Efficiency and surplus bounds in Cournot competition

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Abstract

We derive bounds on the ratios of deadweight loss and consumer surplus to producer surplus under Cournot competition. To do so, we introduce a parameterization of the degree of curvature of market demand using the parallel concepts of ρ-concavity and ρ-convexity. The "more concave" is demand, the larger the share of producer surplus in overall surplus, the smaller is consumer surplus relative to producer surplus, and the lower the ratio of deadweight loss to producer surplus. Deadweight loss over total potential surplus is at first increasing with demand concavity, then eventually decreasing.

Keywords: Cournot equilibrium, social surplus analysis, deadweight loss, market performance.

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1 Introduction

Imperfect competition distorts market allocations by raising the equilibrium price above marginal cost. The size of the distortion depends upon the industry demand curve and the number of competing firms. We quantify this distortion according to various surplus benchmarks, as a function of the number of competitors and the curvature of the demand curve for Cournot interaction. We show that the fraction of potential (first-best) social surplus captured by producers increases as demand becomes more concave. We also provide bounds on consumer surplus and deadweight loss which depend on (potentially) observable magnitudes, such as producer surplus. These bounds depend on two parameters that measure the generalized concavity and convexity of demand.

The paper complements three bodies of literature on imperfect competition. The first addresses market performance under imperfect competition, and traces its lineage back through Mankiw and Whinston (1986), through Spence (1976) and Dixit and Stiglitz (1977), and ultimately to Chamberlin (1933). The emphasis has been on the long-run equilibrium, with the number of firms used to measure market performance, but there has been no attempt to quantify deadweight loss. By contrast, our work is a short-run analysis, with the number of firms fixed. We consider the size of the various surpluses reaped (producer surplus and consumer surplus) and unreaped (deadweight loss) in the market.

The second literature concerns estimation of welfare loss due to market power, and goes back to Harberger's (1954) provocative study that estimated monopoly deadweight loss as 0.1% of GNP. This famous study of distortionary "triangles" has been criticized in several respects, including the use of the profit data, the assumptions of linear demand and unit elasticity of demand for all industries. Subsequent studies (also criticized heavily) have used profit and cost data differently, and typically have assumed linear demand or a constant elasticity. Cowling and Mueller (1978) have suggested that welfare loss could be up to 14% of GNP. We do not further investigate the use of profit data, but we do specify a consistent theoretical model that starts with the equilibrium oligopoly pricing condition and uses it to derive bounds on deadweight loss that depend on the curvature of demand.

The third complementary body of literature uses extended concavity concepts to establish equilibrium existence and uniqueness in the Cournot model. This literature goes back through Novshek (1985) to McManus (1964). Most recently, Deneckere and Kovenock (1999) have synthesized previous results and recast them in terms of demand properties.

The present analysis uses the concept of ρ-concavity that was introduced into economics by Caplin and Nalebuff (1991a) and applied to (Bertrand) oligopoly in Caplin and Nalebuff (1991b). The larger is ρ, the "more concave" the demand function. To obtain a tighter characterization of demand curvature we also use the parallel concept of ρ-convexity whereby the lower ρ the "more convex" is demand.

Section 2 presents a general background to ρ-concavity and ρ-convexity and delivers relations between functions and their inverses. Section 3 constitutes the core of the paper. For n firms in a Cournot oligopoly and an observed equilibrium price and quantity, we first determine bounds on the actual demand curve given that it must lie between two curvature bounds. These bounds on the demand function then determine the bounds on several surplus measures, such as consumer surplus, deadweight loss, and the fraction of producer surplus in the total potential surplus (perfectly competitive benchmark). Ratio forms (and often tighter bounds) are given for the symmetric cost case, and intuition is then provided for ρ-linear demands. Section 4 concludes with comments on the welfare costs of excessive entry.

2 Demand curvature

The degree of concavity of a function can be parameterized using the concept of ρ-concavity as explained and applied in Caplin and Nalebuff (1991a and b). We also use the parallel concept of ρ-convexity to parameterize the degree of convexity of a function. We show that any demand function is both ρ'-concave and ρ''-convex.

