prefers this.

This leads us into the advertising game analysis. Throughout the text we emphasize the

[3] We show that the strong firm will never want to *completely* price the rival out under full information, because the extra sales are not worth the lower price on infra-marginal units.

importance of firms' market shares by stressing two extreme cases, which allow for clean analytical arguments. A specific example helps fill in intermediate cases.

Similarly sized firms share the same incentive to provide extensive horizontal match information to maximize perceived differentiation and relax competition. Then, comparative advertising has no specific role insofar as full product information is provided regardless.

Dissimilar firms have quite different incentives to disclose horizontal match information. Hence, allowing comparative advertising may have a significant impact. This is best illustrated when the match distribution is such that the lower market share falls to zero when information is only one-sided:[4] we show that shares are *always* positive with full information. As noted above, the strong firm prefers no information to one-sided information, which in turn it prefers to full information, and it serves the whole market *unless* its competitor can achieve full information through comparative advertising. Hence, neither firm advertises (provides information) when comparative advertising is banned (because the weaker firm can get no market sales by using direct advertising for its own product alone). When comparative advertising is allowed, the weaker firm will provide full product information to give itself some market share (and hence profits).

If firms are sufficiently different, it is socially optimal that all consumers buy from the stronger firm. Then, comparative advertising deteriorates social welfare by letting the weaker firm make positive sales: full information relaxes the strong firm's price incentive to capture the whole market. Consumer welfare is improved though: full information brings down the price of the strong product (which is otherwise consumed by all) and some consumers choose to buy the cheap weak product which must therefore yield them higher surplus.

The text shows that these insights may remain valid even when the weak firm always retains some strictly positive market share with one-sided information. Under fairly general conditions, a sufficiently strong firm always prefers less information to more. We also show

[4] Technically, the condition is that the match density is strictly positive at the top of its support: $f(b) > 0$.

that there is a range of size differences for which the weak firm uses comparative advertising to achieve full product information, which the strong firm prefers obscured.

These results indicate that the *benefits of comparative advertising accrue to the weak firm, and to consumers, with so much damage to the large firm that total surplus goes down.* Although we noted above that more information can raise prices (and may even hurt consumers to the profit of firms, despite better consumer matching), this happens when comparative advertising is irrelevant in the sense that incentives to divulge own product information are already strong enough. When comparative advertising *is* relevant, the strong firm is attacked by the weak one, and we are in the regime when more information actually reduces prices (and enables better choices). This substantiates the FTC position that *comparative advertising improves competition*, even though one might have worried a priori that more match information would entail higher prices.

We provide an example with a Laplace distribution for match values with a complete characterization of the equilibrium outcome and welfare properties as a function of firm strengths. Results are fully consistent with those where the weak firm's market share vanishes to zero under one-sided information. This example shows that the weak firm's profit can be substantial enough to cover advertising expenses for comparative advertising.

Section 2 gives general background results for Bertrand duopoly with product differentiation. We outline the model in Section 3, describe demand under different degrees of product information, and find the corresponding equilibrium prices. These prices and profits are compared in Sections 4 and 5, which paves the way for the equilibrium information disclosure determined in Section 6. Section 7 shows some key surplus properties on the desirability of comparative advertising. Section 8 covers the Laplace example, and Section 9 discusses quality revelation, other extensions, and interpretation of the model. Section 10 concludes. The longer proofs are collected in the Appendix.

2 Some preliminary results

We first give some results for duopoly pricing that are used quite extensively in the analysis that follows: demands will satisfy the properties used here. The results pertain quite generally to differentiated product Bertrand duopoly with covered markets.

Consider a duopoly where firms 0 and 1 set prices p_0 and p_1. Define Δ as Firm 1's net quality advantage, $\Delta = p_0 - p_1 + Q$, where $Q \in \mathbb{R}$ may be understood as a Firm 1's gross quality advantage. Demand for Firm i's product is given by $D_i(\Delta)$, $i = 0, 1$, where D_1 is an increasing function taking values in $[0, 1]$ defined on $\mathbb{R}$. Further assume that $D_0 = 1 - D_1$, which may be understood as a covered market assumption: if heterogenous consumers have unit demands, each consumer must buy either product (and total demand is normalized to 1). Production costs are assumed to be zero.

Assume that $D_1(0) = D_0(0) = \frac{1}{2}$ so that, if $Q = 0$ and firms charge the same price, they share demand equally. Thus, $Q = 0$ may be viewed as a symmetric case, whereas when $Q \neq 0$, one firm has a competitive advantage over the other one in the sense that it may charge a larger price than its competitor and still serve at least half the market (Firm 1 having the competitive advantage if $Q > 0$). This competitive advantage is presented for the exposition as a quality difference, but could also be (and is formally equivalent to) a marginal production cost difference (marginal costs being constant). To see this, simply reinterpret p_i as a mark-up over marginal cost. More generally the firm with the competitive advantage is the one that has the larger difference between its quality and its marginal cost.[5]

We first establish a general result characterizing Firm 1's equilibrium net competitive advantage, Δ, and how it relates to the gross competitive advantage Q.

[5] To see this, let c_0 and c_1 denote firms' (constant) marginal costs, and define $m_i = p_i - c_i$ as Firm i's mark-up. Redefine $Q = (q_1 - c_1) - (q_0 - c_0)$ and set $\Delta = m_0 - m_1 + Q$. Then Firm 1's profit is $\pi_1 = m_1 D_1(\Delta)$ and now view firms as choosing their mark-ups. Hence the formal analysis is unaffected: note that $\Delta = q_1 - p_1 - (q_0 - p_0)$ so that the demand functions are just as before. The advantaged firm is now seen as the one with the higher quality-cost, and all results that follow can be appropriately re-interpreted.

Lemma 1 *Assume* $D_0 = 1 - D_1$*, and* $D_1(\Delta) = \frac{1}{2}$ *if and only if* $\Delta = 0$*. Then in any pure strategy (simultaneous choice) price equilibrium,* Δ *has the sign of* Q *and* $0 \leq |\Delta| \leq |Q|$*, with equality only if* $Q = 0$*. Therefore, in equilibrium, if* $Q = 0$ *then* $p_0 = p_1$ *and* $D_0 = D_1$*; while if* $Q \gtreqless 0$ *then* $p_0 \lesseqgtr p_1$ *and* $D_0 \lesseqgtr D_1$*.*

Lemma 1 is proved in the Appendix using revealed preference arguments. Whichever firm has a competitive advantage retains that advantage in equilibrium and thus has higher demand (Δ has the same sign as Q) but this advantage is somewhat mitigated because the weaker firm charges a lower price ($|\Delta| < |Q|$ if $Q \neq 0$). It also states that in the symmetric case where $Q = 0$, firms must share the market equally. The present results extend those of Anderson and de Palma (2001), who show such properties hold with the multinomial logit demand model (and $n \geq 2$ competing firms). We now put some additional structure on demand in order to tighten the characterization of equilibrium pricing.

Assumption 1 *There exist two, possibly infinite, real numbers* Δ_ℓ *and* Δ_u *such that* $D_1(\Delta) = 0$ *if and only if* $\Delta \leq \Delta_\ell$ *and* $D_1(\Delta) = 1$ *if and only if* $\Delta \geq \Delta_u$*. Furthermore,* D_1 *is differentiable on* $[\Delta_\ell, \Delta_u]$ *and* $D_1' > 0$ *on* (Δ_ℓ, Δ_u)*.*

Since D_1 is increasing and $D_1(0) = \frac{1}{2}$, we must have $\Delta_\ell < 0 < \Delta_u$. The differentiability assumption does not rule out a non-differentiable point of D_1 at either bound but merely guarantees that there is a right derivative at Δ_ℓ and a left derivative at Δ_u. Henceforth we use $D_1'(\Delta_\ell)$ and $D_1'(\Delta_u)$ to denote inside derivatives at these points. As we will see below, differentiability of D_1 at Δ_ℓ or Δ_u has important implications for the market outcome.

A key property of the demand derivatives is that $D_0'(\Delta) = -D_1'(\Delta)$. Since $\Delta = p_0 - p_1 + Q$ and $D_0 = 1 - D_1$, the derivatives of each firm's demand with respect to the firm's own price are equal and given by $-D_1'(\Delta)$.

For $\Delta \in (\Delta_\ell, \Delta_u)$, equilibrium prices must satisfy standard first-order conditions setting profit derivatives to zero. Thus prices may be written as

$$p_1 = \frac{D_1(\Delta)}{D_1'(\Delta)} \qquad \text{and} \qquad p_0 = \frac{D_0(\Delta)}{-D_0'(\Delta)} = \frac{1 - D_1(\Delta)}{D_1'(\Delta)}. \tag{1}$$

From equation (1) we derive a simple fixed point condition that fully characterizes Δ as a function of Q.[6] Differencing the price expressions in (1) we have Firm 0's price premium as

$$g(\Delta) \equiv \frac{1 - 2D_1(\Delta)}{D_1'(\Delta)}; \tag{2}$$

recalling that $\Delta = p_0 - p_1 + Q$, we then have

$$Q = \Delta - g(\Delta). \tag{3}$$

In order for equations (1) and (3) to be relevant, it is necessary that in equilibrium Δ falls strictly between Δ_ℓ and Δ_u. As we prove in the Appendix, this will be the case for all Q, if D_1 is differentiable at Δ_ℓ and Δ_u, which means if the inside derivatives are zero there.

Lemma 2 *Under Assumption 1, in any pure strategy Nash equilibrium, $\Delta \in [\Delta_\ell, \Delta_u]$. Furthermore, if for some $k \in \{\ell, u\}$, D_1 is differentiable at Δ_k, then $\Delta \neq \Delta_k$ for all $Q \in \mathbb{R}$.*

The above results characterize the equilibrium provided that it exists. Further regularity conditions must be imposed on D_1 to guarantee existence as well as uniqueness of an equilibrium. We thus assume the following.

Assumption 2 *D_1 and D_0 are strictly log-concave on $[\Delta_\ell, \Delta_u]$.*

This means that $\ln D_i$ is strictly concave, so D_i'/D_i is strictly decreasing.

[6] Equation (1) also yields a quick proof (given differentiability) for Lemma 1. When both demands are positive, the first-order conditions are $p_i = \frac{-D_i(\Delta)}{D_i'(\Delta)}$ (as per (1)). Since in any equilibrium, $D_0'(\Delta) = -D_1'(\Delta)$, then higher prices are associated to higher demands. But since $D_0(0) = 1/2$, the firm with the lower demand has a higher net quality. That is, $p_0 < p_1$ holds if and only if $D_0 < D_1$ and if and only if $\Delta > 0$. Taking the first and last inequalities, this can be only true if $Q > 0$. The equality results follow along similar lines. Intuitively, a quality (or cost) advantage is reflected in a higher mark-up and yet higher demand since the quality advantage is only partially offset with a higher price.

