# Shouting to be heard in advertising

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**Abstract**

Advertising competes for consumer attention, but attention is scarce. More profitable senders send more messages to break through the clutter. There may be multiple equilibria: more messages in aggregate induce more "shouting to be heard" among senders trying to break through the advertising clutter, which creates a "lottery ticket" dimension for profit dissipation. Equilibria can involve a small range of loud shouters or large range of quiet senders. All senders prefer there to be less "shouting." Increasing the cost of sending messages can make all senders better off. A new technique is given for describing multiple equilibria, by determining how much examination is consistent with given marginal sender.

JEL CLASSIFICATION: D11, D60, L13.

KEYWORDS: information overload, congestion, advertising, common property resource, overfishing, lottery, junk mail, e-mail, telemarketing, multiple equilibria.

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# 1 Introduction: more than one message?

The volume of conversation at a cocktail party is low when people are talking normally, but high when everyone has to shout to be heard over the din caused by the others trying to do the same thing. A parallel can be drawn with the volume of advertising messages trying to reach through to consumers: firms must send many ads to break through into the consumer's consideration set. Indeed, observed advertising levels can take various intensities, which can lead to wasteful duplication when they are high.[1]

Consumers are increasingly bombarded with advertisements enticing them to buy wares. Advertising messages are embedded in radio programs during the morning commute, in the newspaper read in the coffee break, as spam in email, and as pop-up ads while net-surfing. When driving home, they are present in billboards and neon signs; bulk mail and telemarketing calls await at home, and ads finance commercial television programs watched in the evening. Consumers can scarcely process all the incoming information, and typically screen out a lot of the messages projected at them. Advertisers need their messages to break through the advertising clutter of others' messages in order to attract attention and eventually get to a sale. In such a world, advertisers are more likely to make a connection if they send several messages. This paper is about multiple equilibria in information overflow, emphasizing duplication and repetition, the possibility of higher volume of messages with fewer senders, and possible gains from raising the cost of sending messages.

> Well, it's a non-stop blitz of advertising messages. Everywhere we turn we're saturated with advertising messages trying to get our attention. We've gone from being exposed to about 500 ads a day back in the 1970's to as many as 5,000 a day today. We have to screen it out because we simply can't absorb that much information. We can't process that much data and so no surprise, consumers are reacting negatively to the kind of marketing blitz; the kind of super saturation of advertising that they're exposed to on a daily basis. All of this marketing saturation that's going on is creating this kind of arms race between marketers where they have to up the ante the next time out because their competitors have upped the ante the last time they were out. And the only way you can win is to have more saturation.[2]

[1] See Terui, Ban, and Allenby (2010) for recent work on consideration sets in marketing.

[2] President Jay Walker-Smith of the Marketing Firm Yankelovich (Excerpted from Cutting Through Advertising Clutter, CBS News.) http://www.cbsnews.com/stories/2006/09/17/sunday/main2015684.shtml

Once it is recognized that firms can send multiple messages brings up the possibility that there is message overcrowding in both depth (messages per sender) and width (number of senders). The depth dimension provides a further perspective on the common property resource problem of free access to consumer attention analyzed by Van Zandt (2004), and by Anderson and de Palma (2009). These authors assumed that each sender can send at most one message to the receiver. The marketing maxim contends that several hits per consumer (estimates vary from 3 to 16) are needed to get a reasonable chance of a sale. This maxim is consistent with the idea that a message needs to break through the clutter into the individual's attention span, as per our model. It is also consistent with the idea that messages build on each other to reach a critical mass to be recognized, or indeed that more messages build upon each other to persuade more gradually.

Most of the economics literature on advertising looks at performance within an industry (see Bagwell, 2007, for a fine review). Butters' (1977) model of informative advertising and price dispersion assumes that senders within the same industry send messages announcing the (availability and) price of a good. Messages are sent randomly to consumers, so each consumer will get a different profile of messages. In this context, each sender sends a single message, which is a zero-profit activity under the assumption of free-entry of senders. In Grossman and Shapiro (1984), senders form an oligopoly (also within the same industry), and each firm sends multiple messages, again randomly distributed over receivers, in order to reach a desired customer base. All messages are assumed to be read by the recipient, and senders are homogenous. By contrast, our approach emphasizes the advertising communication technology and looks at heterogenous senders from different industries interacting through a bottleneck of limited individual attention span. In this context, senders send multiple messages even though they know to whom they are sending their messages.[3]

The model describes a continuum of senders of messages facing a potential consumer whose attention is restricted to only process a limited number of messages (see Eppler and Mengis, 2004, for a review of the interdisciplinary literature on information overload, and the early work by Miller, 1956). Sending more messages is more costly for a sender, but increases the probability of breaking through. With a fixed consumer attention span, sending messages is like a lottery where there are multiple winners. Participants typically value a winning connection differently because they enjoy different profits. Buying more tickets

[3]Extending our model to allow for uncertainty about whether the recipient got the message (which is the reason for sending multiple messages in Grossman and Shapiro, 1984) ought not greatly alter our conclusions, and we suppress this to focus on the limited attention span motive for multiple messages.

(sending more messages) improves the odds of breakthrough, but reduces the chances for other senders. The paper proceeds by first analyzing a single sender choosing an integer number of messages, taking as given the mass of messages sent by the others. In trying to overcome congestion by duplication, senders exacerbate it. More profitable senders send more messages. We then solve the model by starting with the marginal sender just indifferent between sending one message or none at all. This sender determines an information congestion level, which then determines the critical marginal senders for all other numbers of messages (steps). Integrating up yields the aggregate number of messages sent, and hence the examination level consistent with the original congestion level and the seed value of the first marginal sender. This examination value is a continuous function whose range is the positive real line. We are then able to prove that a first marginal sender (and hence an equilibrium) exists for any value of examination cost. Multiple equilibria can exist, with a higher marginal sender corresponding to *more* messages. This underlies the property of "shouting to be heard": fewer senders shout at higher volume.

In this context, increasing the cost of sending each message might be expected to help in two dimensions. First, one might expect there to be fewer sender types (the low end drops off) and less competition (in terms of messages sent) at any given level. Instead, however, we show it is possible that raising the cost of sending messages causes more senders to send, and fewer messages in total. This cost increase actually makes all senders better off, even if the extra cost is not rebated to senders. Even highly profitable sender types that send a lot of messages see profits rise through an increase in the probability they are examined. The high types now are more "prominent" in the sense that their messages get through more clearly (less clutter from other types), and efficiency is improved.

The next section sets out the model and provides the equilibrium determination of the integer problem of how many messages are sent per sender. We then use a novel technique to construct the equilibrium in Section 3 and show there exists an equilibrium in Section 4. We characterize the multiplicity of equilibria in Section 5 and show that equilibria with more message volume involve a smaller number of senders, who are all worse off. Section 6 addresses the effects of changing the cost of transmission, and shows when it is possible to make all senders better off by raising the price of sending messages, even without rebates. Section 7 offers some conclusions.

## 2 Breaking through Advertising Clutter

A single receiver will examine a fixed number of anonymous messages, $\phi$, from the $n$ messages received, so $\phi < n$ indicates information congestion. The assumption of a constant $\phi$ is broadly consistent with some empirical evidence from the marketing literature.[4] We assume that if a message is examined, a further message from the same sender has no impact: two successful ads are no better than one. Hence effort is wasted by duplication and cannibalization, but the benefit to the individual sender from duplication is a higher communication probability.

There is a continuum of senders, with total mass $M$, and they are ranked by their profitability conditional on getting a message through to the receiver. This conditional profitability is the product of the probability the receiver is interested (will buy the product) and the sender's profit conditional on being interested, and there is no contagion across senders. A sender's rank (or type) is denoted by $\theta \in [0, 1]$, with associated expected conditional profit $\pi(\theta) > 0$.[5] Because this profit is independent of which other messages are examined, we thus assume that the receiver's purchase decisions are independent of whether she has purchased other products, and which other products. This simplifying assumption allows us to concentrate on the congestion of messages in the receiver's attention span, without worrying about direct "business stealing" across messages.

We assume $\pi(\theta)$ is continuously differentiable and strictly increasing with $\pi(0) < \gamma < \pi(1)$, where $\gamma$ is the cost to sending a message. These bounds on $\pi(.)$ ensure the market is neither unserved nor completely covered. It is straightforward to immediately deal with types which are never present and with the condition under which there is an equilibrium with no congestion ($\phi \geq n$).