Definition 1 *Consider a strictly positive function $\tilde{D}$ with a convex domain $B \subseteq \Re_+$.*

For $\rho \neq 0$, $\tilde{D}$ is ρ-concave if, for all $p_0, p_1 \in B$,

$$\tilde{D}(p_\lambda) \geq \left[(1-\lambda)\tilde{D}(p_0)^\rho + \lambda\tilde{D}(p_1)^\rho\right]^{1/\rho}, \qquad 0 \leq \lambda \leq 1, \tag{1}$$

where $p_\lambda = (1-\lambda)p_0 + \lambda p_1$. For $\rho = 0$, $\tilde{D}$ is 0-concave if

$$\ln \tilde{D}(p_\lambda) \geq (1-\lambda)\ln \tilde{D}(p_0) + \lambda \ln \tilde{D}(p_1), \qquad 0 \leq \lambda \leq 1. \tag{2}$$

A ρ-convex function is defined analogously by reversing the inequalities in (1) and (2).

The larger is ρ, the more stringent the concavity restriction: if $\tilde{D}$ is ρ-concave, it is also ρ'-concave for all $\rho' < \rho$. We refer to one function as "more concave" than another when its ρ value is higher. For ρ-convex functions, the smaller is ρ, the more stringent the convexity restriction. Hence if $\tilde{D}$ is ρ-convex, it is also ρ''-convex for all $\rho'' > \rho$. A logconvex function ($\rho = 0$) is also a convex function ($\rho = 1$), which in turn is also quasiconvex ($\rho = \infty$).

Claim 1 *Consider a strictly positive and decreasing function, D with a convex domain $B \subseteq \Re_+$. There is a pair of values in the extended real line, ρ' and ρ'', such that D is ρ'-concave and ρ''-convex. If D is ρ'-concave and ρ''-convex, then $\rho' \leq \rho''$.*

Proof. The first part follows since decreasing functions are both quasiconvex and quasiconcave. Second, suppose instead that $\rho' > \rho''$. Then D is ρ''-concave and ρ'-convex. But then (for $\rho' \neq 0$ and $\rho'' \neq 0$) $D^{\rho''}$ and $D^{\rho'}$ are linear, which is clearly impossible. A similar argument applies if either ρ' or ρ'' is zero. ■

The ρ-concavity properties of D also imply restrictions on its inverse.[1]

Proposition 1 *Let D be strictly positive and decreasing on its (convex) domain, B. Let P be the inverse of D, with P defined over A which is the range of D. Assume both D and P are twice continuously differentiable. Then*
$-\frac{P''(Q)Q}{P'(Q)} \leq (1-\rho)$ *iff* $[1-\rho](D')^2 - D''D \geq 0$ *iff D is ρ-concave.*

Proof. If D is ρ-concave, then D^ρ/ρ is concave ($\ln D$ for $\rho = 0$). Then $\frac{D'}{D}D^\rho$ is decreasing, or

$$(\rho - 1)(D')^2 + D''D \leq 0 \tag{3}$$

Now, set $D(p) = Q$, so that $D'(p) = 1/P'(Q)$ and $D''(p) = -P''(Q)/[P'(Q)]^3$. Replacing these expressions in condition (3) gives the condition $P''(Q)Q + (1-\rho)P'(Q) \leq 0$. ■

We explore the implications of this result in the context of Cournot competition in the next section (and we justify the notation D and P for the functions at that point).[2]

[1] The analogous statment relating $P(Q)$ and $D(p)$ is $-\frac{D''(p)p}{D'(p)} \leq (1-\rho)$ iff $[1-\rho](P')^2 - P''P \geq 0$ iff P is ρ-concave. Corresponding statments for ρ-convexity are also readily written.

[2] The condition on P concerns the slope elasticity and is analogous to measures of risk aversion.