Proposition 1 *Under Assumptions 1 and 2, there is a unique pure strategy price equilibrium, such that p_0, p_1, Δ, and profits satisfy*

1. *If $\Delta_\ell - \frac{1}{D_1'(\Delta_\ell)} < Q < \Delta_u + \frac{1}{D_1'(\Delta_u)}$, then Δ is given by (3) and p_0 and p_1 are given by (1). Furthermore Δ is strictly increasing in Q with $\frac{d\Delta}{dQ} < 1$;*
2. *if $Q \leq \Delta_\ell - \frac{1}{D_1'(\Delta_\ell)}$, then $\Delta = \Delta_\ell$, $p_1 = 0$ and $p_0 = \Delta_\ell - Q$;*
3. *if $Q \geq \Delta_u + \frac{1}{D_1'(\Delta_u)}$, then $\Delta = \Delta_u$, $p_0 = 0$ and $p_1 = Q - \Delta_u$;*
4. *$\lim_{Q\to\infty} \Delta = \Delta_u$;*
5. *whenever Firm 1's demand is strictly positive, its profit is strictly increasing in Q and whenever Firm 0's demand is strictly positive, its profit is strictly decreasing in Q.*

Thus, whether or not equilibrium entails positive demands for both firms for all Q boils down to whether or not the demand derivative is zero at the point where one demand becomes zero. To understand this, suppose there is a zero derivative at (a finite) Δ_u.[7] Then, even for very high Q, it will not be worth Firm 1 pricing out Firm 0 when $p_0 = 0$, because the mass of last customers to get on board becomes vanishingly small at a high price (approaching $Q - \Delta_u$) and loses revenue on the existing consumer base. With a finite derivative, the trade-off becomes attractive at a high enough quality.

3 The model

Consumers are interested in buying one unit of one of two goods, which are sold by separate firms. The intrinsic benefit of the product class is large enough that all consumers buy one of the two products. Each product's specification is summarized by consumer valuations, which

[7] Of course, the argument also holds for an infinite Δ_u but the finite case is more striking and bears better juxtaposition with the case of a finite derivative.

are assumed independently, identically, and symmetrically distributed around zero with log-concave density $f(.)$, distribution function $F(.)$, and support $[-b, b]$. Hence $f(x) = f(-x)$ and $F(0) = 1/2$. We write consumer utility as

$$u_i = q_i - p_i + r_i, \qquad i = 0, 1, \tag{4}$$

where q_i is product i's quality (identical for all consumers), p_i its price, and r_i is the consumer's (idiosyncratic) match value. Without loss of generality, we assume that $Q \equiv q_1 - q_0 \geq 0$, which amounts to labelling the higher quality firm as Firm 1. We consider an experience good so purchases depend only on expectations of match values.

In the information disclosure game analysis below, we study firms' equilibrium choices of whether to impart information about own match values, and about match values with rivals when comparative advertising is permitted. Information is disclosed in the first stage, and price competition is the second stage. We therefore need first to analyze the price sub-games, as a function of the information available. These sub-games have independent interest as they indicate how prices and performance depend on the extent of information available to consumers.

Throughout, we assume that consumers observe prices (for example, in the store where purchases are made). They also know qualities: this can either be viewed as a direct assumption or else it follows from the analysis of Section 6 that if qualities are unknown to consumers, firms will reveal them in equilibrium (this is an extension of the basic "persuasion game" of Milgrom, 1981). However, absent advertising, consumers do not know their match valuations. Firms can advertise their own product specifications if they so wish. Such advertising will allow consumers to know their realizations of r_i. If firms are allowed to advertise rival product specifications, either they only do that and consumers know their realizations of r_j, or else a firm can advertise both product specifications, so consumers then know both r_i and r_j, a situation we refer to as "comparative advertising." Even though there is nothing untruthful in comparative advertising, or in advertising a rival's characteristics, this may be information that Firm j may choose not to reveal on its own.[8] If there is no information on Firm i's product specification (and hence the value of r_i), consumers must form expectations of their benefits from buying from Firm i. Consumers cannot otherwise acquire any information through search.[9] We next describe demand under the alternative consumer information states that might arise from advertising.

3.1 No information

If neither firm advertises, the consumers know only the expected value from purchasing from either of them. Since the mean match value is zero (by the assumption of symmetry of f), expected utility is

$$u_i = q_i - p_i.$$

Products are ex-ante homogenous except for the quality differential. Firm 1's demand is then zero if $\Delta < 0$, and one if $\Delta > 0$. If $\Delta = 0$, we invoke a standard tie-breaking rule that assigns all demand to Firm 1: since we assume $Q \geq 0$, this corresponds to an efficient allocation when $Q > 0$, and has no bite when $Q = 0$.

The price equilibrium under no information is quite straightforward. It follows from a standard Bertrand equilibrium argument that the low quality firm sets a zero price in equilibrium and the high-quality one serves the whole market at a price of Q.[10] Since the

[8] And in that sense may be viewed as rather negative.

[9] As shown in the online version, an alternative phrasing of the model with a search good instead of an experience good, as in Wolinsky (1986) and Anderson and Renault (1999, 2000), gives rise to an equivalent formulation as long as the search cost is high enough.

[10] Without further restriction, any price premium of Q with the price of the high quality firm between 0 and Q, is an equilibrium outcome to this game. Anderson and de Palma (1987) show that the equilibrium we select is the unique limit of equilibria in a horizontally differentiated market, as product heterogeneity goes to zero. This eliminates equilibria where the weak firm (which makes no sales) prices below marginal cost.

market size is normalized to unity, Q is also Firm 1's equilibrium profit.

3.2 One-sided information

Here we characterize the demand (indicated with a bar) that ensues when the information advertised concerns only one of the products (for example, only one firm advertises its product specifications). Suppose that Firm 0's match is known. Then, since the expected match value with Firm 1 is zero, the relevant utilities are

$$u_0 = q_0 - p_0 + r_0 \qquad \text{and} \qquad u_1 = q_1 - p_1.$$

These expressions give rise to a demand facing Firm 1 given by $\Pr(u_0 < u_1)$ or

$$\bar{D}_1 = \Pr(r_0 < \Delta) = F(\Delta). \tag{5}$$

Since the support of r_0, the random variable underlying $F(.)$, is $[-b, b]$ we have here that $\bar{D}_1(\Delta) = 1$ for $\Delta \geq \Delta_u = b$, and $\bar{D}_1(\Delta) = 0$ for $\Delta \leq \Delta_l = -b$, and Assumptions 1 and 2 are satisfied for this demand. Hence Proposition 1 holds.

We now argue the demands in such a situation are independent of which firm's match values are known (so the result is the same). Indeed, if Firm 1's match is known and Firm 0's match is unknown, the utilities relevant to choices are $u_0 = q_0 - p_0$ and $u_1 = q_1 - p_1 + r_1$. The demand facing Firm 1 is then $\bar{D}_1 = 1 - F(-\Delta)$. However, this demand expression is the same as (5) since symmetry of f implies $F(x) = 1 - F(-x)$. In summary:

Lemma 3 *If information is one-sided, each firm's demand does not depend on which firm's match values are known.*

Thus it makes no difference which firm's matches are known (given only one firm's are). This means there is no systematic bias in the model to favor advertising one's own match, or one's rival's horizontal match.[11]

[11] This result depends on the symmetry of f. Skewness would bias the incentives to reveal or not.

Under one-sided information, from the analysis of Section 2, whether or not one firm is excluded from the market in equilibrium depends on whether Q is high enough and whether the derivative of demand is positive or zero at the upper bound. Since that derivative is simply $f(b)$ (where $b = \Delta_u$ in the earlier notation), then we immediately have the following result as a corollary to Lemma 2 and Proposition 1. Denote equilibrium values of variables with an overbar.

Corollary 1 *Suppose information is one-sided. Then equilibrium demands are both positive regardless of Q if $f(b) = 0$. If $f(b) > 0$, then for $Q \geq b + \frac{1}{f(b)}$, Firm 1 serves the whole market in equilibrium and sets a price $\bar{p}_1 = Q - b$ while $\bar{p}_0 = 0$; for lower Q the market is shared and $\bar{p}_0$, $\bar{p}_1$, and $\bar{\Delta}$ are given by equations (1)-(3). Lastly, $\bar{\Delta} \to b$ as $Q \to \infty$.*

Note that the case $f(b) = 0$ covers the case when the support of match values is the extended real line. With a finite support and $f(b) > 0$, the quality-advantaged firm (1) prices so as to just retain the individual enjoying the highest regard for Firm 0, which is the individual who has a match $r_0 = b$. This compares to the mean value of 0 for Firm 1.[12]

3.3 Full information

Consumers know exactly their match values with both products if they have been advertised. Indeed, for what follows, it suffices that they only know the difference in values, $r_1 - r_0$, that is, the *comparison* between products. Arguably, this information might be easier to disclose than an absolute match value.

A consumer with full information purchases product 1 if and only if

$$q_1 - p_1 + r_1 \geq q_0 - p_0 + r_0$$

or equivalently $r_1 + \Delta \geq r_0$. The probability a consumer with a realization r_1 buys from Firm 1 is $F(r_1 + \Delta)$. Integrating over all possible values of r_1 gives Firm 1's demand as

[12] Equivalently, as per Lemma 3, Firm 1 must price so that the consumer least enamoured of it (holding $r_1 = -b$) nonetheless buys against an expected value of 0 with Firm 0.

$$\widetilde{D}_1(\Delta) = \int_{-b}^{b} F(r_1 + \Delta) f(r_1)\, dr_1 \tag{6}$$

and $\widetilde{D}_0(\Delta) = 1 - \widetilde{D}_1(\Delta)$, where full-information demands are characterized with a tilde.[13]

The range of values of Δ for which $\widetilde{D}_1(\Delta)$ is strictly between 0 and 1 is from $\Delta_l = -2b$ to $\Delta_u = 2b$, and Assumptions 1 and 2 are satisfied for this demand. These bounds arise because these are the values for which at least some value of $F(r_1 + \Delta)$ in (6) is neither zero nor one. For example, if $\Delta = 2b$, even the consumer who least likes product 1 and most likes product zero (that is, $(r_0, r_1) = (b, -b)$, or, indeed, $r_0 - r_1 = 2b$) will just switch to buying product 1 because its net quality advantage is so high.

Although this demand function has the whole market served by Firm 1 if $\Delta \geq 2b$, this never happens in equilibrium. This property is a direct corollary of Lemma 2 and Proposition 1. Since for $\Delta > 0$ we can write (6) as $\widetilde{D}_1(\Delta) = \int_{-b}^{b-\Delta} F(r_1 + \Delta) f(r_1)\, dr_1 + 1 - F(b - \Delta)$, then $\widetilde{D}_1'(\Delta) = \int_{-b}^{b-\Delta} f(r_1 + \Delta) f(r_1)\, dr_1$ and this expression is zero for $\Delta = 2b$. This puts us always in Case 1 of Proposition 1, which proves the next point. Denote equilibrium values with a tilde.

Corollary 2 *Suppose full information prevails. Then equilibrium demands are positive regardless of Q. Equivalently, the equilibrium Δ (i.e., $\tilde{\Delta}$) is always below $2b$, and $\tilde{p}_0$, $\tilde{p}_1$, and $\tilde{\Delta}$ are given by equations (1)-(3). Lastly, $\tilde{\Delta} \to 2b$ as $Q \to \infty$.*

Even though Firm 1 has the ability to price 0 out of the market, it never exercises the option because the marginal gain in consumers is so small even when the stakes (the price) are very large.

[13] By Lemma 1 we can concentrate on the case $\Delta \geq 0$. Then (6) becomes $\tilde{D}_1(\Delta) = \int_{-b}^{b-\Delta} F(r_1 + \Delta) f(r_1)\, dr_1 + 1 - F(b - \Delta)$. Demand can be visualized as the area of the unit square of consumer valuations accorded to each firm. The division line (indifferent consumer type) satisfies $r_1 = r_0 - \tilde{\Delta}$, which is a diagonal line. If $\tilde{\Delta} > 0$, Firm 1 attracts all those consumers for whom $r_0 < \tilde{\Delta}$ irrespective of their valuation of r_1: hence the final term in the demand function.