**Lemma 1** *(i) Types $\theta < \pi^{-1}(\gamma)$ do not send messages in any equilibrium. (ii)There is an equilibrium with no congestion and all types $\theta \geq \pi^{-1}(\gamma)$ sending a single message if $\phi \geq \left[1 - \pi^{-1}(\gamma)\right] M$.*

**Proof:** (i) Profit of type $\theta$ is at most $\pi(\theta) - \gamma$ because its profit is greatest if its message was examined for sure. This profit is negative for $\theta < \pi^{-1}(\gamma)$.

[4] Brown and Rothschild (1993) attribute this finding to Webb and Ray (1979). They then reconsider it in the light of their own experiments, and suggest there may be a less severe congestion effect at high levels of clutter. We emphasize the attention span bottleneck by assuming $\phi$ is constant, but we conjecture the main results still hold true if $\phi/n$ were decreasing in the number of messages, $n$. Note that $\phi/n$ is measured as the ad recall rate in the marketing studies.

[5] The possibility that a consumer is not interested in the product is folded into the expected conditional product.

(ii) If all types $\theta \geq \pi^{-1}(\gamma)$ send a single message, all their messages are examined (so there is no point to sending further messages) if $\phi \geq \left[1 - \pi^{-1}(\gamma)\right] M = n$, and they all make non-negative profits. **Q.E.D.**

We now deal with congestion. Suppose that a sender transmits an integer number $\ell$ messages to the receiver. The probability that at least one of its messages is examined by the receiver is given as follows. First, since messages are anonymous (their provenance is undetermined ex ante), the probability that a given message is examined by the receiver is $\phi/n < 1$. Then, since there is a continuum of senders and a continuum of messages examined, the probability that none of the $\ell$ messages sent by the sender is examined by the receiver is $(1 - \phi/n)^{\ell}$. Hence the breakthrough probability, $\mathbb{P}(\ell, \phi/n)$, that at least one of the sender's messages is examined by the receiver is[6]

$$\mathbb{P}(\ell, \phi/n) = 1 - [1 - \phi/n]^{\ell}, \tag{1}$$

which is an increasing and concave function of the number of messages, $\ell$, sent by the sender.

With a continuum of senders, an individual sender's choice of $\ell$ has infinitesimal impact on aggregate message volume. Therefore senders consider $n$ as fixed when choosing how many messages they should send. This is akin to a monopolistic competition assumption: each sender is a small contributor to the total volume of messages sent. Even though (as we show below) the individual sender's problem has a unique solution for a given total number of messages, differences in the anticipated number of messages change senders' choices and can thence support different equilibria. The marketing maxim concerning how many messages (3 to 16) should be sent is thus specific to the context and depends on how much crowding there is from others.

# 3 Constructing the equilibrium solution

As we will see below, there are multiple equilibria. We need a technique to find them, given that any given $\phi$ may be associated to several values of the marginal active sender, $\theta_1$. However, as we show, any particular value of $\theta_1 > \pi^{-1}(\gamma)$ can be supported only by one value of $\phi$ (by Lemma 1, there can be no equilibrium with a lower $\theta_1$, and an equilibrium with $\theta_1 = \pi^{-1}(\gamma)$ can be supported by a range of values of $\phi$), and this is the key to our technique.

The solution algorithm we use constructs the solution in 6 steps. Briefly, for a given $\theta_1$, we first find the congestion level, $n/\phi$, needed to support $\theta_1$, then determine how many messages, $n$, are sent given the

[6] This is similar to Butters' (1977) classic model of letterbox advertising.

congestion level. We then back out the attention span, $\phi$, required to support such a solution.

**Step 1. Select a value of $\theta_1 \in (\pi^{-1}(\gamma), 1]$**

Here $\theta_1$ is the sender indifferent between sending one message or none. As we see below, this marginal type will be strictly worse off by sending more than one message.

**Step 2. Calculate congestion ratio**

The marginal sender type, $\theta_1$, being indifferent between sending a message or not, makes zero profit, so

$$\pi(\theta_1)\frac{\phi}{n} = \gamma, \tag{2}$$

while types $\theta < \theta_1$ do not transmit (Lemma 1). The (inverse) congestion ratio is given by (2) as: $\frac{\phi}{n} = \frac{\gamma}{\pi(\theta_1)} < 1$.

**Step 3. Calculate the number of messages sent by different senders**

Recalling that $\gamma$ is the cost of sending a message, sender $\theta$'s problem is

$$\max_{\ell=0,1,2,...} \{\pi(\theta)\mathbb{P}(\ell, \phi/n) - \gamma\ell\}, \tag{3}$$

where we take explicit account of the constraint that the number of messages must be an integer. Sender $\theta$ prefers to send $\ell$ rather than $\ell - 1$ messages if the extra benefit of doing so exceeds the extra cost, i.e., if $\pi(\theta)[\mathbb{P}(\ell, \phi/n) - \mathbb{P}(\ell - 1, \phi/n)] > \gamma$, or

$$\pi(\theta)\left(1 - \frac{\phi}{n}\right)^{\ell-1}\frac{\phi}{n} > \gamma, \qquad \ell = 0, 1, 2, ... \tag{4}$$

The L.H.S. is decreasing in $\ell$, indicating that the profit function is concave. A sender of type $\theta$ will transmit at least one message if $\pi(\theta)\frac{\phi}{n} > \gamma$.

On the L.H.S. of (4), $\left(1 - \frac{\phi}{n}\right)^{\ell-1}$ is the probability that sender $\theta$'s first $(\ell-1)$ messages are not examined, and $\frac{\phi}{n}$ (the inverse level of congestion) is the probability that the last message (the marginal $\ell$-th one) *is* examined. Since the L.H.S. is increasing in $\theta$, higher $\theta$ senders send out more ads. With decreasing returns to messages sent, there are rents to higher $\theta$ senders.[7]

The sender $\theta_j \in (0, 1)$ which is indifferent between sending $j - 1$ and $j$ messages has type $\theta_j$ satisfying

$$\pi(\theta_j) = \frac{\gamma}{\left(1 - \frac{\phi}{n}\right)^{j-1}}\frac{n}{\phi}. \tag{5}$$

[7]This property can be seen readily by applying the envelope theorem to (3).

The R.H.S. of this expression is increasing in $j$, so that $\theta_1 < \theta_2 < ...$ The number of messages sent is thus an increasing step-function with unit step size. Notice that the marginal sender type, $\theta_1$, given in (2) follows directly from (5). Substituting (2) into (5) yields an implicit expression for $\theta_j$ in terms solely of $\theta_1$:[8]

$$\pi(\theta_j) = \frac{\pi(\theta_1)}{\left(1 - \frac{\gamma}{\pi(\theta_1)}\right)^{j-1}}. \tag{6}$$

We use a linear profit function example, $\pi(\theta) = \bar{\pi}\theta$, to illustrate. Figure 1 displays the critical $\theta$'s as a function of $\theta_1$ using (6) where $\bar{\pi}/\gamma = 20$).

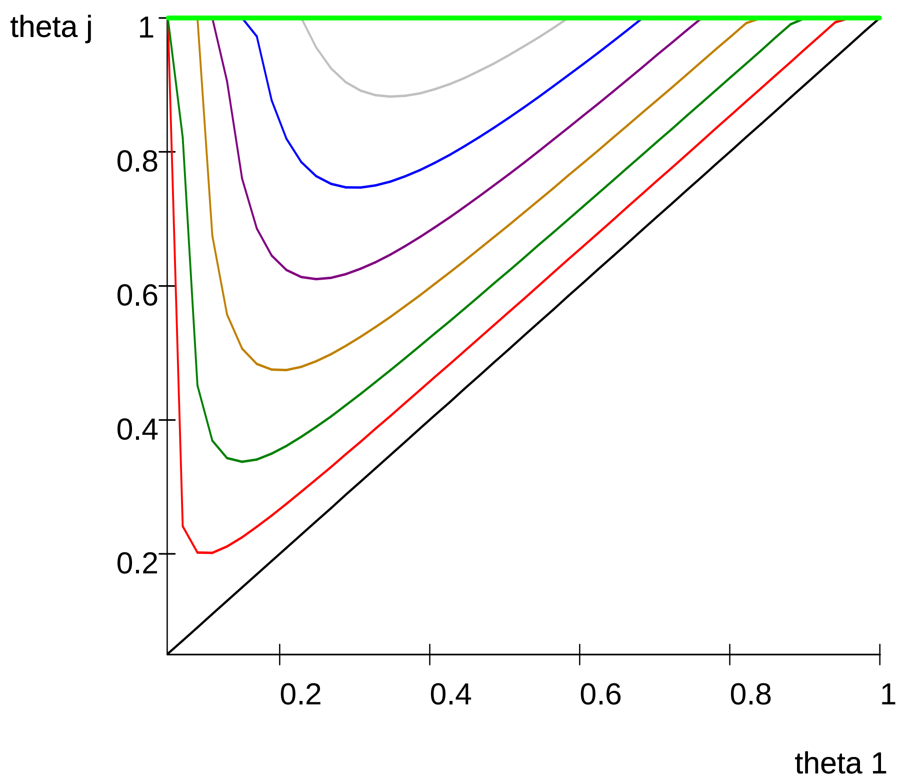


Figure 1. Critical points for steps, $\theta_j$, $j = 1, ..., 7$. Red is lowest $\theta$ for 2 steps (i.e., $\theta_2$ as a function of $\theta_1$), green for 3 steps, etc.