3 Cournot equilibrium

Let there be n firms producing a homogeneous product. Let demand be given by $D(p)$, where D is a strictly decreasing and twice continuously differentiable function on $[0, \overline{p}]$, and is zero on $[\overline{p}, \infty)$. Further suppose that $D' < 0$ on $[0, \overline{p}]$. Hence inverse demand, $P(Q)$, is twice continuously differentiable on $[0, D(0)]$, where Q is total output.

Let Firm i's marginal cost be constant at c_i ($< \overline{p} = P(0)$) per unit, label firms so that $c_1 \leq c_2 \leq ... \leq c_n$, and assume that all firms are active in equilibrium ($c_n < p^c$ in equilibrium suffices, where p^c is the Cournot price). The individual firm's profit function is $\pi_i = [P(Q) - c_i]\, q_i$, where q_i is the individual firm's output, $i = 1, ..., n$. Below we relate the direct demand curve to the inverse one to focus on the relevant ρ-curvature properties, but for now we continue in the standard manner. The standard first-order conditions are

$$P'(Q)q_i + P(Q) = c_i \qquad i = 1, ..., n. \tag{4}$$

Summing up these conditions yields:

$$P'(Q)Q + nP(Q) = n\overline{c}, \tag{5}$$

where $\overline{c} = \frac{1}{n}\sum_{i=1}^{n} c_i$ is mean unit production cost. It is readily shown that the condition:

$$P''(Q)Q + 2P'(Q) \leq 0 \tag{6}$$

ensures both existence (since the profit functions are then concave) and uniqueness (since the LHS of (5) then slopes down for $n > 1$[3]) of equilibrium.[4] Deneckere and Kovenock (1999, Theorem 1) give this condition (with a strict inequality) as "the Cournot equilibrium existence result with the least restrictive conditions on demand known to us." From Proposition 1 we have the counterpart condition on direct demand (see also Deneckere and Kovenock, 1999): (-1)*-concavity of D ensures the existence and uniqueness of a Cournot equilibrium.*

We now follow through with the D-version of Cournot pricing. From (5), the equilibrium is $P'(Q)Q + n[P(Q) - \overline{c}] = 0$, or, in terms of the direct demand function,

$$n(p^c - \overline{c}) = \frac{-D(p^c)}{D'(p^c)} \tag{7}$$

[3]For monopoly, a strict inequality in (6) guarantees uniqueness. From Proposition 1, the inequality is strict as long as D is ρ-concave for some $\rho > -1$, no matter how close, which is what we assume in the bounds analysis below.

[4]The referee noted that the weaker condition of $-(1/n)$-concavity of $P(Q) - c$ ensures existence and uniqueness.

This version of Cournot pricing is important below. We restrict ourselves to $\rho' > -1$, and the surplus bounds depend on the values of ρ' and ρ'' that bound demand curvature. For what follows, let $PS = \sum_{i=1}^{n}[p^c - c_i]q_i$ denote producer surplus at the Cournot equilibrium, and CS denote consumer surplus. It is helpful to use a benchmark of "mean-cost" industry profit $MPS = (p^c - \overline{c})\, Q$ which is the profit that would be earned in the industry if the same total output, Q, were produced, and each firm had the same (mean) cost, $\overline{c}$.

Lemma 1 *Consider a Cournot oligopoly with n firms producing at constant (but different) marginal cost. Then $MPS \leq PS$. This holds with equality when marginal costs are equal.*

Proof. Letting $\overline{q} = Q/n$ be average output, (4) and (5) imply

$$q_i > \overline{q} \Leftrightarrow c_i < \overline{c}: \tag{8}$$

a firm produces above average output if and only if its cost is below the mean. We need to show that $MPS = D(p^c)[p^c - \overline{c}] \leq \sum_{i=1}^{n}[p^c - c_i]q_i = PS$, or $\sum_{i=1}^{n} q_i[\overline{c} - c_i] \geq 0$. Subtracting $\sum_{i=1}^{n} \overline{q}[\overline{c} - c_i]$ $(= 0)$ from the L.H.S. of the last inequality yields $\sum_{i=1}^{n}[q_i - \overline{q}][\overline{c} - c_i] \geq 0$, which is necessarily true by property (8). ■

We use MPS extensively below. An alternative interpretation of MPS comes from noting that $MPS = \frac{-TIR}{n\eta}$, where TIR is total industry revenue (p^cQ) and η is the price elasticity of demand. This derives from writing (7) as the Lerner rule, $\frac{p^c - \overline{c}}{p^c} = -\frac{1}{\eta}\frac{1}{n}$.