4 Equilibrium pricing for equal qualities

We now consider pricing sub-games conditional on information states induced by advertising decisions in the first stage of the game. Since the match density function, f, is log-concave, arguments from Caplin and Nalebuff (1991) guarantee that Assumption 2 holds so that existence and uniqueness under one-sided or full information follows from Proposition 1.

We first compare equilibrium prices under symmetry of qualities ($q_0 = q_1$). If there is no information, products are viewed as perfect substitutes and a standard Bertrand argument gives prices both equal to marginal cost, which we recall is zero.

With one-sided information, demand for Firm 1 is given by (5) which, for $Q = 0$ yields $\bar{D}_1 = F(p_0 - p_1)$. Thus demand for Firm 0 is $\bar{D}_0 = 1 - \bar{D}_1 = 1 - F(p_0 - p_1)$. Using Lemma 1, $\Delta = 0$; by Proposition 1, prices are equal (to $1/2\bar{D}'(0)$) and are given by

$$\bar{p} = \frac{1}{2f(0)} \ . \tag{7}$$

With full information, $\widetilde{D}_1 = \int_{-b}^{b} F(p_0 - p_1 + r_1) f(r_1)\, dr_1$ (see (6)). Again from Corollary 2, both prices are equal to $1/2\widetilde{D}'(0)$, or

$$\widetilde{p} = \frac{1}{2\int_{-b}^{b} f^2(r)\, dr} \ . \tag{8}$$

These prices are compared below.

Proposition 2 *If $Q = 0$, then $\tilde{p} \geq \bar{p} > 0$, where the first inequality is strict for distributions other than the uniform. That is, prices are higher under full information than under one-sided information than under no information.*

Proof. Since $f(.)$ is symmetric with maximum at $r = 0$, then

$$\int_{-b}^{b} f^2(r)\, dr \leq f(0) \int_{-b}^{b} f(r)\, dr = f(0)\,,$$

or, from (7) and (8), $\bar{p} \leq \widetilde{p}$. Equality is only attained with a uniform density, so then prices are equal. Otherwise, $\bar{p} < \widetilde{p}$. Since demand is split 50–50, profits are also higher (unless f is uniform) under full information. Both prices exceed the no-information zero price. ■

With equal qualities, profits are simply equal to half the equilibrium prices. This makes the case that more product information is better for firms (see also Meurer and Stahl, 1994). Another way to think of this is to note that full information (when $r_1 - r_0$ is known) is a mean preserving spread of r_1 alone (one-sided information), which in turn is a mean preserving spread of 0 (no information). This progression reveals less product differentiation and hence lower prices.

Full match information is the information state (for equal qualities) that gives the highest total welfare. This is because there is no allocation distortion due to unequal prices, and full information enables consumers to reach the first-best solution that each consumer buys her highest match.

The symmetric quality case underscores the product differentiation advantage imparted by full information over limited information. However, the result that a higher degree of information delivers higher equilibrium prices may lead to welfare losses if the model is broadened to allow for non-purchase.[14] This is one potential strike against the benefits of informative advertising generally (and not just comparative advertising.)

In conclusion, a symmetric setting delivers no distinctive role for comparative advertising. Nor does it enable much useful debate about the welfare benefits of comparative advertising. For that, we turn to the case of quite different qualities. Note that results under symmetry will hold in the neighborhood of $Q = 0$ since the various profit and surplus functions are continuous and comparison inequalities are strict at $Q = 0$.[15] This enables us below to state

[14] For another example, consumers might have price sensitive demands that depend also on their match values. With full information, those that like the product a lot can buy a lot, and conversely: there is a welfare loss when there is less match information since "one-size fits all" in demand.

[15] Except for the uniform, where the price equality between one-sided and full information requires us to evaluate explicitly what happens for $Q > 0$.

results for situations where firms are roughly of equal size.

5 Asymmetric qualities and information states

We first establish a (non-equilibrium) result which will be used later for the asymmetric cases, and helps build intuition for the results. The expressions compared are demands under full and one-sided information (see (5) and (6)) for given prices.[16]

Proposition 3 *The firm with higher net quality has higher quantity demanded under one-sided information than full information:*

$$\widetilde{D}_1(\Delta) = \int_{-b}^{b} F(r_1 + \Delta) f(r_1) dr_1 \leq F(\Delta) = \bar{D}_1(\Delta)$$

if and only if $\Delta \geq 0$*, with equality if and only if* $\Delta = 0$*.*

One-sided information gives a demand advantage to the firm with a net quality advantage because consumers impute the average valuation (0) to the unknown match value. The situation can also be interpreted as that of a monopolist facing a (known) outside option: a "strong" monopolist will therefore prefer not to advertise specific match values. This insight underpins results in Anderson and Renault (2006) where high search costs play effectively the role of an attractive outside option (so the monopolist is more likely to want to advertise). The result also suggests that were we to allow more firms, strategic effects aside, lower quality ones are more likely to wish to prefer fuller information. We return to this point below.

While the Proposition suggests that Firm 1 is better off when information is one-sided because it has higher demand for given $\Delta > 0$, it is less clear whether the *equilibrium* Δ also favors it, since one might also suspect that it could support a higher price under full

[16] This inequality is related to Jensen's inequality. We compare the expected value of some functional transformation of a random variable with the value of that function evaluated at the expected value of the random variable; Jensen's inequality compares these quantities under a convexity assumption on the functional whereas we consider a functional that is first convex and then concave. For the uniform distribution, $F(r + \Delta)$ is piecewise linear and a quick proof of the result may be obtained by applying Jensen's inequality.

information insofar as this means greater perceived product differentiation. The profit results are proved next, first determining the effect on profit of rival price and then own price.

For the next two results, a *strong demand state* refers to the information state (i.e., full information or one-sided information) for which a firm's demand is higher for all Δ, given the advantage conferred by Proposition 3. Let $\tilde{\pi}_i$ and $\bar{\pi}_i$, $i = 0, 1$ denote a firm's equilibrium profit under full and one-sided information respectively.

Lemma 4 *Assume $Q > 0$. A firm's profit in its strong demand state is strictly higher than in its weak demand state if its rival's equilibrium price is higher in the strong state. Hence:*

1. *if $\tilde{p}_1 \geq \bar{p}_1$ then $\tilde{\pi}_0 > \bar{\pi}_0$.*

2. *if $\bar{p}_0 \geq \tilde{p}_0$ then $\bar{\pi}_1 > \tilde{\pi}_1$.*

Proof. First assume that $\tilde{p}_1 \geq \bar{p}_1$. Under full information, Firm 0 can always select a price such that $\Delta = \bar{\Delta}$ so that, by Proposition 3 and since $\bar{\Delta} > 0$ for $Q > 0$, its demand is higher than with one-sided information. Furthermore, the corresponding price is $\bar{p}_0 + \tilde{p}_1 - \bar{p}_1 \geq \bar{p}_0$ so that its profit is strictly higher than with one-sided information. A symmetric argument shows the other part of the Lemma. ■

Intuition for this lemma is simple. In either informational state, our model has the standard property that a firm may achieve higher profits if its rival charges a higher price because the firm can achieve the same price difference and hence the same demand while charging a higher price. If in addition the state at which the rival's price is higher coincides with that at which demand is higher at any given price difference, then that state is clearly more profitable. However, a fundamental tension is that rivals may charge lower prices in firms' strong states, meaning that we have to go deeper into the model to resolve firms' information revelation incentives, which we do in the next lemma and the next section.

Lemma 5 *Assume $Q > 0$. A firm's profit in its strong demand state is strictly higher than in its weak demand state if its own equilibrium price is strictly higher in the strong state. Hence:*

1. *if $\bar{p}_1 > \tilde{p}_1$ then $\bar{\pi}_1 > \tilde{\pi}_1$.*

2. *if $\tilde{p}_0 > \bar{p}_0$ then $\tilde{\pi}_0 > \bar{\pi}_0$.*

Proof. Assume $\bar{p}_1 > \tilde{p}_1$. If $\bar{D}_1(\bar{\Delta}) \geq \tilde{D}_1(\tilde{\Delta})$ then the result clearly holds. Thus assume $\tilde{D}_1(\tilde{\Delta}) > \bar{D}_1(\bar{\Delta})$. Using the first order conditions we have $\bar{p}_1 = \frac{\bar{D}_1(\bar{\Delta})}{\bar{D}_1'(\bar{\Delta})} > \tilde{p}_1 = \frac{\tilde{D}_1(\tilde{\Delta})}{\tilde{D}_1'(\tilde{\Delta})}$ so that we must have $\tilde{D}_1'(\tilde{\Delta}) > \bar{D}_1'(\bar{\Delta})$. Furthermore we must also have $\tilde{D}_0(\tilde{\Delta}) < \bar{D}_0(\bar{\Delta})$. Using these two inequalities and the first order conditions for Firm 0 we have $\bar{p}_0 > \tilde{p}_0$. This implies, applying Lemma 4, that one-sided information is more profitable for Firm 1 than full information, which proves the first part of the lemma. The other part of the lemma is proved in a similar fashion. ■

This lemma tells us that if we show that a firm charges a higher price in its more favorable information state then this is enough to guarantee that it earns a higher profit in that state. This result follows from Lemma 4 along with both firms' first-order conditions for prices. We next use the results above to determine profit relations for Q large.

5.1 Price and profit relations for large quality differences

The next two results use the property that log-concavity of $1-F$ (as implied by log-concavity of f) means $\frac{1-F(\Delta)}{f(\Delta)}$ is decreasing. Since it is positive, it reaches a finite limit $\ell \geq 0$ as $\Delta \to b$.

Lemma 6 *Assume b is finite. Then as Q tends to infinity, $\bar{p}_0$ and $\tilde{p}_0$ tend to the same finite limit and $\bar{p}_1 - \tilde{p}_1$ tends to b.*

Proof. Since $\bar{\Delta} \to b$ and $\tilde{\Delta} \to 2b$ as $Q \to \infty$, we have $\lim_{Q\to\infty}(\bar{p}_1 - \tilde{p}_1) + (\tilde{p}_0 - \bar{p}_0) = b$. Showing that $\tilde{p}_0 - \bar{p}_0$ tends to zero as $Q \to \infty$ therefore suffices to establish the result. Since

$\bar{p}_0 = [1 - F(\bar{\Delta})/f(\bar{\Delta})$, then $\bar{p}_0 \to \ell$ as $Q \to \infty$. Furthermore we have:

$$\begin{aligned}\tilde{p}_0 &= \frac{\int_{-b}^{b-\tilde{\Delta}}[1-F(r+\tilde{\Delta})]f(r)dr}{\int_{-b}^{b-\tilde{\Delta}} f(r+\tilde{\Delta})f(r)dr} \\ &= \frac{\int_{-b}^{b-\tilde{\Delta}} \frac{[1-F(r+\tilde{\Delta})]}{f(r+\tilde{\Delta})} f(r+\tilde{\Delta})f(r)dr}{\int_{-b}^{b-\tilde{\Delta}} f(r+\tilde{\Delta})f(r)dr}.\end{aligned}$$

From the log-concavity of f we therefore have $\frac{1-F(b)}{f(b)} \leq \frac{[1-F(r+\tilde{\Delta})]}{f(r+\tilde{\Delta})} \leq \frac{1-F(-b+\tilde{\Delta})}{f(-b+\tilde{\Delta})}$, so

$$\ell \leq \tilde{p}_0 \leq \frac{1-F(-b+\tilde{\Delta})}{f(-b+\tilde{\Delta})}.$$

Since $\tilde{\Delta} \to 2b$ as $Q \to \infty$, $\tilde{p}_0 \to \ell$, and the result follows. ■

Note that the limit, ℓ, is 0 if $f(b) > 0$ or indeed if any higher-order derivative is non-zero (by l'Hopital's rule). Then both $\bar{p}_0$ and $\tilde{p}_0$ tend to zero.