The black line (45 degree line) is $\theta_1$, the next line up (red) is the locus for $j = 2$, followed by $j = 3$ (dark

[8] In the sequel we will use a linear profit function example, $\pi(\theta) = \bar{\pi}\theta$, to illustrate various properties. The properties of (6) are described below.

green), etc. For example, any candidate solution with $\theta_1 = 0.4$, entails 5 or more messages being sent for $\theta$ around 0.8, and the maximum number of messages reaching 6 for all senders whose profitability exceeds (approximately) $0.9\bar{\pi}$. The $\theta_j$'s are single-troughed functions of $\theta_1$, which is a general property of these functions.

The pattern in Figure 1 suggests that only one message will be sent in equilibrium if either $\theta_1$ is low or high; intermediate cases have more disparity across senders. This we will establish below.

**Step 4. Find the maximal number of messages per sender**

The maximum number of messages sent per sender (denoted by $k$ below) is the number of messages sent by type $\theta = 1$. This is seen from Figure 1 to be first increasing and then decreasing in $\theta_1$ (from 1 to 6 and then back down to 1: track the light green line at the top of the Figure). The idea is that with $\theta_1$ small the marginal sender has a low profitability, and can only be induced to send a message if there is very little congestion, which in turn means that each other sender does not want to send many messages because there is a high chance the first one will get through. At the other extreme, if the marginal active type (indifferent between sending zero and one message) is high, then there are few higher types and they are unlikely to want to send several messages given the marginal type did not (unless the profit as a function of type becomes very elastic).[9]

Formally, the maximal number of messages sent by any sender is the largest integer $k$ that satisfies $\pi(\theta_k) \leq \pi(1)$. Using (6) determines the value of $k$ as the largest integer satisfying

$$\frac{\pi(\theta_1)}{\left[1 - \frac{\gamma}{\pi(\theta_1)}\right]^{k-1}} \leq \pi(1). \tag{7}$$

Taking logarithms, $k$ is the largest integer that satisfies $F(\theta_1) + 1 \geq k$ with

$$F(\theta_1) \equiv \frac{\ln \pi(1) - \ln \pi(\theta_1)}{\ln \pi(\theta_1) - \ln(\pi(\theta_1) - \gamma)}. \tag{8}$$

Equivalently, $k = \lceil F(\theta_1) \rceil$ where $\lceil . \rceil$ denotes the ceiling function. Note that $k-1$ is the amount of *duplication* of messages for the sender with the most messages (it sends $k$), and recall that the *number* of active senders is $M(1-\theta_1)$.

[9]With high $\theta_1$, there must be a relatively high congestion level, which at first glance would seem to suggest a high level of messages per sender. However, recall that the congestion level is a ratio of messages sent to messages examined, so that a high congestion level can be consistent with a low volume of messages sent, and that is what is happening here.

**Proposition 1** *Consider a candidate equilibrium with all senders $\theta \geq \theta_1$ active. The maximum duplication $k-1$ first increases and then decreases with the number of senders. Duplication tends to zero when $\theta_1 \downarrow \pi^{-1}(\gamma)$ or $\theta_1 \uparrow 1$.*

**Proof:** As shown in the Appendix, $F(\theta_1)$ is quasi-concave and tends to zero at both limits $\theta_1 \downarrow \pi^{-1}(\gamma)$ and $\theta_1 \uparrow 1$, and so $k$ is unity approaching these limits. Hence, either the number of steps, $k$ is always unity, or else it increases and then decreases with $\theta_1$. **Q.E.D.**

The function $F(\theta_1)$ and the construction of the associated ceiling function is given in Figure 2 for the uniform profit example, $\pi(\theta) = \bar{\pi}\theta$.

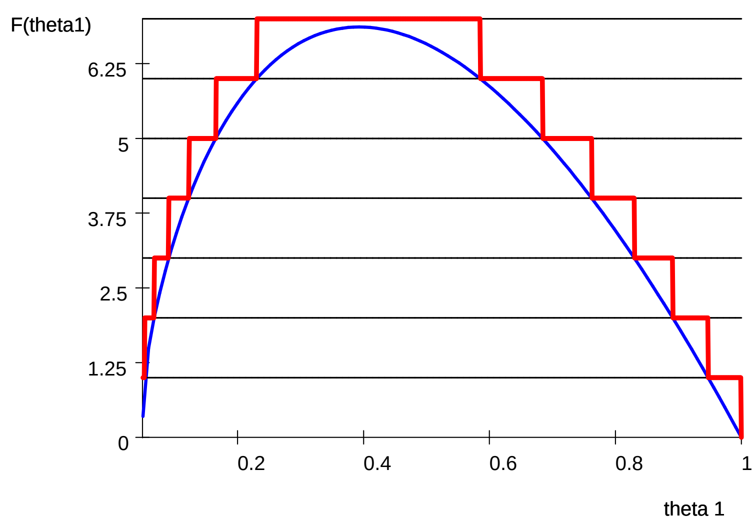


Figure 2. The function $F(\theta_1)$ (blue) and its ceiling function $\lceil F(\theta_1) \rceil$ (red) for $\bar{\pi}/\gamma = 20$.

Here the maximum number of steps (i.e., different numbers of messages sent across senders) is seen to be for middling values of $\theta_1$. In conjunction with Figure 1, only one message is sent for low $\theta_1$ (little congestion and many senders) and also for high $\theta_1$ (high congestion and few senders). In the middle there are many different per sender message levels, corresponding to a large degree of endogenous differentiation across sender types. The higher sender types engage in more shouting to be heard to break through the clutter produced by the other senders.

**Step 5. Aggregate message volume**

Given we have determined how many steps there are ($k$), and where they are, for any given $\theta_1$, we can now determine the aggregate number of messages, $n$, for any given $\theta_1$. The number of messages transmitted is given by adding up the number of messages on each level, $j = 1, ...k$, so

$$n(\theta_1) = M[1-\theta_1] + M[1-\theta_2] + ...M[1-\theta_k] > 0, \tag{9}$$

with $M[1-\theta_1] \leq n(\theta_1) \leq Mk[1-\theta_1]$, so $n(\theta_1)$ is bounded above. Note that $n(\theta_1) = M[1-\theta_1]$ when $k = 1$. Therefore $n(\theta_1) = Mk - M\sum_{j=1...k}\theta_j$ , so that

$$n(\theta_1) = Mk - M\sum_{j=1...k}\pi^{-1}\left(\frac{\pi(\theta_1)}{\left[1-\frac{\gamma}{\pi(\theta_1)}\right]^{j-1}}\right), \tag{10}$$

For the linear profit function used in the examples, (10) reduces to:

$$n(\theta_1) = Mk - M\theta_1\sum_{j=1...k}\left[1-\frac{\gamma}{\bar{\pi}\theta_1}\right]^{1-j}. \tag{11}$$

**Step 6. Determine the level of examination consistent with $\theta_1$**

Define now $\Phi(\theta_1)$ as the value of $\phi$ that would support an equilibrium with $\theta_1$ as the marginal sender type. Using the value of $n(\theta_1)$ above, we can recover the value of $\Phi(\theta_1)$ from (2):

$$\Phi(\theta_1) = \frac{n(\theta_1)}{\pi(\theta_1)}\gamma. \tag{12}$$

Note that $\Phi(\theta_1) \leq n(\theta_1)$ for $\theta_1 > \pi^{-1}(\gamma)$, so that $\Phi(\theta_1)$ is bounded above in this case.