4 Surplus bounds

We now derive bounds on consumer surplus and deadweight loss. We first prove a key proposition that restricts where the demand function may lie if we know the Cournot equilibrium price and quantity and the bounds on demand curvature $\rho'' \geq \rho'$.

Proposition 2 *Let D be ρ'-concave and ρ''-convex, with $\rho'' \geq \rho'$. Then*

$$D(p^c)\left[1 + \frac{\rho''}{n}\frac{(p^c - p)}{(p^c - \overline{c})}\right]^{1/\rho''} \leq D(p) \leq D(p^c)\left[1 + \frac{\rho'}{n}\frac{(p^c - p)}{(p^c - \overline{c})}\right]^{1/\rho'}$$

if neither ρ' nor ρ'' is zero. If one is zero, the appropriate bound is $D(p^c)\exp\left[\frac{1}{n}\frac{(p^c - p)}{(p^c - \overline{c})}\right]$.

Proof. Suppose that D is ρ'-concave, with $\rho' > 0$ so that

$$D^{\rho'}(p) \leq D^{\rho'}(p^c) + \rho' D^{\rho'}(p^c)\frac{D'(p^c)}{D(p^c)}[p - p^c] \tag{9}$$

which says simply that a concave function lies below its tangent (at p^c), where p^c is the Cournot equilibrium price. Substituting in from the oligopoly equilibrium condition (7) and raising both sides to the power $1/\rho'$ yields:

$$D(p) \leq D(p^c)\left[1 + \frac{\rho'}{n}\frac{(p^c - p)}{(p^c - \overline{c})}\right]^{1/\rho'}. \tag{10}$$

Notice that the same expression applies for $\rho' < 0$ (since the inequality in (9) is reversed but then raising both sides to the power $1/\rho' < 0$ then again reverses the inequality). The case $\rho' = 0$ is attained by taking the appropriate limit of the right-hand side of (10) to give

$$D(p) \leq D(p^c)\exp\left[\frac{1}{n}\frac{(p^c - p)}{(p^c - \overline{c})}\right]. \tag{11}$$

The lower bounds follow from similar arguments using ρ'' with the inequalities reversed. ■

The proposition first uses the restriction that the demand function must lie between two $\rho-$linear functions. Given an equilibrium price and industry output, there is an infinite set of $\rho-$linear functions that go through this point and that could be used to bound demand. The oligopoly first-order condition ties down the bounding $\rho-$linear demand function as the tangent to demand at the equilibrium point.

Proposition 2 provides bounds on output restriction due to imperfect competition. Under symmetry $(c_i = \overline{c})$, the ratio of competitive to Cournot output is $D(\overline{c})/D(p^c)$ and is bounded above by $\left[1 + \frac{\rho'}{n}\right]^{1/\rho'}$ (or $\exp\frac{1}{n}$ when $\rho = 0$). For monopoly, output is cut back by at most one half for concave demand, and $1 - \frac{1}{e}$ for a logconcave demand. Under oligopoly, the numbers are $\frac{1}{n+1}$ and $1 - e^{-1/n}$. We now turn to surplus analysis.