Firm 1 charging a higher price with one-sided information than with full information implies from Lemma 5 that it prefers one-sided to full information for Q large. In the following Proposition we show that it prefers no information under some mild technical assumptions.

Proposition 4 *Assume b is finite and Q is sufficiently large. Firm 1 strictly prefers one-sided information to full information. If in addition $\ell < b$, then Firm 1 strictly prefers no information to one-sided information.*[17]

Proof. The first part follows directly from Lemmas 5 and 6. With no information, Firm 1 serves the whole market at a price of Q and thus earns a profit of Q. With one-sided information, Firm 1 charges $\bar{p}_1 = \bar{p}_0 - \bar{\Delta} + Q$. As $Q \to \infty$, $\bar{\Delta} \to b$ and $\bar{p}_0 \to \ell$ so that, if $\ell < b$, then Firm 1's price is strictly below Q, which proves the second part. ■

[17] The limit ℓ is 0 if $f(b) > 0$ or if any higher-order derivative is non-zero.

What this means is that the quality advantaged firm may want to hold back information and keep consumers uninformed.[18] Nevertheless, the other firm's incentives may lie in the opposite direction. As shown next, this would happen if the density f is differentiable at b.

Proposition 5 *For $Q > 0$, Firm 0 weakly prefers one-sided information to no information, and strictly prefers full information to none. If in addition $f'(b)$ is finite or $f(b) = 0$, and Q is sufficiently large, then Firm 0 strictly prefers full information to one-sided information, which is indifferent to no information if and only if $f(b) > 0$.*

Proof. For any $Q > 0$, the first part holds since Firm 0 has no sales under no information and has strictly positive sales at a strictly positive price with full information.

To prove the second part, recall that as Q goes to infinity, $\bar{\Delta}$ goes to b and $\tilde{\Delta}$ goes to $2b$. Also recall that Firm 1's equilibrium demand derivatives with respect to Δ may be written as $\bar{D}_1'(\bar{\Delta}) = f(\bar{\Delta})$ with one-sided information and $\tilde{D}_1'(\tilde{\Delta}) = \int_{-b}^{b-\tilde{\Delta}} f(r+\tilde{\Delta})f(r)dr$. For Q large, $-b+\tilde{\Delta} > 0$ and hence, $f(r+\tilde{\Delta})$ is decreasing in r on $[-b, b-\tilde{\Delta}]$ (because f is log-concave and symmetric with respect to zero) and hence $\tilde{D}_1'(\tilde{\Delta}) \leq f(-b+\tilde{\Delta})F(b-\tilde{\Delta})$.

Now, as Q tends to infinity, both $\bar{\Delta}$ and $-b+\tilde{\Delta}$ tend to b. Hence, using the symmetry of f with respect to 0, $f(-b+\tilde{\Delta})F(b-\tilde{\Delta})$ may be approximated by $[f(\bar{\Delta}) + f'(\bar{\Delta})(-b+\tilde{\Delta} - \bar{\Delta})][F(-\bar{\Delta}) + f(\bar{\Delta})(b-\tilde{\Delta}+\bar{\Delta})]$. Since $\frac{F(-\bar{\Delta})}{f(\bar{\Delta})} = \frac{1-F(\bar{\Delta})}{f(\bar{\Delta})}$ has a finite limit as $\bar{\Delta}$ tends do b, assuming that $f'(b)$ is finite implies that $\frac{f(-b+\tilde{\Delta})F(b-\tilde{\Delta})}{f(\bar{\Delta})}$ tends to zero as Q tends to infinity. This proves that for Q large enough, $\bar{D}_1'(\bar{\Delta}) > \tilde{D}_1'(\tilde{\Delta})$.

From Lemma 6, for Q large we have $\bar{p}_1 > \tilde{p}_1$ which, from Firm 1's first-order conditions is consistent with the above result only if $\bar{D}_1(\bar{\Delta}) > \tilde{D}_1(\tilde{\Delta})$, which implies that $\bar{D}_0(\bar{\Delta}) < \tilde{D}_0(\tilde{\Delta})$. Hence, from the above inequality between demand derivatives and Firm 0's first-order conditions we have $\tilde{p}_0 > \bar{p}_0$: full information being Firm 0's strong demand state, it

[18] This may not hold if individual demand were price elastic. Then consumers with a good match would buy more – but those with bad matches would buy less. Without match information, all consumers purchasing from 1 would buy an average amount, and so it is not clear which way this effect would play.

follows from Lemma 5 that for Q large enough, Firm 0 earns more profit with full information.

Finally, with $f(b) = 0$ Firm 0's demand and profit are strictly positive for any Q with one-sided information, whereas with $f(b) > 0$, Firm 0 has no market under one-sided information for Q large and is therefore indifferent between one-sided and no information, which are both dominated by full information even if $f'(b)$ is infinite. ■

The above proposition says that for a large firm asymmetry, the small firm prefers more information and this preference is strict except in the special case where it retains a positive market share only with full information (when $f(b) > 0$) which makes it indifferent between no information and one-sided information. While the weak firm faces a lower rival price under full information, the direct information effect of Proposition 3 dominates.

Comparative advertising can permit the low-quality firm to reach its preferred information state (full), as is shown in the following section.

6 Equilibrium information disclosure

We can now address the equilibrium advertising strategies starting with the case when comparative advertising is debarred. Firms are viewed as setting advertising content before the price sub-game is resolved, so there is a two-stage game in ad content and then pricing. In the context of consumers going to the store for a pain reliever, they see all ads before going, and they see all prices on arrival.[19]

If consumers have no information about a product's attributes, they use the expected match value of zero. Otherwise, they know the match value communicated from any ads.

[19] The game structure is also motivated by the empirical implausibility of the mixed strategy equilibria that would result for some Q values if prices and advertising are chosen simultaneously. Indeed, there is no pure strategy equilibrium for Q close to zero, whether comparative advertising is allowed or not. To see this note that for $Q = 0$, firms share the market equally and $\Delta = 0$ in any pure strategy equilibrium (regardless of information disclosed). Each firm would clearly deviate from no information, so the relevant candidates involve one-sided or full information. Then the analysis in Section 3 indicates that demand has a larger derivative with respect to Δ with partial information which implies that each firm wishes to deviate from one-sided to full information while increasing its price, and each firm would deviate from full to one-sided information while decreasing its price.

Recall that advertising is costless. We start with close qualities.

Proposition 6 *If $Q > 0$ is small enough and comparative advertising is barred, then each firm reveals its match in the only equilibrium. This is still an equilibrium when comparative advertising is allowed.*

Proof. For $Q = 0$, for any distribution apart from the uniform, from Proposition 2, each firm's profit is strictly higher when it reveals information than when it does not, regardless of the strategy of the other firm. By continuity, this property still holds for $Q > 0$ small enough, hence the equilibrium stated is unique. For the uniform, the analysis in the online version shows both firms strictly better off revealing than not. Both revealing own matches is still an equilibrium when the comparative advertising is allowed since all information is revealed by the firms separately, and so there is no extra information to be revealed. ■

We exclude $Q = 0$ from the statement, but it holds for every distribution apart from the uniform at this point. Even for the uniform, the equilibrium would be as stated if we applied the tie-break rule in favor of divulging MORE information.[20] We would not expect a difference to this result for more firms if they were roughly equal, because the price effect of greater product differentiation would encourage information revelation and the quality-advantage effect noted in Proposition 3 would be small. An outside option would even increase the incentive to provide information if it were relatively attractive.

Turning now to large Q, we first deal with banned comparative advertising.

Proposition 7 *If comparative advertising is barred, $\ell < b$ finite and Q is large enough, then:*

1. *if $f(b) > 0$, neither firm reveals its match;*

[20] Given our rule, the uniform admits other equilibria since firms are indifferent between one-sided and full information (see Proposition 2). In particular, either firm alone revealing is also an equilibrium.

2. if $f(b) = 0$, only Firm 0 reveals its match.

Proof. (1) From Proposition 4, Firm 1's profit is higher when it does not reveal its match information than when it does, regardless of the strategy of Firm 0 (note that if Firm 0 does not reveal, Firm 1 is better off not revealing because it serves the entire market in both cases and price is higher when not revealing since it does not need to get on board the consumer most disliking it). Given Firm 1 does not reveal, Firm 0 gets zero profits regardless; by the advertising tie-breaking rule, it will not reveal (see Proposition 5).

(2) From Proposition 4, for Q large, Firm 1 strictly prefers one-sided information. Furthermore, since $f(b) = 0$, Firm 0 earns a strictly positive profit with one-sided information, but nothing with no information. ■

Firm 0's incentive not to disclose any information is weak if $f(b) > 0$: it is indifferent between revealing and not revealing because in any case it is kept out of the market. When $f(b) = 0$, though, it earns a strictly positive profit by revealing.

The next result shows that comparative advertising by the high quality firm is irrelevant to the market outcome. The result makes concrete the idea that comparative advertising is done by the small firm. (The exponential example below gives an indication of the range of values for which various equilibria arise.)

Proposition 8 *Assume $Q > 0$. If one-sided information is weakly more profitable than full information for Firm 0 then it is strictly more profitable for Firm 1. Hence, whenever there is a unique equilibrium involving comparative advertising, the weak firm (0) deploys it against the strong firm (1).*

Proof. From the contrapositive of Lemma 4, if one-sided information is weakly more profitable for Firm 0, then $\bar{p}_1 > \tilde{p}_1$. Then, from Lemma 5, Firm 1's profit is strictly higher with one-sided information. For the second statement, suppose the only equilibrium had comparative advertising by 1. But the first statement implies that Firm 0 too would strictly prefer full information to one-sided, and we know that 0 prefers full information to none (Proposition 5). This means comparative advertising by 0 must also be an equilibrium, a contradiction to the postulated uniqueness. ■

In general, although comparative advertising has no impact on the market outcome for similar market shares (Q small) since information is fully disclosed anyway, it changes the situation quite strikingly for big asymmetries (Q large).

Proposition 9 *If comparative advertising is allowed, and b is finite, there exists a range of quality differences, Q, for which Firm 0 discloses all the horizontal match information using comparative advertising, while Firm 1 does not advertise match information at the unique equilibrium. If in addition, $f'(b)$ is finite or $f(b) > 0$, this situation prevails for Q large enough.*

Proof. From Proposition 4, for Q sufficiently large, Firm 1's full information profit is lower than for one-sided information. Since the reverse inequality holds for $Q = 0$ and profits are continuous functions of Q, there is at least one crossing point where both situations are equally profitable for Firm 1. Then, at the first such crossing point Firm 0 earns strictly more profits with full information (from the contra-positive to Proposition 8) and this remains true for a slightly larger Q. Thus, for such a quality difference, Firm 1 always prefers no information while Firm 0 prefers full information. Therefore, if comparative advertising is allowed, the unique equilibrium is as claimed. The final part follows from Proposition 5. ■

Therefore comparative advertising promotes full information when firms are asymmetric enough: the adverse price effect on the low-quality firm is overtaken by the beneficial demand information effect. The same tensions are likely to arise with several firms. Insofar as information effects dominate, then we might expect the lowest quality firms to benefit most from comparative advertising. However, the fact that lowest qualities are not likely to see much absolute improvement in demand means that it may be the middle range of firms who most indulge in the practice: but this needs a dedicated model to address.