The ability of firms to send several messages enables them to differentiate themselves. Sending more messages increases the probability of a hit, but erodes the profits of other competitors. More profitable firms (in terms of higher $\pi$) send more messages and have thus more chances of making a hit. Even though their advertising spending is higher, they still have higher profits: the empirical prediction is that more profitable firms send more ads.

# 4 Equilibrium existence

We have shown above that there exists an attention span which supports an equilibrium characterized by the marginal type $\theta_1 > \pi^{-1}(\gamma)$. This attention span function, $\Phi(\theta_1)$, is continuous on its domain, although it is not differentiable everywhere since it involves different regimes characterized by the maximal number of messages sent, $k$. The function $\Phi(\theta_1)$ (given by (12)) allows us to back out the value or values of $\theta_1$ that can be sustained as equilibria for any $\phi$.

The continuity and limit properties of $\Phi(\theta_1)$ are proved in the Appendix, giving us the following result.

**Lemma 2** *For any $\theta_1 \in (\pi^{-1}(\gamma), 1]$, there exists a unique, continuous, and differentiable examination value $\Phi(\theta_1)$ given by (12) that supports a congested equilibrium with marginal sender $\theta_1$. As $\theta_1 \downarrow \pi^{-1}(\gamma)$ or as $\theta_1 \uparrow 1$, $k(\theta_1) = 1$ and $\Phi(\theta_1) = M[1 - \theta_1]$.*

The properties described in the first two Lemmas enable us to show the existence of an equilibrium and some key features of this equilibrium in the next Proposition. It may be helpful for the reader to refer to Figure 3 which illustrates the function $\Phi(\theta_1)$ (for the linear example, $\pi(\theta) = \bar{\pi}\theta$, with $\bar{\pi}/\gamma = 20$). The precise details of the construction of this Figure will be described below: it is provided here for convenience in following the argument of the Proposition. Note that the black line continues up the vertical axis (at $\theta_1 = \pi^{-1}(\gamma) = 0.05$).

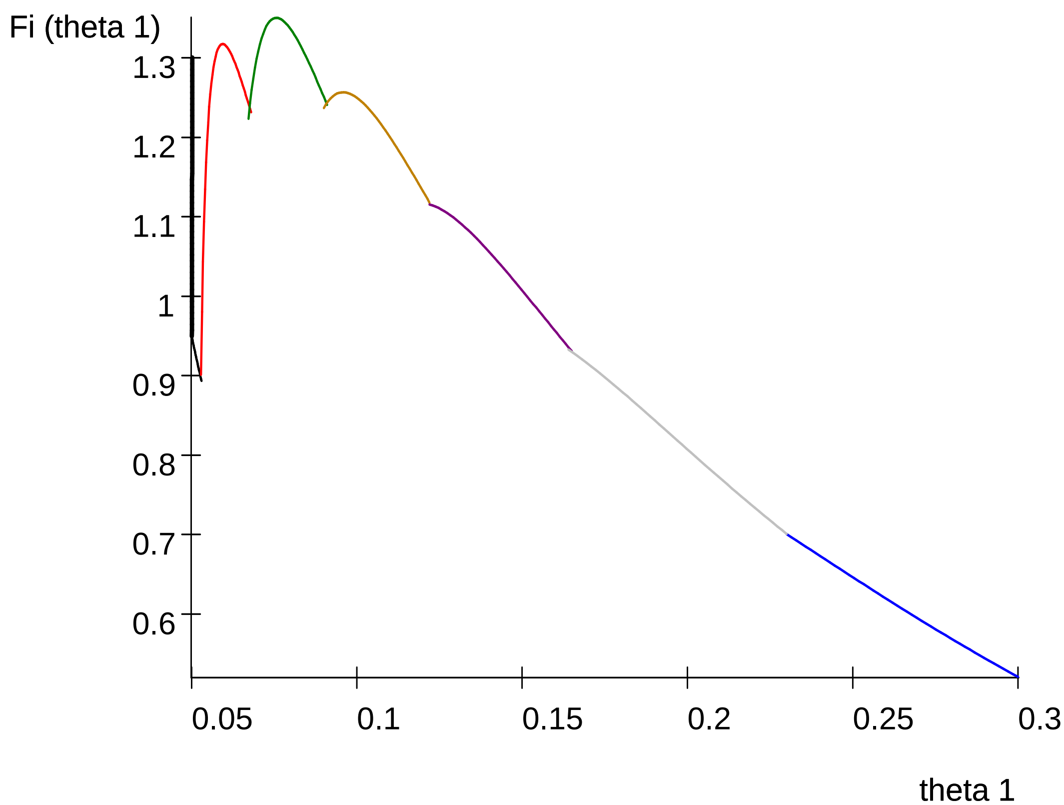


Figure 3. The function $\Phi$ for a linear profit function.

Equilibrium existence is proved in the next Proposition; the Lemmas above give the characterization of the different types of equilibrium (congested and not).

**Proposition 2** *For any $\phi$ there exists an equilibrium. It is unique for $\phi$ large enough.*

**Proof:** The existence of an equilibrium for $\phi \geq \left[1-\pi^{-1}(\gamma)\right] M$ follows from Lemma 1. For large enough $\phi$, this is the only equilibrium type because $\Phi$ is bounded on $(\pi^{-1}(\gamma), 1]$.

From Lemma 2, $\Phi(\theta_1)$ is continuous on $(\pi^{-1}(\gamma), 1]$ and tends to $\left[1-\pi^{-1}(\gamma)\right] M$ and 0 as it approaches its limits. Hence any $\phi \in [0, \left[1-\pi^{-1}(\gamma)\right] M)$ gives rise to a solution (or solutions) $\theta_1 \in (\pi^{-1}(\gamma), 1]$. Notice that there exists a solution with $\theta_1 \to \pi^{-1}(\gamma)$ as $\phi \to \left[1-\pi^{-1}(\gamma)\right] M$ since the continuous function $\Phi(\theta_1) \to \left[1-\pi^{-1}(\gamma)\right] M$ as $\theta_1 \to \pi^{-1}(\gamma)$ and a similar argument establishes that the solution tends to 1 as $\phi \to 0$. **Q.E.D.**

We show below that the function $\Phi(\theta_1)$ is not necessarily monotone and this feature gives rise to multiple equilibria and discontinuities in the mapping from $\phi$ to $\theta_1$.

# 5 Equilibrium properties

## 5.1 Conditional examination functions

To describe multiple equilibria, it is helpful to define the construction of the function $\Phi(\theta_1)$ from its component pieces. In particular, we will show that the function $\Phi(\theta_1)$ is the upper envelope of functions $\Phi_i(\theta_1)$, where each of these sub-functions is defined by modifying (12) to

$$\Phi_i(\theta_1) \equiv \frac{n_i(\theta_1)}{\pi(\theta_1)}\gamma, \tag{13}$$

where $n_i(\theta_1)$ is the number of messages sent under the restriction $j < i$ in (9):

$$n_i(\theta_1) = M[1-\theta_1] + M[1-\theta_2] + ...M[1-\theta_i]. \tag{14}$$

In terms of the 6-step procedure described above, the functions $\Phi_i(\theta_1)$ are given by skipping step 4 (in which we endogenously determined $k$). The functions $\Phi_i(\theta_1)$ can therefore be viewed as those of a constrained problem, where at most $i$ messages per sender are allowed.

The domain of $\Phi_i(\theta_1)$ is the set of $\theta_1$ values for which some senders want to send at least $i$ messages. By analogy to (7), the domain is the interval of $\theta_1$ values satisfying

$$\frac{\pi(\theta_1)}{\left[1 - \frac{\gamma}{\pi(\theta_1)}\right]^{i-1}} \leq \pi(1);$$

recalling the definition of $F(\theta_1)$ in (8), this is the set of $\theta_1$ values such that $i \geq \lceil F(\theta_1) \rceil$. Since $F(\theta_1)$ is quasi-concave, the domain of $\Phi_i(\theta_1)$ strictly contains the domain of $\Phi_j(\theta_1)$ for all $j > i$.