Proposition 3 *Let D be ρ'-concave and ρ''-convex, with $\rho'' \geq \rho' > -1$. Then*

$$MPS\frac{n}{\rho'' + 1} \leq CS \leq MPS\frac{n}{\rho' + 1}.$$

Proof. Consumer surplus is $CS = \int_{p^c}^{\infty} D(p)dp$. When $\rho' \neq 0$, the upper bound in Proposition 2 is $D(p) \leq D(p^c)\left[1 + \frac{\rho'}{n}\frac{(p^c-p)}{(p^c-\overline{c})}\right]^{1/\rho'}$. For $\rho' > 0$, the expression on the right has an intercept, α, that satisfies $1 + \frac{\rho'}{n}\frac{(p^c-\alpha)}{(p^c-\bar{c})} = 0$. For $\rho' < 0$, this goes to zero as p goes to infinity, and define α as infinity in this case. Hence $CS \leq \int_{p^c}^{\alpha} D(p^c)\left[1 + \frac{\rho'}{n}\frac{(p^c-p)}{(p^c-\bar{c})}\right]^{1/\rho'} dp$. The assumption that $\rho' > -1$ ensures that this integral is well-defined. Thus

$$CS \leq \left[-\frac{nD(p^c)(p^c-\bar{c})}{1+\rho'}\left(1 + \frac{\rho'}{n}\frac{(p^c-p)}{(p^c-\bar{c})}\right)^{1+1/\rho'}\right]_{p^c}^{\alpha}$$

For $\rho' > 0$, the anti-derivative term is zero at $p = \alpha$ by definition, while for $\rho' \in (-1, 0)$, the anti-derivative term goes to zero as p goes to infinity. Hence we have $CS \leq MPS\frac{n}{1+\rho'}$. In a similar fashion, for $\rho' = 0$ we have $CS \leq \int_{p^c}^{\infty} D(p^c)\exp\left[\frac{1}{n}\frac{p^c-p}{p^c-\bar{c}}\right] dp = nMPS$. Analogous arguments with reversed inequalities yield the lower bound. ■

From Lemma 1, we can write a looser upper bound as $CS \leq \frac{nPS}{\rho'+1}$; and hence we can determine a bound on the distribution of surplus as $\frac{CS}{CS+PS} \leq \frac{n}{n+\rho'+1}$. For symmetric costs, we can also find an analogous lower bound:

Corollary 1 *Let costs be symmetric and D be ρ'-concave and ρ''-convex, with $\rho'' \geq \rho' > -1$. Then*[5]

$$\frac{n}{n+\rho''+1} \leq \frac{CS}{CS+PS} \leq \frac{n}{n+\rho'+1}.$$

The bound expression is an decreasing function of the concavity-convexity index ρ, so that the consumer share in social surplus is smaller for more concave demand. The intuition is best captured by looking at ρ-linear demands. A useful way to parameterize ρ-linearity is $D(p) = K[1+\rho(a-bp)]^{1/\rho}$, for $p \in [0, \bar{p}]$, while $D(p) = 0$ for $p \geq \bar{p}$, where $\bar{p} = \frac{1}{\rho b}(1+\rho a)$ for $\rho > 0$ and $\bar{p} = \infty$ otherwise. We impose $K > 0, b > 0, a > 0$, and $1 + \rho a \geq 0$ for D to be a demand function. These conditions ensure that demand is positive and strictly decreasing on $[0, \bar{p}]$, and that $\bar{p} > 0$ for $\rho > 0$. Keeping K, a, and b constant, we can generate a set of ρ-linear functions for $\rho \in [\frac{-1}{a}, \infty)$. All demand curves pass through the price-quantity pair $(\frac{a}{b}, K)$. At this point, the elasticity of demand is $-a$ for any ρ. For any given n and c, this means that we can set $a = bc + \frac{1}{n}$ so that the equilibrium price is always $\frac{a}{b}$ independently of the value of ρ.[6] Equilibrium quantity, K, and producer surplus, $K\left(\frac{a}{b} - c\right)$, are then also independent of demand curvature.

[5]Under cost symmetry, with a concave demand, $\rho' = 1$ and consumer surplus is at most $n/2$ of producer surplus. It reaches this upper bound for a linear function, $\rho' = \rho'' = 1$.

[6]This follows from writing (7) as $\frac{p^c-c}{p^c} = \frac{-1}{\eta}\frac{1}{n}$ and substituting in the parameter values given.