Who gains and who loses is underscored in the next section.

7 Welfare and consumer surplus

We consider two standards, total surplus and consumer surplus. We shall see that consumers are better off (with Q large) when comparative advertising is allowed and used, so there is full information: they get better matches and tend to have lower prices too. But total surplus is lower (interestingly, despite a smaller distortion in prices; optimality would have equal prices). This underscores the fact that the high quality firm is worse off.

7.1 Total surplus

We start with the case of similar firms.

Proposition 10 *For Q close enough to 0, total surplus is highest under full information, and this is implemented in equilibrium whether or not comparative advertising is allowed.*

Proof. In equilibrium, all information is revealed (Proposition 6). This is optimal, given firm pricing, because in the neighborhood of $Q = 0$ prices are arbitrarily close so that the consumer allocation is arbitrarily close to optimal. This indeed ensures the full social optimum is arbitrarily close to being attained. ■

In this case there is no special role for comparative advertising and also no role for expanding (or restricting) advertising. Matters are different for high Q.

Proposition 11 *For Q large enough with b finite and either $f'(b)$ finite or $f(b) > 0$, total surplus is lower when comparative advertising is allowed*

Proof. First note that for Q large enough it is optimal that all consumers consume good 1. Indeed, as long as $Q \geq 2b$, demand is a sufficient statistic for welfare since no matter who consumes which product, the greatest horizontal match difference is $2b$ (which is bounded by assumption) and this is dominated by Q. Therefore, the situation yielding the highest social surplus is the one for which demand for product one is higher.

From Propositions 9 and 7, the equilibrium changes from one-sided information to full information with comparative advertising allowed. The proof of Proposition 5 shows that for Q large, with $f(b) > 0$ or $f'(b)$ finite, Firm 0's sales are higher with full information, which establishes the result. ■

Comparative advertising leads to full information revelation,[21] which harms welfare. This is not because full information yields higher prices, since in this model high prices are not intrinsically harmful since there is no deadweight loss from non-purchase.[22] It is not even the case that net prices are more distorted (in the sense of being more different for given Q) under full information than one-sided or no information. Indeed, the equilibrium Δ(which is always below the optimal value of Q) is larger, and hence closer to optimal under full information (Corollaries 1 and 2 to Proposition 1)! It is the information structure that causes the welfare loss: from Proposition 3, one-sided information distorts the allocation of consumers in favor of the high quality product and this outweighs the adverse impact of equilibrium pricing that favors the low quality product whereas under full information, price distortion is the only source of inefficiency.

The above result is to be contrasted with the result for $Q = 0$ where full information unambiguously improves social surplus over one-sided or no information. In that case however, the socially optimal outcome of full information arises in equilibrium whether comparative

[21] This result does not depend on the tie-breaking assumption.

[22] The lack of an outside option is instrumental to this property. With enough quality asymmetry, prices are actually higher under one-sided information than full information (Lemma 6). This means that *consumers* may nonetheless be better off with comparative advertising, as we see next.

advertising is debarred or not. By contrast, for Q large, since Firm 1 strictly prefers one-sided information, full information may arise in equilibrium only if comparative advertising is allowed. Firm 0 deploys it whenever it prefers full information, and with Q large, this is always harmful. Comparative advertising though, by increasing Firm 0's profit, may enable it to enter a market it could not enter otherwise. Hence, *comparative advertising enhances the ability to enter of low quality entrants but such entry is detrimental to social surplus.* This rather goes against the FTC's position of encouraging comparative advertising, although it is important to think of consumers too: the next results validate the position.

7.2 Consumer surplus

Predictions on consumer surplus are in stark contrast with those on social surplus. For identical qualities, predictions on which informational state would be preferred by consumers are ambiguous since *more information improves the match but leads to higher prices.* For a uniform distribution, full information is superior to one-sided information since prices are the same in both regimes. With the Laplace distribution, one-sided information is better for consumers than full information. However, no information is better for consumers under both distributions.

For a large quality difference we now show that consumer surplus is higher with full information so that comparative advertising by the low quality firm is desirable for consumers. This is easily seen in the extreme case where $f(b) > 0$.[23] Then with one-sided information, consumers all buy from the high quality firm while with full information, that firm charges a lower price which improves the situation of those who still buy product 1 and some of them switch to product 0, implying that they increase their surplus by doing so. We now show that this result holds more generally.

Proposition 12 *For Q large enough and either $f'(b)$ finite or $f(b) > 0$, consumer surplus*

[23] The next proof extends this result to $f(b) = 0$.

is higher when comparative advertising is allowed.

Proof. The restriction implies that allowing comparative advertising changes the equilibrium disclosure from one-sided to full information (Propositions 7 and 9). We now show that consumer surplus is higher with full information under the milder condition that b is finite. Recall from Lemma 6 that for Q large, a switch from one-sided information to full information induces a drop in product 1's price. Hence all consumers buying product 1 with one-sided information gain from such a switch and the corresponding increase in consumer surplus is bounded below by $\bar{D}_1(\bar{\Delta})(\bar{p}_1 - \tilde{p}_1)$. Furthermore, since only customers of Firm 0 with one-sided information stand to lose from the switch, the loss in consumer surplus is bounded above by $\bar{D}_0(\bar{\Delta})(\tilde{p}_0 - \bar{p}_0)$. Since $\bar{D}_0(\bar{\Delta}) \to 0$ and $\bar{D}_0(\bar{\Delta}) \to 1$ as $Q \to \infty$, the result is a direct consequence of Lemma 6 ■

The proof shows that consumer surplus is higher with full information even if $f(b) = 0$ for Q large enough. Most consumers then buy the high quality product which is cheaper with full information. The only consumers who might be hurt by a move from one-sided to full information are those who buy from Firm 0 in both instances, which is a vanishing population for Q large. This means that comparative advertising enhances consumer surplus whenever it is used. Better matches and lower prices are enabled for large firm asymmetries, in contrast to the fundamental conflict between these forces which can arise under fairly symmetric firm strengths, which we highlighted at the start of this sub-section.

The full range of outcomes for the Laplace density is illustrated next.

8 An example: the Laplace distribution

Several of the results for the main text have been given for bounded densities. We have also concentrated on results in the neighborhood of symmetric qualities, and for a sufficiently large quality difference. The Laplace example shows what can happen for intermediate quality differences.[24] Let the density and distribution of consumer tastes be given by:

$$f(x) = \frac{1}{2}e^{|x|} \quad \text{and} \quad F(x) = \frac{1}{2}e^{x} \text{ for } x < 0, \text{ with } F(x) = 1 - \frac{1}{2}e^{-x} \text{ for } x > 0.$$

Since the density has full support, Firm 0 retains a toehold under one-sided information, no matter what the quality advantage of Firm 1. The symmetric equilibrium prices when $Q = 0$ are readily calculated to be 2 for full information, 1 for one-sided information, and 0 for no information: profits are half this amount because the market splits equally.

The profits are illustrated as a function of $Q \in [0, 5]$ in Figure 1. The 45-degree line is Firm 1's profit under no information; Firm 0 nets zero. Clearly Firm 0 prefers full information to one-sided to none. Simulations indicate that the profit dominance of full-information over one-sided information prevails for all values of $Q \geq 0$. This is consistent with our theoretical results for $Q = 0$ and Q large. This pattern also implies that there can *never* be no information in equilibrium, and full information ensues whenever comparative advertising is legal.

Firm 1 prefers full to one-sided to no information for low Q, and the opposite for high Q. Both concur with the earlier results. In the middle range, there are two patterns (no information on top or in the middle), but since no information is not a relevant market outcome, we shall not dwell on these. Hence the relevant cases are those considered already for low and large Q. In equilibrium, then, all information is revealed for low Q regardless of the legality of comparative advertising. For large Q, only the low quality firm advertises, and it will comparative advertise if that is legal.

Consumer surplus is illustrated in Figure 2, where we hold q_0 constant and raise q_1. As expected, surplus is increasing in Q (given q_0 is fixed). This means that the high quality

[24] The uniform example given on the web version shows some slightly different patterns pointed out below.

firm cannot extract the full value of the extra quality in equilibrium, which is also in line with Lemma 1: it is hurt by the competitive response of a lower p_0. Consumer surplus for low qualities is highest for no information, and lowest for full information. This is surprising because one might expect the better matching effect of more information to not be fully extracted by firms. However, no general result here is available, since consumer surplus for the uniform example is highest under full information and lowest with no information.

For large Q we see here that full information gives highest consumer surplus (and no information the lowest). In accord with the equilibrium analysis above, this shows that full information is best for consumers for Q large enough, and so the possibility of comparative advertising must raise their welfare. However, it is important that Q be large enough. The example illustrates that *consumers are actually worse off if comparative advertising is legal* for intermediate Q between around 3.7 and 5.8: in this range, one-sided information is the equilibrium arrangement without comparative advertising, and this gives higher surplus than full information, which Firm 0 uses if comparative advertising is legal.

From Figure 2, total welfare is highest under full information for low Q, and under one-sided information for high Q (see Proposition 11). Here comparative advertising reduces total welfare when used: the point where full information total surplus becomes smaller (around $Q = 4.3$) exceeds where full information becomes less profitable for Firm 1 (around $Q = 3.7$), which is where Firm 0 starts using comparative advertising.

Finally, this numerical example illustrates how profitable comparative advertising may be for the smaller firm. For $Q = 3.7$ where comparative advertising becomes a relevant practice, Firm 0 nearly doubles its profit from .21 to .39 if it is allowed to use comparative advertising. Note that its profit without comparative advertising is about 7% of its competitor's profit (which is 3.03), so the incremental profit needs to cover any advertising cost in equilibrium.

9 Discussion

It was assumed above that qualities are known to consumers beforehand. We first show that if qualities are known to firms but not consumers, then firms will advertise qualities if they can, so the basic set-up still holds.[25] We then discuss some background to comparative advertising in practice.

9.1 Quality disclosure

Let us now consider the possibility that qualities as well as horizontal attributes are unknown. A standard result in the literature due to Milgrom (1981) and Grossman (1981), is that a monopoly firm that may disclose certifiable information about its product's quality, always discloses it in equilibrium. We now show that practically the same result holds for our duopoly setting with horizontal differentiation as well as vertical (quality), provided that disclosure on quality information induces no updating on match values. Assume that qualities for the two firms are independently drawn from the same distribution and that realizations are initially known only by firms. First note that no information disclosure is not an equilibrium. In such an equilibrium, firms would engage in symmetric Bertrand competition in the second stage and earn zero profit. It would be profitable for a firm to deviate and disclose its horizontal attributes thus creating some product differentiation. Second, there is no equilibrium where only the low quality firm discloses its quality. Recall that, independent of what information is revealed about horizontal attributes, the high quality firm earns some strictly positive profit that is strictly increasing in its quality. Suppose that for some given low quality and horizontal attributes information disclosed, there is some non-zero subset of high qualities that are not disclosed in equilibrium. The consumers form some conditional expectation as to the quality of a firm that does not disclose so that any firm with a quality above that conditional expectation is better off disclosing its quality.

[25] If vertical qualities cannot be disclosed, expected qualities are used throughout the analysis.