Finally, lower $i$ implies lower $n_i(\theta_1)$ (see (14)). Since $\Phi_i(\theta_1) \equiv \frac{n_i(\theta_1)}{\pi(\theta_1)}\gamma$, a lower $n_i(\theta_1)$ implies a lower $\Phi_i(\theta_1)$. Since $k$ corresponds to the highest possible value of $i$ for any $\theta_1$, then $\Phi(\theta_1) = \Phi_k(\theta_1)$. In summary,

**Proposition 3** *The function $\Phi(\theta_1)$ is the upper envelope of the restricted functions $\Phi_i(\theta_1)$, $i = 1, ..., k$.*

Figure 4 gives the $\Phi_i$ functions for the linear profit example with $\bar{\pi}/\gamma = 20$: recall that $\Phi(\theta_1)$ is the upper envelope of these functions, as given in Figure 3.

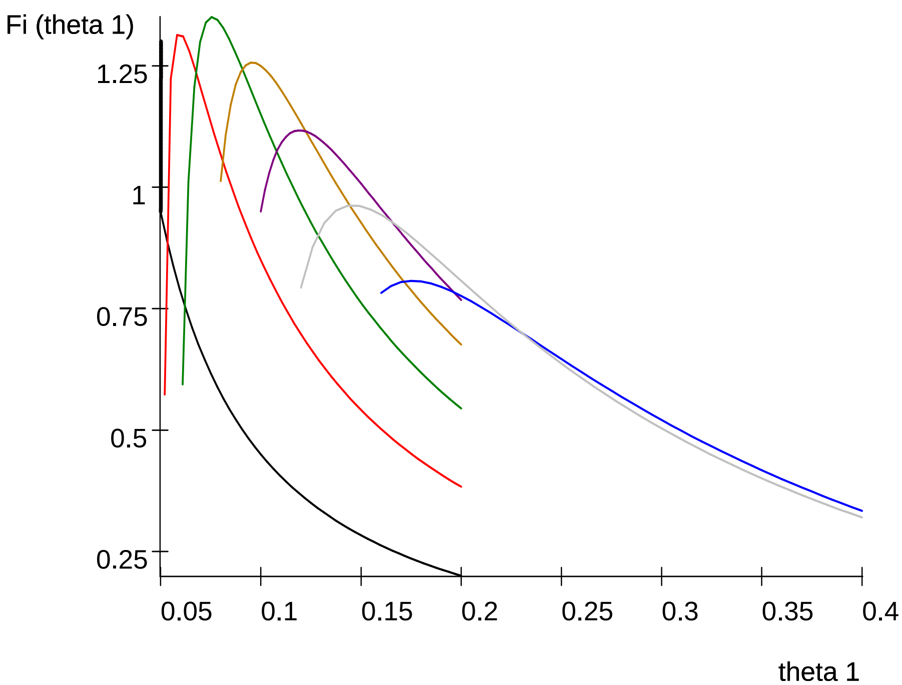


Figure 4. The functions $\Phi_i$ and their upper envelope.

The linear example, $\pi(\theta) = \bar{\pi}\theta$, affords some further characterization results, described in Section 5.3 below.

One interest of this construction via conditional examination functions is that it describes equilibria at which the number of messages per sender is capped, for example, with voluntary restrictions, regulation, or a spam filter that blocks multiple copies of the same message. We give some implications at the end of this Section.

## 5.2 Characterization

As is clear from the Figures for the linear example, there is an odd number of equilibria for almost all $\phi$. More generally, Lemmas 1 and 2 show that $\Phi$ is continuous on $(\pi^{-1}(\gamma), 1]$. At $\theta_1 = \pi^{-1}(\gamma)$, $\Phi$ takes all values greater than or equal to $M\left[1 - \pi^{-1}(\gamma)\right]$, and at $\theta_1 = 1$, $\Phi = 0$. Hence, any value of $\phi$ cuts the function $\Phi(\theta_1)$ an odd number of times, except when $\phi$ corresponds to a turning point of $\Phi$.

Ranking the equilibria from lowest to highest $\theta_1$ values, the "odd-numbered" ones are the ones for which the examination curve, $\Phi(\theta_1)$, is locally falling. These are readily seen to be the stable equilibria in that a perturbation in the neighborhood of an "even" equilibrium will lead away from it. Indeed, a rise in $\phi$ keeping $n$ constant will raise $\phi/n$ and so decrease $\theta_1$: this change is consistent with an equilibrium on the downward-sloping part of $\Phi(\theta_1)$ in that the direction of movement of $\theta_1$ is towards the new intersection of $\phi$ and $\Phi(.)$ on the same segment, but it leads away on the upward-sloping part.

Recall (from (12)) that $n(\theta_1)/\Phi(\theta_1) = \pi(\theta_1)/\gamma$. Hence the congestion rate $n(\theta_1)/\Phi(\theta_1)$ is strictly increasing in $\theta_1$ along the curve $\Phi(\theta_1)$ since $\pi(\theta_1)$ is strictly increasing in $\theta_1$. Even though the function $\Phi(\theta_1)$ may be locally increasing, the corresponding increase in $n$ is always large enough to raise the congestion rate, $n(\theta_1)/\Phi(\theta_1)$, as $\theta_1$ rises. Thus if a more profitable sender (a sender with a higher $\theta$) is the marginal one, then congestion will be higher despite the fact that there are fewer senders! Put another way, if the attention span function, $\Phi(.)$, is locally increasing, then an increase in $\phi$ locally raises the total number of messages sent and the congestion rate. However, if $\Phi(.)$, is locally decreasing, an increase in $\phi$ locally decreases the congestion rate. Then a larger attention span eases the congestion problem as the volume of messages falls and allows in lower marginal sender types.

When there are multiple equilibria, we can have solutions with many active senders all sending few messages and others with fewer senders but a larger number of messages in total. The higher $k$ is what enables the total number of messages to be higher, as higher value senders transmit more messages to get through the clutter created by their competitors. However, if $k = 1$ throughout the whole range of $\theta_1$, $\Phi(\theta_1)$ is strictly decreasing and equilibrium is unique in this case.[10] Notice that this also covers the case when senders are restricted to send at most one message.

## 5.3 More volume with fewer shouting

The main characterization result of different equilibria is the following:

**Proposition 4** *Consider a set of equilibria, for given $\phi$, ranked from low to high $\theta_1$ values. Then the maximal number of messages per sender, $k$, is weakly higher for higher $\theta_1$ and more messages are sent. Hence, the amount of congestion is higher: there is more shouting to be heard. Profits of each active sender are strictly decreasing across equilibria, as too are aggregate profits.*

**Proof:** From (2) we have $n = \phi\pi(\theta_1)/\gamma$, so that higher $\theta_1$ imply higher $n$ since $\phi$ is given. Thus, there is a higher amount of congestion (more "shouting to be heard"). We now show that the corresponding $k$ cannot decrease. From (6), $\frac{d\theta_j}{d\theta_1}$ has the same sign as $\left[1 - \frac{j\gamma}{\pi(\theta_1)}\right]$. Hence, if $\frac{d\theta_j}{d\theta_1} > 0$, then $\frac{d\theta_i}{d\theta_1} > 0$ for all $i < j$. However, if $\frac{d\theta_k}{d\theta_1} > 0$, all the values of $\theta_j$ have increased and there must be fewer messages sent for the higher level of $\theta_1$, a contradiction. Hence it must be that $\frac{d\theta_k}{d\theta_1} < 0$, and at least as many message levels must be present to generate a larger message volume at higher $\theta_1$. The last result follows from the property that equilibrium profits per sender decrease in $n$ (see (3) and applying the envelope theorem); the aggregate result then follows directly. **Q.E.D.**

This means that either there is the same number of levels of message-sending (same $k$) as we consider equilibria with higher $\theta_1$, and the $k^{th}$ level kicks in earlier (i.e., at a lower level of $\theta$); or else the number of messages on the top step is bigger than before. Both cases lead to a higher number of messages sent. For a large enough sending cost, there is but one equilibrium:

**Proposition 5** *If $\gamma \geq \frac{\pi(1)}{4}$, then for any $\phi$ there is only one equilibrium: in equilibrium, only one message is sent by each active sender. If $\gamma < \frac{\pi(1)}{4}$ there exists some $\phi$ for which an equilibrium exists with more than one message being sent by some senders.*

[10] If $k = 1$, from (12), $\phi(\theta_1) = M[1 - \theta_1]\gamma/\pi(\theta_1)$, which is strictly decreasing.