Since producer surplus is tied down, its share in social surplus depends on how consumer surplus varies with ρ. With price held fixed at $\frac{a}{b}$, consumer surplus depends upon how demand changes with ρ for prices above $\frac{a}{b}$. Differentiating the log of the parameterized demand function with respect to ρ gives $\frac{1}{\rho^2}\left[\frac{x}{1+x} - \ln(1+x)\right]$ where $x = \rho(a - bp)$. This expression is zero when $x = 0$, increasing for negative values of x, and decreasing for positive values. It is therefore always negative. This means that consumer surplus falls as demand becomes more concave, and so the share of producer surplus in total surplus rises. Intuitively, think of a demand curve that bows in more when demand is more concave.

Hence with ρ-linear functions, a more concave function (larger ρ) delivers a lower ratio of consumer to producer surplus. The argument extends to functions that are not ρ-linear: consider a ρ-convex demand function, and compare to another demand function sufficiently more concave that it is ρ-concave. Then CS/PS is smaller for the more concave one. In this sense, CS/PS is lower the more concave the demand function, and the argument holds because the bounds decrease with ρ. With this justification we henceforth analyze comparative static properties by considering the bound expressions.

Our measure of deadweight loss uses the cost of the most efficient firm (see also Daskin, 1991). At the optimum, this firm serves the whole market at unit cost, c_1.

Proposition 4 *Let D be ρ'-concave and ρ''-convex, with $\rho'' \geq \rho' > -1$. Then*

$$\left[\left(1+\rho''\frac{q_1}{Q}\right)^{1+\frac{1}{\rho''}} - 1\right]\frac{n}{\rho''+1}MPS - PS \leq DWL \leq \left[\left(1+\rho'\frac{q_1}{Q}\right)^{1+\frac{1}{\rho'}} - 1\right]\frac{n}{\rho'+1}MPS - PS$$

if neither ρ' nor ρ'' is zero. If one is zero, the appropriate bound is $\left[ne^{q_1/Q} - n\right]MPS - PS$.

Proof. Deadweight loss at a Cournot equilibrium is $DWL = \int_{c_1}^{p^c} D(p)dp - PS$. For $\rho' \neq 0$, Proposition 2 implies

$$DWL \leq \int_{c_1}^{p^c} D(p^c)\left[1+\frac{\rho'}{n}\frac{(p^c-p)}{(p^c-\overline{c})}\right]^{1/\rho'} dp - PS.$$

Evaluating the expression on the RHS gives the desired upper bound after noting that $nD(p^c)(p^c - \overline{c}) = MPS$ and $\frac{p^c-c_1}{n(p^c-\overline{c})} = \frac{q_1}{Q}$. A similar argument holds for $\rho' = 0$ using $DWL \leq \int_{c_1}^{p^c} D(p^c)\exp\left[\frac{1}{n}\frac{p^c-p}{p^c-\overline{c}}\right]dp - PS$. The lower bounds follow from analogous arguments with the inequalities reversed. ■

From Lemma 1, an upper bound follows as $\frac{DWL}{PS} \leq \left[(1+\rho')^{1+\frac{1}{\rho'}} - 1 \right] \frac{n}{\rho'+1} - 1$. Under cost symmetry, we can write the upper bound as $\frac{n+\rho'}{\rho'+1} \left(1+\frac{\rho'}{n}\right)^{\frac{1}{\rho'}} - \frac{n}{\rho'+1} - 1$ with the lower bound given by an analogous expression evaluated at ρ''. These bounds are decreasing in ρ. To see this note that for any two values ρ' and ρ'' such that $\rho' < \rho''$, there exists a demand function which is both ρ'-concave and ρ''-convex. (For example, a ρ-linear decreasing function with $\rho \in (\rho', \rho'')$). The bounds imply that the bound expression evaluated at ρ'' must be less that the bound expression evaluated at ρ'.

Our next result combines the findings above for symmetric costs. Let $TS = DWL + CS + PS$ denote total potential surplus available in the market. For ease of comparison, we present the results in terms of TS/PS, bearing in mind that we are interested in the inverse of this ratio (which indicates how much producers are able to extract of the total gains available). More producers imply a lower price and producer surplus, so that TS/PS increases with n, which is corroborated by the bounds below.