Now consider the choice of a low quality firm. With no horizontal attributes disclosed or if only one product's attributes are disclosed and the quality difference is large enough, it earns zero profits. Otherwise, its profit is strictly positive and strictly increasing in its quality (Proposition 1). Whenever the latter situation arises, then an argument analogous to that used for the high quality firm shows that the low quality is always revealed. The only situation when the low quality firm cannot guarantee itself some strictly positive profit is when the quality difference is very large and comparative advertising is not allowed. Then, the low quality is not disclosed but consumers update their beliefs accordingly and anticipate the low quality is very low relative to the high quality. The only information disclosed in that case is the higher quality and the high quality firm serves the whole market.

To summarize, the only situation where the market outcome would not be fully identical to that obtained while assuming that qualities are known is when the quality difference is large and comparative advertising is not allowed. Then the low quality is not revealed but it is anticipated by consumers to be much lower than the high quality. The market outcome is qualitatively similar to that derived in previous sections where the quality difference should be replaced by the difference between the high quality and some expected low quality.

9.2 On comparative advertising

Our theory focuses on firms' incentives to disclose information on horizontal match characteristics. The firm with the smaller market share uses comparative advertising against its competitor, only if its market share is significantly lower. The asymmetry in market shares that we have modeled as a large Q may be due to factors other than a quality or marginal cost advantage, such as consumer loyalty to a brand. Below we discuss the relevance of focussing on horizontal match information rather than on quality information, and we argue that results concur with some empirical regularities.

A typical comparative advertisement includes a claim that the product performs better than some competing product(s). In one classic case, Subway claimed its sandwiches were healthier than McDonald's; Advil claims it is faster and stronger than Tylenol. At first blush, these appear to be vertical quality claims. Whether they might be interpreted as horizontal claims depends critically on the heterogeneity in consumer tastes and on the consumers' perception of the potential product space. When a consumer learns that Subway food is healthier, she may lean to Subway if she is strongly health conscious but she may veer to McDonald's if she wants to get fed at a low cost and worries that Subway food is not filling enough. This latter argument was actually used by Quiznos in a comparative advertising campaign against Subway.[26] Similarly, a consumer who learns that Advil is faster and stronger than Tylenol might (reasonably, as it turns out) worry that the former could cause harsher gastro-intestinal side effects. Whether strength and speed correspond to higher quality depends on how different consumers value these attributes relative to the potential perceived risks associated with taking the drug.

We argued above that if qualities were not known initially, each firm would certify its own quality information so there is no specific role for comparative advertising on quality. An obvious reason why a firm might not disclose its quality is that certification is imperfect or costly (and this point applies to both horizontal and vertical quality). Perhaps too firms may only certify relative qualities in practice because consumers are unable to evaluate absolute quality claims. Then disclosing quality information requires using comparative advertising as in Barrigozi, Garella, and Peitz (2007). Allowing for imperfect certification (see Shin 1994) is one research direction to explore, for both vertical quality and horizontal match information. The analysis should also allow for disclosure of partial product information as in Anderson and Renault (2006), since actual claims in comparative ads usually concentrate

[26] "Quizno's is using marketing jujitsu effectively by attacking Subway's core value, low-calorie healthfulness. Quizno's compares the generous amount of meat and cheese on their sandwiches to the skimpy portions that make Subway low-fat, low calorie." from "Comparative advertising: Marketing jujitsu." at http://www.brainposse.com/archivejujitsu2.html

on one dimension of the product space, which might be selected for strategic reasons. It would also be worthwhile to introduce costly advertising reach into the model.

The idea that comparative advertising is successful only from small against large ("ju-jitsu"), and not in the other direction, has been termed the Iron Law of marketing, and examples abound.[27] Anderson, Ciliberto, and Liaukonyte (2008) code TV commercials for non-prescription (OTC) analgesics, a product category for which advertising expenditures represent a large percentage of revenue. Comparative advertising is widely used. Tylenol has the highest market share followed by Advil. The latter is the industry leader in comparative advertising spending, and Tylenol is by far the main target of comparative advertising by other brands. Comparative advertising represents only a small percentage of Tylenol's advertising expenditure which is the largest in the industry. The behavior of Tylenol is quite consistent with our model, although it does use some comparative advertising. This might be partly explained by allowing a persuasive component to advertising, which is missing from our model. To properly account for the other brands' behavior, it would be useful to extend the model to oligopoly to predict the relation between market shares and comparative advertising activity (both advertisers and targets). As noted in the text, there is a conflict between expanding demand and triggering lower rival prices when there is more information, and with several firms there are multiple subsets of information that can be revealed. A free rider problem might arise among the smaller firms, and they also are likely to gain less than some larger rivals (by dint of their lower initial qualities). This means that an oligopoly extension does not follow trivially from the results here, although we hope to have identified the main tensions at play.

[27] http://www.brainposse.com/archivejujitsu1.html

10 Conclusions

Comparative advertising involves informing consumers of characteristics of rival products. On the surface, the practice would appear socially beneficial (assuming of course that the advertising is not misleading) and should lead to better informed choices. It has though been pointed out that it may relax price competition (and lead to higher prices) because it increases product differentiation. However, this is also true for direct advertising, so a useful theory should also explain when it is used and not.

The theory proposed here does this by focussing on intrinsic quality differences in the products sold. If these qualities are quite similar, firms have enough incentive to advertise their own products and comparative advertising plays no role. This is true in a balanced market with firms that have similar market shares. Only if market shares are sufficiently different does comparative advertising come into play. If it is illegal, the strong firm may not need to advertise, and the weak firm may be overwhelmed. If comparative advertising is legal though, the weak firm can improve its consumer base and survive by using advertising that targets the dominant product and compares characteristics. Thus, the model predicts that comparative advertising is used by weaker firms targeting market leaders. This is in line with most instances in practice.

The model also delivers a salutary message for comparative advertising. It enables weaker firms to increase sales, and, in some instances, to survive. The dominant firm effectively parlays its quality advantage into both a high mark-up and high sales, although this is more acute when only one product's information is advertised, a case where the weaker firm may be driven out of the market. The paper shows that the informational benefits of comparative advertising may be overwhelmed by excessive sales by the low-quality firm, which is harmful for very large quality differences. However, some caveats are worth drawing. First, even when total welfare falls, it may be that consumer welfare rises since comparative advertising (full information) may be associated with lower prices when quality (or cost) differences are large enough. Second, such lower prices might entail a lower deadweight loss if the model were extended to allow for non-purchase options.

The modeling approach is based on truthful informative advertising of horizontal characteristics with rational consumers. The approach was chosen to portray comparative advertising in a favorable light by allowing the conveyance of more hard information. If consumers were not rational (rationality is embodied in the model in the assumption that consumers form correct expectations of mean valuations in the absence of information), they might be manipulated by misleading advertising. The legal system may play an important role in ensuring truthfulness in this context.

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11 Proofs

11.1 Lemma 1

Assume that $Q \geq 0$. We first show that $\Delta = Q$ or $\Delta = 0$ imply that $Q = 0$. Assume first that $\Delta = Q$, so that $p_0 = p_1 = p$. Then, in order for Firm 0 not to wish to deviate, for any real number $\delta \geq -p$ we must have

$$p[1 - D_1(Q)] \geq (p + \delta)[1 - D_1(Q + \delta)],$$

which is equivalent to

$$(p - \delta)D_1(Q + \delta) \geq pD_1(Q) + \delta[1 - 2D_1(Q + \delta)].$$

If $Q > 0$, then for any $\delta \in (-Q, 0]$, $D_1(Q + \delta) > \frac{1}{2}$ and thus $\delta[1 - 2D_1(Q + \delta)] > 0$. Then Firm 1 could deviate from p to $p - \delta$ so as to earn a profit of $(p - \delta)D_1(Q + \delta)$ which strictly exceeds $pD_1(q)$. So we must have $Q = 0$ in order for Δ to be equal to Q in equilibrium.

Now suppose that $\Delta = 0$ so that $p_0 = p_1 - Q$. Then, for any $\delta > 0$ we must have

$$\frac{1}{2}(p_1 - Q) \geq (p_1 - Q + \delta)[1 - D_1(\delta)],$$

or, equivalently,

$$(p_1 - \delta)D_1(\delta) \geq \frac{1}{2}p_1 + (\delta - \frac{Q}{2})[1 - 2D_1(\delta)].$$

If $Q > 0$, since $\delta > 0$ so that $D_1(\delta) > \frac{1}{2}$, for δ sufficiently small, the right hand side strictly exceeds $\frac{1}{2}p_1$. Firm 1 would therefore be better off charging $p_1 - \delta$ rather than p_1. Thus in order for Δ to be zero in equilibrium we must have $Q = 0$.

We now show that in equilibrium $0 \leq \Delta \leq Q$ which, along with the results above, proves the Lemma for $Q \geq 0$. First, it is necessary that Firm i prefers p_i to its rival's price so that

$$p_0[1 - D_1(\Delta)] \geq p_1[1 - D_1(Q)]$$

and

$$p_1 D_1(\Delta) \geq p_0 D_1(Q).$$

Adding these two inequalities and rearranging yields

$$p_0 - p_1 \geq (p_0 - p_1)[D_1(Q) + D_1(\Delta)]. \tag{9}$$

Since $Q \geq 0$, if $p_0 > p_1$, then $D_1(\Delta) > D_1(Q) \geq \frac{1}{2}$. Thus $D_1(Q) + D_1(\Delta) > 1$ which contradicts inequality (9). So we must have $p_0 \leq p_1$, or equivalently $\Delta \leq Q$.

It must also be the case that Firm i prefers charging p_i than a price that would set Δ to zero, so that

$$p_0[1 - D_1(\Delta)] \geq \frac{1}{2}(p_1 - Q)$$

and

$$p_1 D_1(\Delta) \geq \frac{1}{2}(p_0 + Q).$$

Adding these two inequalities yields

$$[1 - D_1(\Delta)]p_0 + D_1(\Delta)p_1 \geq \frac{1}{2}(p_0 + p_1). \tag{10}$$

We know from above that $p_1 \geq p_0$. If $p_1 > p_0$, inequality (10) requires that $D_1(\Delta) \geq \frac{1}{2}$ and therefore $\Delta \geq 0$. If $p_1 = p_0$ then $\Delta = Q \geq 0$. This completes the proof for $Q \geq 0$.

Similar arguments establish the result for $Q \leq 0$. Q.E.D.

11.2 Lemma 2

If $\Delta < \Delta_\ell$ or $\Delta > \Delta_u$, then whichever firm has a demand of 1 could increase its price without losing any demand and thus, increase it profit; this proves the first part of the Lemma.

We now show that differentiability at Δ_k implies that $\Delta = \Delta_k$ cannot be an equilibrium. For instance for $k = u$, differentiability at Δ_u implies that the left derivative of D_1 at Δ_u is 0 (since the right derivative is zero). Then Firm 1's profit derivative is $D_1(\Delta_u) = 1 > 0$ so that Firm 1 would deviate and increase its price. Similarly, if $\Delta = \Delta_\ell$, Firm 0 would wish to increase its price from the candidate equilibrium. Q.E.D.

11.3 Proposition 1

The argument for existence is standard (see Caplin and Nalebuff, 1991).