**Proof:** Recall first from (7) that if $\pi(1) \leq \frac{\pi(\theta_1)}{\left[1-\frac{\gamma}{\pi(\theta_1)}\right]}$ then at most one message will be sent (the sender $\theta = 1$ is just indifferent to sending a further message if this holds with equality). Define $X = \frac{\gamma}{\pi(\theta_1)}$, and so this condition is

$$X[1-X] \leq \frac{\gamma}{\pi(1)}.$$

The LHS is maximized, at $X = 0.5$. Therefore, all senders will send at most one message if $\gamma \geq \frac{\pi(1)}{4}$ (and there will be some senders if $\gamma < \pi(1)$). As noted above, for $k = 1$ there can only be one equilibrium for any $\phi$. If $\gamma < \frac{\pi(1)}{4}$ then (7) implies that $\Phi(\theta_1)$ involves $k = 2$ for some $\theta_1$. Choosing the corresponding value of $\phi$ suffices to sustain such $\theta_1$ as an equilibrium. **Q.E.D.**

In the case of a linear profit function, the equilibrium involves one message per sender (with some active senders) if $\gamma \in \left(\frac{\bar{\pi}}{4}, \bar{\pi}\right)$. Some further properties of the model can be derived for the special case of a linear profit function, and are proved in the Appendix.

**Proposition 6** *Assume that $\pi(\theta) = \bar{\pi}\theta$. For any $\theta \in (\gamma/\bar{\pi}, 1]$ the examination value $\Phi(\theta_1)$ is piecewise quasi-concave: the functions $\Phi_i(\theta_1)$ are quasi-concave.*

Figure 5 below illustrates multiple equilibria for a parametric example with high $\phi$ values (for the linear example, $\pi(\theta) = \bar{\pi}\theta$, with $\bar{\pi}/\gamma = 10$). At the level of $\phi$ given by the blue line, there are 5 equilibria. The first one has congestion with only one message per sender. For a higher level of $\phi$ (for example, $\phi = 0.95$) there are 3 equilibria. The first has one message per sender and no congestion. But the other two have two messages per sender and congestion.

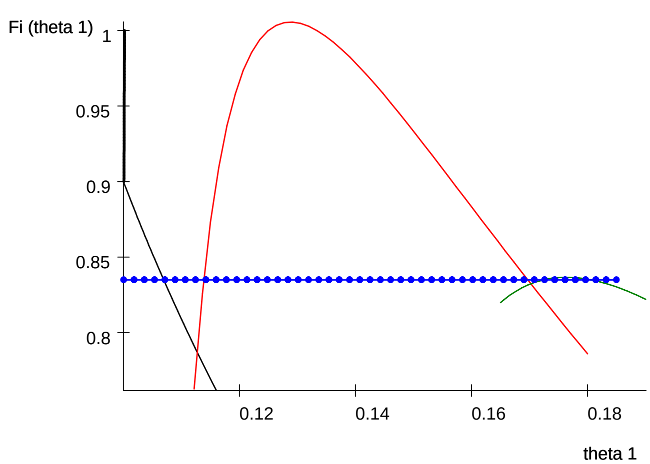


Figure 5. Illustration of 5 equilbria $k = (1,2,2,3,3)$, linear profit function.

The impact of message caps can also be seen from Figure 4 in the light of the above discussion. For example, if $\phi$ is slightly bigger than 1.25, the number of equilibria drops from 5 to 3 with a cap of 2 messages per sender. As per Proposition 4, sender welfare rises if the equilibrium in the absence of a cap had some senders transmitting 3 messages. However, if $\phi = 0.6$, say, then the only equilibrium had up to 7 messages transmitted per sender. If a cap of 2 messages per sender were imposed, then the new equilibrium would be at the value of $\theta_1$ where the red curve has a value of 0.6 on its descending part. More senders would transmit, because of less crowding by the louder shouters, which gives a clear gain to low-$\theta$ types - some of these become more profitable and enter the market, while others benefit directly from the reduction in congestion. However, the erstwhile loud shouters (high $\theta$-types) may be worse off. Despite less congestion, they now are constrained by the cap to send only two messages and thus have a lower chance of getting their very profitable message across. Thus the gains from less congestion and a broader sender base may be offset by the loss of breakthrough probability to the most profitable types.

## 6 Raising the cost of sending messages

Surprisingly, there may be significant benefits to senders from higher message transmission costs. This is because of the reduction in "shouting." We now show when it is possible to make all active senders better off with a higher transmission cost.

In what follows, we will make use of the following property, which we recall from Lemma 1(ii). Namely, a marginal sender type $\theta_1 = \pi^{-1}(\gamma)$ can be supported as an equilibrium if and only if $\phi \geq [1 - \theta_1] M$, and that such an equilibrium entails a transmission cost $\gamma = \pi(\theta_1)$ and $k = 1$, i.e., a single message per sender. In Figure 6 below, the bright green line represents the equation $\phi = [1 - \theta_1] M$. Any $(\phi, \theta_1)$ below it must have congestion, even if senders sent only one message. Above or on it, there would be no congestion if senders only sent one message.

In Figure 6, which repeats the upper envelope function, $\Phi(\theta_1)$, from Figure 3, $\Phi$ is vertical at $\pi^{-1}(\gamma)$ down to its intersection with the line $\phi = [1 - \theta_1] M$ [11]. (The particular function drawn sets $\gamma = 0.1$, and the vertical segment corresponds to the vertical axis above $\phi = 0.9$.) Higher values of $\gamma$ correspond to functions

[11] The black line, for one message per firm, is always below the green line because the black line involves congestion.

with the vertical segment of $\Phi(\theta_1)$ to the right.

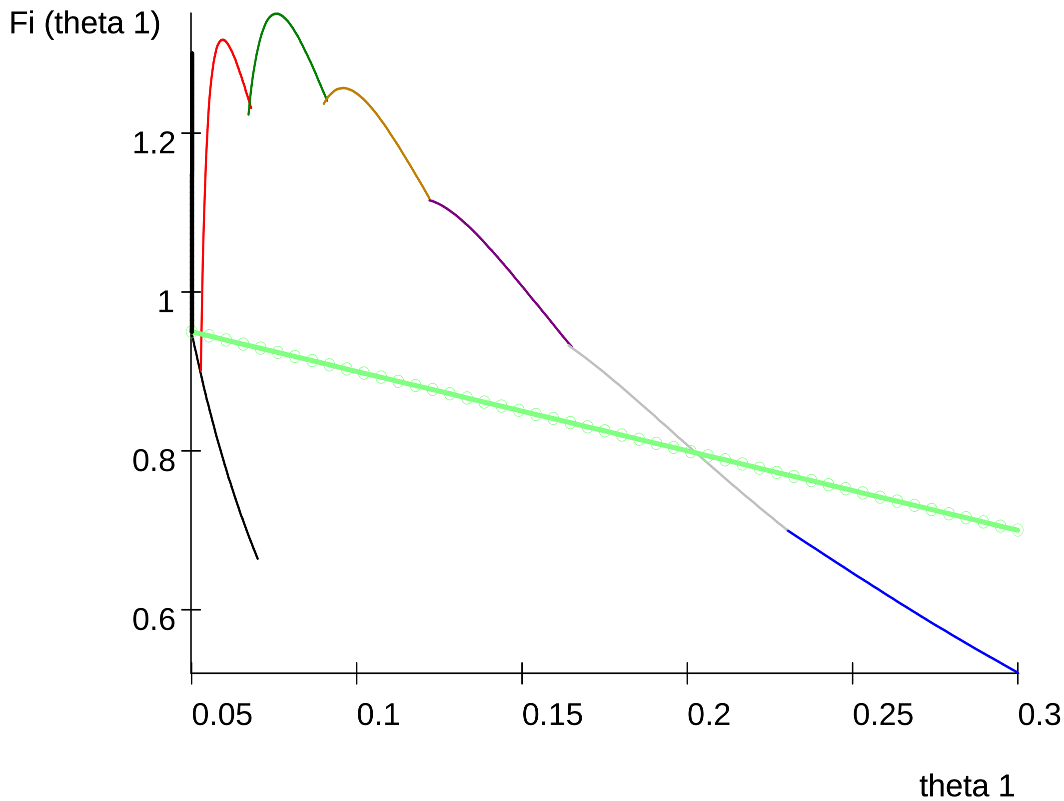


Figure 6. The upper envelope function $\Phi$ and the locus $\phi = 1 - \theta_1$.

The experiment of the next sub-section takes an equilibrium point above this line, and effectively drops it down to the line by raising $\gamma$.

## 6.1 Improvements

There is a simple sufficient condition for it to be possible to make all senders better off than at an equilibrium.