Proposition 5 *Let D be ρ'-concave and ρ''-convex, with $\rho'' \geq \rho' > -1$; let costs be equal. Then*

$$\frac{n+\rho''}{\rho''+1}\left(1+\frac{\rho''}{n}\right)^{\frac{1}{\rho''}} \leq \frac{TS}{PS} \leq \frac{n+\rho'}{\rho'+1}\left(1+\frac{\rho'}{n}\right)^{\frac{1}{\rho'}}$$

if neither ρ' nor ρ'' is zero. If one is zero, the appropriate bound is $ne^{1/n}$.

Since each bound is the sum of those following Propositions 3 and 4, they are decreasing in ρ. The intuition again follows from the ρ-linear parameterization after Corollary 1. Increasing ρ tightens the demand curve around its anchor price, and in the limit as ρ goes to infinity, it becomes a rectangular (step) demand where consumers inelastically buy K units up to a price $\frac{a}{b}$. This illustration underlies the fact that the limit of the upper bound in the proposition as ρ' goes to infinity is 1, and producers extract the full potential surplus. As was the case for the share of producer surplus in social surplus, the fraction of the first-best total surplus captured by producers is larger if demand is more concave.

We now study how market efficiency is affected by demand curvature. In line with standard welfare analysis, we relate deadweight loss to the total potential surplus that may be generated by the market. Equivalently we consider the ratio of the total potential surplus to the social surplus generated by the market equilibrium. For the symmetric case, using Corollary 1 and Proposition 5 we have

$$\frac{(n+\rho'')(\rho'+1)}{(\rho''+1)(n+\rho'+1)}\left(1+\frac{\rho''}{n}\right)^{\frac{1}{\rho''}} \leq \frac{TS}{PS+CS} \leq \frac{(n+\rho')(\rho''+1)}{(\rho'+1)(n+\rho''+1)}\left(1+\frac{\rho'}{n}\right)^{\frac{1}{\rho'}} \tag{12}$$

Further insight can be gained by considering ρ-linear functions for which $\rho''=\rho'=\rho$. We can then determine the impact of changing demand curvature on the relative deadweight loss, $\frac{DWL}{TS}$ (the terminology follows Tirole, 1988). Clearly relative deadweight loss moves the same way as $\frac{TS}{PS+CS}$. For the special case of isoelastic demands (see Tirole, 1988, Exercise 1.4), it can be readily shown that the more elastic the demand, the larger the relative deadweight loss under monopoly (given an isoelastic demand). In our setting, this translates to relative deadweight loss increasing for $\rho \in (-1,0)$. However, the deadweight loss disappears as we approach the limit of rectangular demand of our earlier parameterization of ρ-linear demand, suggesting that an increase in ρ necessarily decreases relative deadweight loss for large values of ρ. The next proposition clarifies how relative deadweight loss depends on ρ.

Proposition 6 *Let D be ρ-linear and costs be symmetric. Then* $\frac{TS}{PS+CS}=\frac{n+\rho}{(n+\rho+1)}\left[1+\frac{\rho}{n}\right]^{\frac{1}{\rho}}$ *is quasiconcave in ρ, increasing for $\rho \in (-1,0)$ and decreasing for ρ large enough.*

Proof. First note that $\ln\left[\frac{TS}{PS+CS}\right] \equiv S(\rho) = \ln(n+\rho) - \ln(n+\rho+1) + \frac{1}{\rho}\ln\left[1+\frac{\rho}{n}\right]$. Hence $S'(\rho) = \frac{1}{\rho^2}\frac{1}{(n+\rho)(n+\rho+1)}\left\{\rho(n+2\rho+1) - (n+\rho)(n+\rho+1)\ln\left[1+\frac{\rho}{n}\right]\right\}$.