Before going through the 3 cases it is useful to note that since D_1 and $D_0 = 1 - D_1$ are strictly log-concave, g is strictly decreasing on $[\Delta_\ell, \Delta_u]$ and so the right-hand side of equation (3) (the equation is $Q = \Delta - g(\Delta)$) is strictly increasing on that same interval. This shows that Δ is uniquely defined in Case *1*. Furthermore, because $\frac{D_i'}{D_i}$, $i = 0, 1$ is strictly increasing, prices are uniquely determined by equation (1). It also shows that in this case Δ must be strictly increasing in Q. Implicit differentiation of (3) and Assumption 2 imply $\frac{d\Delta}{dQ} < 1$.

First consider case *3*. If $\Delta \leq \Delta_\ell$, then Firm 1 makes zero profit whereas, since $Q > 0$. it could obtain a strictly positive profit by charging, for instance, a price $p_0 + Q > 0$. Next note that $\Delta_u + \frac{1}{D_1'(\Delta_u)}$ is the right-hand side of (3) evaluated at $\Delta = \Delta_u$. Since Q is at least as large and the right-hand side of (3) is strictly increasing on (Δ_ℓ, Δ_u), there is no Δ in that interval that satisfies (3). Since an equilibrium exists and using Lemma 2, we must have $\Delta = \Delta_u$. We also know from Lemma 2 that this case may arise only if $D_1'(\Delta_u) > 0$ so that Firm 0's profit left derivative with respect to p_0 is $-p_0 D_1'(\Delta_u)$ which would be negative if $p_0 > 0$ and thus Firm 0 would wish to decrease its price. Thus we have $p_0 = 0$ and the expression for p_1 follows.

Case *2* may be treated with symmetric arguments.

Now consider case *1*. We show that we may not have $\Delta = \Delta_u$ and a symmetric argument would show that we cannot have $\Delta = \Delta_\ell$. From Lemma 2, this suffices to complete the proof. Hence suppose that $\Delta = \Delta_u$. As was shown above, we must then have $p_0 = 0$. The right derivative of Firm 1's profit is given by $1 - p_1 D_1'(\Delta_u) = 1 - (Q - \Delta_u) D_1'(\Delta_u)$. Since $Q < D_1(\Delta_u) + \frac{1}{D_1'(\Delta_u)}$ the right derivative of profit strictly exceeds 0. Then Firm 1 could increase its profit by increasing its price.

For part 4, the limit result follows from case *3* if $D_1'(\Delta_u) > 0$. Otherwise, if $D_1'(\Delta_u) = 0$,

(2) and (3) can only hold when $\Delta \to \Delta_u$ for $Q \to \infty$ since $g(\Delta)$ is finite for $\Delta < \Delta_u$.

For part 5, the equilibrium Δ increases in Q and strictly increases in case *1*. Since D_1 increases in Δ, D_1 increases in Q and D_0 decreases in Q. To complete the proof it suffices to show that p_1 strictly increases in Q whenever $D_1 > 0$ and p_0 strictly decreases in Q whenever $D_0 > 0$. This is immediate in cases *2* and *3*. In case *1* prices are given by (1). Since D_1 and D_0 are assumed to be strictly log-concave, p_1 strictly increases in Δ and p_0 strictly decreases in Δ which proves the result since Δ strictly increases in Q in case *1*. Q.E.D.

11.4 Proposition 3

From symmetry of f, equality clearly holds if $\Delta = 0$. We now show that the inequality holds strictly for $\Delta > 0$. Symmetry of f implies

$$\int_{-b}^{b} F(r+\Delta) f(r) dr = \int_{-b}^{0} \left[F(r+\Delta) + F(-r+\Delta) \right] f(r) dr,$$

and

$$F(\Delta) = \int_{-b}^{0} 2F(\Delta) f(r) dr.$$

Hence it suffices to establish that $F(r+\Delta) + F(-r+\Delta) < 2F(\Delta)$ for all $r < 0$. This is equivalent to

$$F(-r+\Delta) - F(\Delta) < F(\Delta) - F(r+\Delta)$$

or

$$\int_{\Delta}^{-r+\Delta} f(s) ds < \int_{r+\Delta}^{\Delta} f(s) ds$$

Using appropriate changes of variables, this condition may be rewritten as

$$\int_{0}^{-r} f(\Delta+t) dt < \int_{0}^{-r} f(\Delta - t) dt.$$

Since $\Delta > 0$, quasi-concavity and symmetry of f around zero implies that $f(\Delta+t) < f(\Delta-t)$, for all $t \in (0, -r]$. This ensures the proper inequality.

Symmetric arguments establish reverse inequalities for $\Delta < 0$. Q.E.D.

12 A search good

Suppose that a consumer can observe the product's attributes before making a purchase at cost $c > b$. She must incur the visit cost c to buy from either firm, but the cost of the first visit is irrelevant since the consumer must buy one of the two products in any case. Hence buying from a second firm or sampling it and not buying costs c. We now show that demands are exactly the same as with the experience good version of the model (and so prices and equilibria are too). This we do by showing that the consumer always buys from the first firm she visits so that the information she obtains when she gets there is irrelevant.

Consider a consumer who, after observing prices and advertised information, decides to visit Firm i first. If information about product i was provided through advertising, then the consumer has not learned anything from her first visit and she will clearly choose to buy product i given that she initially chose to visit Firm i. Let us thus assume that r_i was unknown to her when she chose to visit Firm i. A first possibility is that she was informed about her match with the other product when she made that choice. A standard sequential search argument shows that she would then have chosen to incur search cost c to find out about r_i, if and only if her match with the competing product (r_j, $j \neq i$) augmented by the price difference $p_i - p_j - c$ is strictly less than $-c < -b$.[28] She will then choose to purchase product i even if she finds out that $r_i = -b$. Suppose finally that neither product was known when the consumer decided on her first visit. Since she chose to visit Firm i first, we must have $p_i \leq p_j$. Then the search theoretic argument used above shows that, when she finds out her match with product i, for any r_i, she will not visit Firm j: since $r_i + p_j - p_i \geq -b > -c$ it is not worth incurring search cost c to find out about r_j.[29] The ability of the consumer to obtain product information which has not been advertised before buying therefore has no

[28] Here $p_j + c$ is the price of the known product j.

[29] Here the price of the known product is p_i since the cost of visiting Firm i is already sunk. Furthermore, the exact condition used here assumes that recalling Firm i's offer after visiting Firm j has no cost.

impact on her choice of product since it would be too costly to use that information.

In an earlier version of this paper, we studied product information disclosure with a search good. Assuming that only one of the two products is unknown, we found that the firm selling the unknown product would disclose horizontal attributes if and only if its quality is below the other product's. Furthermore, a known product with low quality uses comparative advertising (if allowed) to disclose information about an unknown high quality product.[30] Hence the predictions of the model with search (and only one product unknown), are qualitatively similar to those derived in the present paper. The full range of outcomes for the uniform density is illustrated next.

13 Equilibrium profits and equilibrium information disclosure; uniform density

We give a full characterization of sub-game outcomes and the full equilibrium for the special case of uniform density on $[-\frac{1}{2}, \frac{1}{2}]$ below.

We first compare firm pricing behavior under one-sided information disclosure with that under full information. Assume first that $Q < 3/2$ so that Firm 0 retains some positive market share in the one-sided equilibrium.[31]

From the general pricing formula (3), Firm 1's equilibrium net quality advantage Δ solves $Q = \Delta - g(\Delta)$. The function g is defined by (2) as $g(\Delta) = \frac{1-2D_1}{D_1'}$, so that

$$\bar{g}(\Delta) = -2\Delta$$

for one-sided information transmission (recalling $\bar{D}_1(\Delta) = F(\Delta) = \frac{1}{2} + \Delta$) and

$$\tilde{g}(\Delta) = -\Delta - \frac{\Delta}{1-\Delta}$$

[30] These results hold as long as a pure strategy equilibrium exists, which is not necessarily the case for all search cost values or quality differences.

[31] This bound comes from applying Proposition 1, Case 1: $\Delta_u + \frac{1}{D_1'(\Delta_u)}$ for the uniform is equa1 to $\frac{1}{2} + 1$.

for full information (recalling $\widetilde{D}_1(\Delta) = \int_{-b}^{b} F(r+\Delta)f(r)dr$ which is $\frac{1+2\Delta-\Delta^2}{2}$ for $\Delta \geq 0$). It is readily verified that both $\bar{g}$ and $\tilde{g}$ are strictly decreasing, and, furthermore, $\bar{g} > \tilde{g}$ for the relevant range of $\Delta \in (0,1)$. These properties establish:

Lemma 7 *For the uniform density and $Q \leq 3/2$, price differences satisfy $\widetilde{\Delta} < \bar{\Delta}$.*

This means that the extra product differentiation involved with full information revelation exacerbates price differences. Nonetheless, they still are less than the quality difference, as per Lemma 1.

From the first-order conditions under the two different information structures, Firm 0's prices satisfy (see the Appendix):

$$\bar{p}_0 = \frac{1}{2} - \bar{\Delta}$$

under one-sided information and

$$\widetilde{p}_0 = \frac{1-\widetilde{\Delta}}{2}$$

under full information disclosure, where $\bar{\Delta}$ and $\widetilde{\Delta}$ are the corresponding equilibrium Δ's.

Lemma 8 *For the uniform density and $Q \leq 3/2$, prices satisfy $\widetilde{p}_0 > \bar{p}_0$ and $\widetilde{p}_1 > \bar{p}_1$.*

Proof. From the two price expressions given above, it is clear that $\widetilde{p}_0 > \bar{p}_0$ (for given $\Delta > 0$). Furthermore, the two price expressions are decreasing in Δ and since Lemma 7 shows in equilibrium, $\widetilde{\Delta} < \bar{\Delta}$, we must have $\widetilde{p}_0 > \bar{p}_0$ in equilibrium. Finally, in order for $\widetilde{\Delta} < \bar{\Delta}$ we must also have $\widetilde{p}_1 > \bar{p}_1$ in equilibrium. ■

Thus, for $Q \leq \frac{3}{2}$, both firms charge higher prices when consumers know both products than when they know only one.[32] The first Appendix Figure plots equilibrium prices against Q, Firm 1's prices are increasing in Q while Firm 0's prices are decreasing. Red is full information, black is one-sided, and purple (the 45-degree line for Firm 1) is zero information.

[32]The result can easily be proved directly from the equilibrium prices given in the Appendix: $\bar{p}_0 = \frac{1}{2} - \frac{Q}{3}$ and $\tilde{p}_0 = \frac{(1-Q)+\sqrt{8+(1-Q)^2}}{8}$. The proof in Lemma 8 holds more generally.

The above Lemmas may be used along with Proposition 3 to establish that the low quality Firm 0 is better off if both products are known.

Lemma 9 *For the uniform density, profits satisfy* $\widetilde{\pi}_0 > \bar{\pi}_0$.

Proof. Both demands for Firm 0 (with one-sided or full information) are decreasing in Δ and, by Proposition 3, demand with full information is strictly larger for a given $\Delta > 0$. Since, by Lemma 7, Firm 1's net quality advantage, Δ, is lower under full information, demand for Firm 0 is larger with full information. Furthermore, since it charges a higher price, it earns a higher profit. ■

The converse to the argument in the proof above is that demand for the high quality firm is lower with full information than when only one product is known. Since it charges a higher price, with full information, equilibrium profits for the high quality firm may not be compared on the basis of the above results. We show below that the high quality firm also prefers full information if Q is not too large but prefers one-sided information for Q large enough. Direct calculation from the values in the Appendix for prices in the various regimes gives:

Lemma 10 *For the uniform density, there exists a quality value* $\hat{Q} > 3/2$ *such that profits satisfy* $\widetilde{\pi}_1 > \bar{\pi}_1$ *for* $0 < Q < \hat{Q}$ *and* $\widetilde{\pi}_1 < \bar{\pi}_1$ *for* $Q > \hat{Q}$.