**Proposition 7** *Consider a congested equilibrium with marginal sender type $\theta_1$. It is possible without lump-sum transfers to make almost all active senders better off by raising the transmission cost to $\hat{\gamma} = \pi(\theta_1)$ $(= \frac{n}{\phi}\gamma)$ if $[1 - \theta_1] M \leq \phi$.*

**Proof.** As per Lemma 1(ii), an equilibrium with just one message per sender can be achieved through appropriate choice of $\gamma$ if and only if $[1-\theta_1]\,M \leq \phi$ for the specified $\theta_1$ (where we recall that $[1-\theta_1]\,M$ is the mass of types sending). Such an equilibrium can then be attained by choosing a transmission rate $\hat{\gamma} = \pi(\theta_1)$. This entails an increase in the transmission cost because originally we had $\gamma = \frac{\phi}{n}\pi(\theta_1)$ determining $\theta_1$, with $n > \phi$, (i.e., congestion): at such a transmission cost, with one message per sender, the number of messages sent is $[1-\theta_1]\,M \leq \phi$.

It remains to prove that all senders are no worse off given $\hat{\gamma}$ when $[1-\theta_1]\,M \leq \phi$. Clearly the *marginal* sender is indifferent between the original cost $\gamma$ and the higher cost $\hat{\gamma}$ because it makes zero profit before and after. We now show that profits are higher for all senders $\theta > \theta_1$; equivalently, we establish that the following relation holds:

$$\pi(\theta) - \hat{\gamma} > \pi(\theta)\left[1 - \left(1 - \frac{\phi}{n}\right)^{l^*}\right] - l^*\gamma \qquad \text{for all } \theta > \theta_1$$

where $l^*$ denotes the number of messages sent by a sender of type $\theta$ at the original equilibrium. The LHS of this expression is the profit per sender at the new equilibrium (when its message is examined for sure, and it sends only one, at cost $\hat{\gamma}$); the RHS is its original profit. We also know that, by construction, $\pi(\theta_1) = \hat{\gamma}$ and $\pi(\theta_1)\frac{\phi}{n} = \gamma$, so that $\hat{\gamma} = \frac{n}{\phi}\gamma$. After replacing this expression, we therefore want to show that

$$\pi(\theta) - \hat{\gamma} > \pi(\theta)\left[1 - \left(1 - \frac{\phi}{n}\right)^{l^*}\right] - l^*\frac{\phi}{n}\hat{\gamma},$$

or

$$\pi(\theta)\left(1 - \frac{\phi}{n}\right)^{l^*} > \left[1 - l^*\frac{\phi}{n}\right]\hat{\gamma},$$

which is true for $\theta > \theta_1$ since $\pi(\theta) > \pi(\theta_1) = \hat{\gamma}$ because the inequality:

$$\left(1 - \frac{\phi}{n}\right)^{l^*} > \left[1 - l^*\frac{\phi}{n}\right]$$

holds strictly for $l^* > 1$. **Q.E.D.**

If instead $\phi < [1-\theta_1]\,M$, then it is not possible to have all active senders transmit a message and have them all read. It may though still be possible to make all senders better off.[12] For the case of the Proposition,

[12] To see this, it suffices to note that when there are multiple equilibria we could move to an equilibrium with less congestion even while leaving $\gamma$ unchanged.

note that the receiver would still examine more messages than the number actually sent (if $\phi > [1 - \theta_1] M$), and so further welfare gains could be realized. It is striking that any higher profits from higher transmission costs need not be redistributed in order to make everyone better off.

The result that senders can be all better off is reminiscent of a result by Van Zandt (2004), although the mechanisms are quite different. In his paper, messages are targeted to receivers with diverse preferences (and each sender transmits a single message). Gains from raising sending costs arise because marginal senders are eliminated from receivers where they are less profitable, so benefiting those remaining through reduced congestion, and eliminated senders benefit on other receivers where they have a profit advantage. Here it is multiple messages that are pruned, even though there is a single receiver type.

# 7 Conclusions

The advantage to the sender from sending multiple messages is that this will improve the likelihood that the recipient does examine at least one of the sender's messages, and so raises the probability of a successful transaction. However, the recipient need only examine one message from a sender in order to make a transaction, so that any further messages received beyond the first one from a particular sender are wasted. This situation is formally analogous to a lottery with a fixed number of prizes, which are more highly valued by some (high $\theta$ types) than others. The chances of winning the lottery depend on the number of tickets bought, and the value of a further prize after winning once is zero (this is the assumption that the consumer need only examine one message from a sender to be informed of the opportunity). As we show, in equilibrium higher $\theta$ types send more ads. This is because they have higher profit from making a connection, and so they "buy more tickets."

We have modeled senders as choosing how many messages to send. Another way of breaking through the advertising clutter is for the sender to capture attention with the advertisement. The Marketing press stresses this strategy. In terms of our model, we could consider the variable $l$ as choice of quality of advertising (which would naturally be a continuous variable). Here, higher "quality" is understood to entail higher breakthrough probability, and not (necessarily) intrinsic artistic merit. Our results suggest higher quality ads from senders with more to gain from a breakthrough; they also suggest a "race-to-the-top" where others are also trying to break through. Because this process engenders more effective clutter, this in turn suggests that spending caps (as in the political arena), or advertising limits or agreements (as with cigarettes) might be desirable. Such caps are already implicitly modeled through the $\Phi_i$ functions in the current set-up.

We have kept the number of messages examined constant (at $\phi$). Endogenous receiver effort levels are addressed in Anderson and de Palma (2009) for the case of a single message per firm. It is straightforward to see that the multiplicity of equilibria survives under endogenous examination (it suffices to take an examination function with a zero marginal opening cost below $\phi$ and a prohibitive one above it).

The issue of time is another interesting generalization. Everything in the current model is static, so that yesterday's decisions do not affect today's purchases. This may be viewed as daily receipt of junk mail, with consumers each day forgetting yesterday's offers. It may also be relevant to TV-watching with transient commercials that are already out of one's consciousness by the next break. Dynamic models of memory retention, advertising pulsing, and reduction on ads as consumers take up offers would be clearly desirable.

We have also assumed that messages are independent, in order to close down business-stealing effects within industries. If there were competition within sectors, higher message costs might cause less competition between firms which would give an even greater advantage to firms from higher message costs (for example, the model of Grossman-Shapiro, 1984, its elaboration by Cristou and Vettas, 2008, and an empirical setting in Goeree, 2008). In parallel research (Anderson and de Palma, 2010) we address this issue by considering sub-groupings of senders ("industries") that compete in the same product market as well as for attention in aggregate (although senders only transmit one message each in that model).

Finally, the framework here can be adapted to allow for targeting of advertising (for more on this topic, see Bergemann and Bonatti, 2010, Johnson, 2010, and Iyer, Soberman, and Villas-Boas, 2005). We have treated a single receiver, but the results apply to different receivers too under targeting. Some results are quite immediate under targeting individuals of different profiles. For example, suppose individuals vary only by their examination values, $\phi$. Suppose too that the equilibrium involves the maximal number of messages, and so is on the downward-sloping part of the function $\Phi(\theta)$ (see Figure 4). A higher $\phi$ then entails a lower $\theta_1$, which in turn means that the congestion rate $(n/\phi)$ is lower for individuals examining more. Other results, for more elaborate patterns of differences across individuals (such as different conditional profits across senders for different individuals) are more difficult to establish. These remain a subject for further investigation.

# 8 Appendix

**Properties of maximal number of messages per sender (Proposition 4)**

The maximum number of messages, $k$, sent by any sender for any $\theta_1$ is determined in the text as $k = \lceil F(\theta_1) \rceil$ with $F(\theta_1) = \frac{\ln \pi(\theta_1) - \ln \pi(1)}{\ln\left(1 - \frac{\gamma}{\pi(\theta_1)}\right)}$ (see (8) and Figure 1).