Except for possibly at $\rho = 0$, this expression has the sign of the term in curly brackets (since $\rho > -1$ and $n \geq 1$), so define this term as $T(\rho)$,

$$T(\rho) = \rho(n+2\rho+1) - (n+\rho)(n+\rho+1)\ln\left(1+\frac{\rho}{n}\right), \tag{13}$$

which is a continuous function of ρ. S is increasing when T is positive, and decreasing when T is negative. We show that T is first positive and then negative, so that S (and therefore $TS/(PS+CS)$) is quasiconcave. The rest of the proof uses three steps:

(i) T is negative for $\rho \geq (e^2-1)n$;

(ii) for $n \geq 2$, T has a local minimum at $\rho = 0$, at which point T is zero (the case $n = 1$ is treated at the end).

(iii) the second derivative of T is decreasing in ρ for $n \geq 2$.

Coupled with (ii), (iii) proves that T must be positive for $\rho < 0$: if it were negative at some $\rho < 0$ then it would have to be concave at some point in order to later have a local minimum at $\rho = 0$, but this contradicts (iii). Finally, from (i), T is negative for ρ large enough, but, from (ii) it has a local minimum at $\rho = 0$. To become negative, it must turn from convex to concave, but by (iii) it cannot become convex again after it has become negative, and so there is a unique value of $\rho > 0$ for which T crosses the line $T = 0$.

For $n = 1$, $T'''(\rho)$ has the sign of $-1-2\rho$, so that T'' is increasing for $\rho \in (-1, -\frac{1}{2})$ and it is decreasing for $\rho > -\frac{1}{2}$. Since $\lim_{\rho \to -1} T(\rho) = 0$, $\lim_{\rho \to -1} T'(\rho) = \infty$, and $\lim_{\rho \to -1} T''(\rho) = \infty$, $T(\rho)$ is positive, increasing, and concave at first: it then becomes convex before falling to 0 at $\rho = 0$, whereafter it is concave and so falling since this is an inflection point. It is thus positive for $\rho \in (-1, 0)$ and negative for $\rho > 0$. ■

The intuition follows our earlier parameterization of ρ-linear demand, whereby we hold producer surplus fixed as we increase ρ. For ρ negative, consumer surplus is very large relative to deadweight loss (it tends to infinity as ρ goes to -1), and bowing in the demand function reduces consumer surplus more than it reduces deadweight loss. This increases relative deadweight loss. For large enough ρ, consumer surplus and deadweight loss become more similar in size and total surplus consists mostly of producer surplus. Because PS remains unchanged, the joint reduction in consumer surplus and deadweight loss leads to a drop in relative deadweight loss.

5 Conclusions

We have presented a set of surplus bounds for Cournot competition. Different surpluses are important in different contexts. To measure monopoly deadweight loss (the harm inflicted by market power), our results on deadweight loss bounds as a fraction of industry profits mean that losses can be inferred from observation of industry profits and tight demand estimates.

Whether a monopoly firm enters a market depends on its profit, but the socially optimal entry decision depends on total surplus generated. When demand is very concave (ρ' high), the firm's incentives are aligned with the optimum and entry is close to optimal. For a very convex demand (ρ'' low) much of the surplus generated remains uncaptured and entry decisions may be far from optimal.

These surplus comparisons are also important under oligopoly. A firm enters the market if it earns a positive profit. The optimal decision depends on the incremental total surplus. An extension of the present research is to quantify the severity in welfare terms of the overentry problem identified by Mankiw and Whinston (1986). Does it become more or less severe as ρ' increases or ρ'' decreases?[7] When ρ' is large (demand is very concave), firms capture almost all of the total surplus. An extra firm will not reduce price much and so its social value is small. Nevertheless, it may still earn substantial profit by simply attracting customers from rival firms (the business stealing effect). This suggests that overentry may indeed be a serious problem for ρ' large, even though there is little deadweight loss for a fixed number of firms, so care is needed in interpreting our welfare results.

Curvature properties are important elsewhere in economics. Two examples are cost functions, for which curvature measures returns to scale, and utility functions under risk where curvature measures risk aversion.

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