The large quality difference result is consistent with the text Proposition. The uniform is rather special because profits are equal under one-sided and full information when $Q = 0$. As Q rises above zero, full information dominates one-sided information as regards Firm 1's profits, and continues to do so whenever Firm 0's equilibrium demand under one-sided information is positive. Firm 1's profit indifference point happens for $\hat{Q}$ at which Firm 0 earns nothing under one-sided information. Hence for low Q the results of Proposition 6 apply, and those of Proposition 7 for large Q (above $\hat{Q}$). In the interim, some extra possibilities arise. These are discussed below.

The next Appendix Figure has Q on the horizontal axis and profits on the vertical. The upward-sloping lines are Firm 1's profits for no information (magenta), full information (green), and one-sided information (blue). While full information always dominates one-sided information for parameter values such that 0 serves some market, no information dominates both for a large quality advantage and loses to both for a small quality advantage. The downward-sloping lines are Firm 0's profits, with full information (black) always dominating one-sided information (red), which in turn always dominates no information (for $Q < 3/2$).

Equilibrium disclosure follows directly from this Figure. In particular, no information is never an equilibrium for $Q < 3/2$ because Firm 0 can provide information, generate product differentiation and get a positive profit. Indeed, it is a dominant strategy (given equilibrium pricing in the sub-games) for 0 to provide information. Given that, Firm 1 will always want to provide information itself. So here there is full revelation, and no role for comparative advertising insofar as any equilibrium still entails full revelation.[33]

Now consider $Q > 3/2$. The driver for the equilibrium is what happens to Firm 1's profit between full information and one-sided information.[34] As per Lemma 10, this depends on which side of $\hat{Q}$ the quality difference Q lies. For $Q > \hat{Q}$ (which exceeds 3/2), the only equilibrium is for there to be no advertising if comparative advertising is not permissible (as per Proposition 7): it is a dominant strategy (among the pricing sub-games) for 1 to NOT reveal, and, in response, since 0 gets nothing either way, it does not reveal either (by the tie-break rule that favors less information over more in case of indifference). Otherwise, the

[33] With comparative advertising allowed, there is an equilibrium with each providing information about matches with its rival ("negative advertising"). There is another equilibrium with either of the firms providing a full comparison and the other doing nothing.

[34] The one-sided information price, given the rival sets $p_0 = 0$ and that Firm 1 serves the whole market, must ensure that 1 gets on board the consumer who least likes it, which is $r_0 = -\frac{1}{2}$; this means a price of $Q - 1/2$ (since 1 delivers expected utility zero to all). Of course, this is less attractive than no information, whereby the price charged is Q (with no product differentiation, the keel is even).

only equilibrium is comparative advertising by Firm 0 (as per the text Proposition), which enables it to survive.

For $Q \in \left[3/2, \hat{Q}\right]$, equilibrium is driven by the twin properties that $\bar{\pi}_1 < \widetilde{\pi}_1 < \pi_1^{zero}$ and $\widetilde{\pi}_0 > \bar{\pi}_0 = \pi_0^{zero} (= 0)$. With comparative advertising debarred, one equilibrium has no information provided, and another has both providing own match information. In the former case, Firm 1 prefers no information to one-sided information and so does not advertise if Firm 0 does not advertise, and Firm 0 will not advertise if Firm 1 does not. In the other equilibrium, each firm prefers to advertise if the other advertises. Allowing now comparative advertising, the latter is still an equilibrium. The former is not because Firm 0 would prefer comparative advertising, and this comparative advertising is the other equilibrium.[35] However, since $Q > 3/2$, comparative advertising allows a weak firm to survive against the optimality rule.[36]

Finally, note that (using the price expressions later in the Appendix) as Q becomes large, the full information price for Firm 1 goes to $Q - 1$, whereas its one-sided information price is $Q - \frac{1}{2}$. This shows that the full information price can be lower for the high quality firm. Moreover, consumer surplus is higher under full information. Under one-sided information all consumers buy from Firm 1, whereas under full information those who still buy from Firm 1 pay a lower price and those choose to buy from Firm 0 are better off.

The next Appendix Figure plots consumer surplus and total surplus. The curves at the bottom are consumer surplus. Note consumer surplus with no information is zero surplus (the expected match value conditional on no information is zero, and so too is price). The interesting feature is the double crossing of the consumer surplus for full and one-sided

[35] Equilibrium strategies cannot involve Firm 1 giving a full comparison and 0 doing nothing, since 1 would deviate to advertising nothing at all. Nor can they involve negative advertising by either alone: both prefer full information to one-sided information, which outcome they can get by either full comparative advertising or indeed reciprocal neagative advertising.

[36] Comparative ads more generally might facilitate toe-hold entry for entrants to become larger later, and this could be desirable in an extended context.

information. Zero surplus (no information) dominates for Q very low, one-sided information is next highest, and full information is lowest. Firms can manage to extract a lot of surplus when the extra information is imparted. Consistent with our analysis, consumers are best off under full information for Q large enough.

Notice in the Figure for surplus that consumers prefer full information (blue line) for Q larger than around 3. However, if we look at the firms' decisions, Firm 1 prefers one-sided to full information for Q larger than about 2.2. This raises the possibility that comparative advertising is bad for consumers in this range because information is revealed when one-sided information is better for consumers. However, the relevant comparison is not between full and one-sided information in that the tie-break rule we used means the equilibrium has no information if there is no comparative ads allowed. To see the brittleness of this solution, suppose instead we assumed information is revealed if a firm is indifferent. Then, the equilibrium with comparative advertising debarred would be one-sided (in this quality range between 2.2 and 3), which is optimal, instead of the equilibrium our tie-break rule selects, which is no advertising at all. Then comparative advertising would be bad because it yields inferior (to consumers) surplus, a result which is effectively due to high prices. Indeed, the Laplace example in the text does illustrate such an outcome (for intermediate Q values) where comparative advertising is used in equilibrium but is detrimental to consumer surplus.

Total surplus is also illustrated in the Figure: the diagonal (in purple) is the zero information case, green is full information, and black is one-sided (which corresponds to no information for $Q > 3/2$ because all consumers buy from Firm 1 anyway, and there is no pricing distortion). Note that the green line converges to the black one as Q gets large because the surplus under full information involves almost everyone buying from Firm 1: this result is verified by plotting total surplus for large values of Q.

13.1 Pricing expressions for the uniform density

We give a free-standing derivation of equilibrium prices for the uniform density $f(x) = 1$ for $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, and zero otherwise.

13.2 One-sided information

Since $\bar{D}_1(\Delta) = F(\Delta)$ in general, for the uniform density we have $\bar{D}_1(\Delta) = \frac{1}{2} + \Delta$ for $\Delta \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ and $\bar{D}_1(\Delta)$ is zero below the lower bound and one above the upper bound. When within the bounds, $\bar{D}_1' = -1$, so we have a simple linear demand system.[37]

We can immediately determine the equilibrium prices for $\Delta \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ as

$$p_1 = \frac{1}{2} + \Delta \qquad \text{and} \qquad p_0 = \frac{1}{2} - \Delta.$$

These prices depend only on net quality differences so we may apply Lemma 1.

Taking the difference of these two equations we can write out and solve for $\Delta = Q/3$. Substituting back gives prices as

$$p_0 = \frac{1}{2} - \frac{Q}{3}, \qquad \text{and} \qquad p_1 = \frac{1}{2} + \frac{Q}{3},$$

which therefore hold for the interior regime, with $\Delta = Q + p_0 - p_1 = Q/3$ (which is consistent with Lemma 1): so that this regime applies when $Q < 3/2$ (recall $Q > 0$). Equilibrium profit levels are given by these prices squared (as is standard for linear demands with unit slopes).

Otherwise, for $Q \geq 3/2$, we have

$$p_0 = 0 \qquad \text{and} \qquad p_1 = Q - \frac{1}{2}.$$

Here the quality-advantaged firm prices so as to just retain the individual retaining the highest regard for Firm 0, which is the individual who has a match $r_0 = 1$, which compares to the mean value of 0 for Firm 1.[38]

[37] The "symmetric" version, with consumers knowing their valuations at both firms, does NOT give a linear demand system. This latter system is determined in the text below.

[38] Equivalently, as per Lemma 3, if Firm 1 reveals its match information while Firm 0 does not, 1 must

13.3 Full information

For the uniform density, recalling that $r_1 \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, (6) becomes

$$\widetilde{D}_1(\Delta) = \int_{-1/2}^{1/2-\Delta} \left(\frac{1}{2} + r_1 + \Delta\right) dr_1 + \Delta = \frac{1 + 2\Delta - \Delta^2}{2}$$

Notice that $\Delta < 1$ for both firms to have positive demands: for $\Delta > 1$, Firm 1's demand is $\widetilde{D}_1(\Delta) = 1$, but, by Lemma 1, this is never relevant.[39] Firm 0's demand is $1 - \widetilde{D}_1(\Delta)$ or:

$$\widetilde{D}_0(\Delta) = \frac{(1-\Delta)^2}{2}$$

Since evaluations for the two products are i.i.d. we have $\widetilde{D}_1(0) = \frac{1}{2}$, and all assumptions of Lemma 2 are satisfied. Thus the firm with the higher quality will set a higher net quality and thus garner a larger share of demand, even though it charges the higher price.[40]

We now find the equilibrium prices conditional on consumers knowing product specifications and qualities of both products (Perloff-Salop, uniform distribution, with asymmetric qualities, effectively). Since $\widetilde{D}_0(\Delta) = \frac{(1-\Delta)^2}{2}$, the first order condition for Firm 0 yields

$$p_0 = \frac{(1-\Delta)}{2};$$

we can immediately substitute in for $\Delta = Q + p_0 - p_1$ to yield a linear reaction function

$$p_0 = \frac{1 - Q + p_1}{3}.$$

For Firm 1, we have the first order condition $1 - \frac{(1-\Delta)^2}{2} - p_1(1-\Delta) = 0$.

Substituting in $p_0 = \frac{(1-\Delta)}{2}$, then $1 - 2p_0^2 - 2p_0 p_1 = 0$.

price so that the consumer least enamoured of it (holding $r_1 = 0$) nonetheless buys against an expected value of 0 with Firm 0.

[39] Demand is convex for $\Delta < 0$ (high prices for Firm 1) and concave for $\Delta > 0$ (low prices). The demand derivative is continuous on its support, and so there is no kink.

[40] Any best reply price for Firm 1 must satisfy $p_1 \in [0, p_0 + q_1 - q_0 - 1]$, where the upper bound is where Firm 1's demand disappears. Hence Firm 1's profit is a continuous function that is defined over a compact set, and so has a maximum. Equilibrium existence follows from Caplin and Nalebuff (1991).

Solving out these equations for prices then gives the solutions as

$$p_0 = \frac{\hat{\omega} + \sqrt{8 + \hat{\omega}^2}}{8}, \qquad \text{and} \qquad p_1 = \frac{-5\hat{\omega} + 3\sqrt{8 + \hat{\omega}^2}}{8},$$

where $\hat{\omega} = 1 - Q$. Note that these prices are equal at $\hat{\omega} = 1$ (symmetry) to one half. They also verify $\Delta > 0$, as desired.