We now prove that this function is quasi-concave in $\theta_1$. At the end-points we have $F\left(\pi^{-1}(\gamma)\right) = 0$ and $F(1) = 0$. $F(.)$ is also continuously differentiable and positive for $\theta_1 \in (\pi^{-1}(\gamma), 1)$; therefore there is a least one turning point. We show that there is a unique solution to $F'(\theta_1) = 0$ so that $F(\theta_1)$ is quasi-concave on its support. Differentiation of $F(.)$ implies that

$$sgn\left[F'(\theta_1)\right] = sgn\left[\frac{\pi(\theta_1)}{\gamma}\left[1 - \frac{\gamma}{\pi(\theta_1)}\right] \ln\left(1 - \frac{\gamma}{\pi(\theta_1)}\right) - \ln \pi(\theta_1) + \ln \pi(1)\right].$$

Define $x = \gamma/\pi(\theta_1)$, with $x \in (0, 1)$. Then we can rewrite

$$sgn\left[F'(\theta_1)\right] = sgn\left[\left[\frac{1}{x} - 1\right] \ln(1 - x) - \ln \pi(\theta_1) + \ln \pi(1)\right].$$

Any solution to $F'(\theta_1) = 0$ solves the equation

$$\left[\frac{1}{x} - 1\right] \ln(1 - x) = \ln \pi(\theta_1) - \ln \pi(1).$$

The LHS of this equation is increasing in $\theta_1$. For the solution to $F'(\theta_1) = 0$ to be unique, it then suffices to prove that the RHS in increasing in $x$ (since $x$ is decreasing in $\theta_1$). Here

$$\frac{d}{dx}\left[\left[\frac{1}{x} - 1\right] \ln(1 - x)\right] = -\ln(1 - x) - x > 0,$$

since $-\ln(1 - x) - x$ is increasing and zero at $x = 0$. Quasi-concavity and continuity of $F(.)$ implies that the maximal number of messages sent is is either always 1 or else is increasing and then decreasing in steps of size 1.

**Proof of Lemma 2.** Existence of a unique solution is given by the 6-step procedure in the text (see expressions (7), (10), and (12)). The key is that any $\theta_1 > \pi^{-1}(\gamma)$ induces a value of $\Phi(\theta_1) = \frac{n(\theta_1)}{\pi(\theta_1)}\gamma$ which is bounded above.

Because $\pi(\theta_1)$ is positive and continuous, to show $\Phi(\theta_1)$ is continuous is proved by showing that $n(\theta_1)$ is continuous. Define a regime by its corresponding value of $k$. The boundary between two regimes as the uppermost number of messages transmitted changes between $k$ and $k-1$ is given by $\theta_k = 1$. Continuity and differentiability within each regime (i.e., for given $k$) follows from the fact that each of the 6 steps preserves continuity and differentiability. Now consider the boundaries between regimes. In regime $k$ with $\theta_k = 1$, we have (from (9)): $n(\theta_1) = M[1-\theta_1] + M[1-\theta_2] + ...M[1-\theta_{k-1}]$, which is clearly equal to the expression for $n(\theta_1)$ in the regime $k-1$. Since the functions (6) defining $\theta_k$ as functions of $\theta_1$ are continuous in $\theta_1$, it is necessarily the case that continuity of $n(\theta_1)$ is preserved between neighboring regimes. Therefore $n(\theta_1)$ is also continuous across regimes. However, it is not generally differentiable at the critical points corresponding to the boundary between $k$ and $k-1$.

The last part is shown as follows. First we prove that $\lim_{\theta_1 \downarrow \pi^{-1}(\gamma)} \Phi(\theta_1) = M\left[1-\pi^{-1}(\gamma)\right]$. Let $\pi(\theta_1) = \gamma + \varepsilon$, $\varepsilon > 0$. Then we have the following inequalities from the bounds defining $k$ (see (7)):

$$\frac{\gamma+\varepsilon}{\left[1-\frac{\gamma}{\gamma+\varepsilon}\right]^{k-1}} \leq \pi(1) < \frac{\gamma+\varepsilon}{\left[1-\frac{\gamma}{\gamma+\varepsilon}\right]^{k}},$$

for which the only solution for $\varepsilon$ small enough is $k=1$ (recall that $\pi(.)$ is continuous and that $\gamma < \pi(1)$). Given $k=1$, then the number of messages sent tends to $n\left(\pi^{-1}(\gamma)\right) = M\left[1-\pi^{-1}(\gamma)\right]$. Furthermore, by (2), the congestion rate, $n/\phi$, is unity (meaning all messages sent are examined). Thus $\phi$ must also tend to $M\left[1-\pi^{-1}(\gamma)\right]$, as was to be proven.

We next show that $\lim_{\theta_1 \uparrow 1} \Phi(\theta_1) = 0$. In this case, we have $\pi(\theta_1) = \pi(1) - \varepsilon$, for $\varepsilon > 0$. The inequalities defining $k$ are now:

$$\frac{\pi(1)-\varepsilon}{\left[1-\frac{\gamma}{\pi(1)-\varepsilon}\right]^{k-1}} \leq \pi(1) < \frac{\pi(1)-\varepsilon}{\left[1-\frac{\gamma}{\pi(1)-\varepsilon}\right]^{k}},$$

which clearly has a unique solution $k=1$ for $\varepsilon$ small enough. Therefore $n(\theta_1) = M[1-\theta_1]$ and the number of messages sent tends to zero as $\theta_1 \uparrow 1$. Since $\phi < n$, $\phi$ must also tend to 0 as $\theta_1 \uparrow 1$, by (2). **Q.E.D.**

**Proof of Proposition 6.**

Note first that

$$\Phi_i(\theta_1) = \frac{n_i(\theta_1)\gamma}{\bar{\pi}\theta_1}.$$

Hence:

$$\frac{\bar{\pi}}{\gamma}\frac{d\Phi_i}{d\theta_1} = \frac{n_i'(\theta_1)\theta_1 - n_i(\theta_1)}{[\theta_1]^2},$$

where, for the linear formulation, we have:

$$n_i(\theta_1) = i - \theta_1 \sum_{j=1...i} \frac{1}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)^{j-1}}.$$

On the interior of the domain of $\Phi_i(\theta_1)$, we have:

$$n_i'(\theta_1) = -\sum_{j=1\ldots i} \frac{1-\frac{\gamma}{\bar{\pi}\theta_1}j}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)^j}.$$

Hence,

$$\frac{\bar{\pi}}{\gamma}[\theta_1]^2 \frac{d\Phi_i(\theta_1)}{d\theta_1} = -i + \frac{\gamma}{\bar{\pi}} \sum_{j=1\ldots i} \frac{j-1}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)^j} = \Omega_i. \tag{15}$$

This expression implies that the function $\Phi_i(\theta_1)$ is quasi-concave since the factor on the LHS is positive and so $\frac{d\Phi_i}{d\theta_1}$ has the sign of the RHS. The RHS is decreasing in $\theta_1$: this implies quasi-concavity because $\frac{d\Phi_i}{d\theta_1}$ is either of the same sign throughout its range so $\Phi_i$ is always decreasing (or increasing) or else it switches sign from positive to negative (so that $\Phi_i$ is increasing and then decreasing). **Q.E.D.**

EXAMPLE. In regime $k = 1$ we have that $\frac{d\Phi}{d\theta_1} < 0$. In regime $k = 2$, we have:

$$\Omega_2 = -2 + \frac{\gamma/\bar{\pi}}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)^2}.$$

We compute this value at the critical point $\theta_2 = 1$. Recall that : $\theta_2 = \frac{\theta_1}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)}$ which is equal to 1 in this case. Therefore:

$$\Omega_2 = -2 + \frac{\gamma/\bar{\pi}}{\theta_1^2},$$

and so $\Omega_2 > 0$ if and only if $\theta_1 < \sqrt{\gamma/2\bar{\pi}}$.

Since the critical value of $\theta_2 = \frac{\theta_1}{\left(1-\frac{\gamma}{\bar{\pi}\theta_1}\right)} = 1$, we solve this equation for its 2 solutions and choose the lower one (since the transition is from the regime with $k = 1$ to that with $k = 2$, i.e., the solution is the lower one solving $\theta_1^2 - \theta_1 + \frac{\gamma}{\bar{\pi}} = 0$, or $\theta_1 = \left(1 - \sqrt{1-4\frac{\gamma}{\bar{\pi}}}\right)/2$. The condition $\theta_1 < \sqrt{\gamma/2\bar{\pi}}$ now reduces to $1 - 3\frac{\gamma}{\bar{\pi}} < \sqrt{1-4\frac{\gamma}{\bar{\pi}}}$or $1 + 9\left(\frac{\gamma}{\bar{\pi}}\right)^2 - 6\frac{\gamma}{\bar{\pi}} < 1 - 4\frac{\gamma}{\bar{\pi}}$, which is satisfied if $\frac{\gamma}{\bar{\pi}} < \frac{2}{9}$. In this case, there are multiple solutions for some values of $\phi$. A necessary condition is therefore that $\theta_1 < 1/3$.

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